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Empirical procedures to predict the shear strength of unsaturated soils

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ABSTRACT: Several empirical procedures have been developed for predicting the shear strength of unsaturated soils in the recent years using the soil-water characteristic curve and the saturated shear strength parameters. Various investigators have also suggested mathematical relationships such as elliptical and hyperbolic functions and other curve fitting techniques to predict the shear strength of unsaturated soils. The philosophy used in each of these proposed empirical procedures is discussed and summarized in this paper. These simple procedures are of significant value to the engineers to put the theories related to shear strength of unsaturated soils into practice.

1 INTRODUCTION

Shear strength forms an important engineering property in the design of numerous geotechnical and geo-environmental structures such as earth dams, retaining walls, pavements, liners, covers etc. Geotechnical structures often do not become saturated during their design life period. There are structures such as soil covers that are designed to remain in an unsaturated state during their entire life period. As a result, the unsaturated soil condition is relevant in the design of these geotechnical and geo-environmental structures.

A theoretical framework for unsaturated soil mechanics that parallels saturated soil mechanics is now available in terms of stress state variables, namely; net normal stress, $(\sigma_n - u_a)$, and matric suction, $(u_a - u_w)$ (Fredlund and Rahardjo, 1993). The framework for unsaturated soil mechanics, however, is based on experimental studies that are costly and time consuming.

Several empirical procedures have been developed for predicting the coefficient of permeability and shear strength of unsaturated soils. The soil-water characteristic curve (SWCC) along with the saturated shear strength parameters have been used to predict the shear strength of unsaturated soils. There are several such procedures available in the literature (Vanapalli et al. 1996a, Fredlund et al. 1996, Oberg & Sallfours 1997, Khallili & Khabbaz 1998 and Bao et al. 1998). Various other investigators have suggested mathematical relationships such as elliptical and hyperbolic functions and other curve fitting techniques to predict the shear strength of unsaturated soils (Abramento & Carvalho 1990, de Campos & Carillo 1995, Escario & Juca 1989, Lu 1992, Shen & Yu 1996, Xu 1997).

The philosophy used in each of the proposed procedures for predicting the shear strength of unsaturated soils is discussed in this paper. These relatively simple procedures are of significant value to the engineers in putting the theories related to shear strength of unsaturated soils into practice.

2 SHEAR STRENGTH OF UNSATURATED SOILS

Haines (1925, 1927) was probably one of the earliest investigators to study the shear strength behavior of unsaturated soils. His studies showed an increase in cohesion with an increase in negative pore-water pressure. Terzaghi (1943) demonstrated that the shear strength of unsaturated soils could be better understood from the studies related to the distribution and geometry of the pore-water combined with the stress within the pore-water.

Bishop (1959) proposed shear strength equation for unsaturated soils by extending Terzaghi's principle of effective stress for saturated soils.

$$\tau = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)]\tan\phi' \qquad [1]$$

where:
τ = shear strength of unsaturated soil
c' = effective cohesion
ϕ' = angle of frictional resistance
$(\sigma_n - u_a)$ = net normal stress
$(u_a - u_w)$ = matric suction
χ = a parameter dependent on the degree of saturation

Bishop et al. (1960) explained the relationship between the degree of saturation, S, and the empirical parameter, χ. The value of χ was assumed to vary from 1 to 0, which represents the variation from a fully saturated condition to a total dry condition. These studies have indirectly provided an important step in understanding the relationship between shear strength and the SWCC behavior (Barbour 1999).

Fredlund et al. (1978) have proposed a relationship to explain the shear strength of unsaturated soils in terms two independent stress state variables as shown below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [2]$$

The shear strength contribution due to matric suction, ϕ^b, was initially assumed to be linear based on the analysis of limited results published in the literature. Later experimental studies performed over a large range of suction values have shown that the variation of shear strength with respect to soil suction is non-linear (Escario & Saez 1986, Gan et al. 1988 and Escario & Juca 1989). The Fredlund et al. (1978) equation can be applied for both the linear and non-linear variation of shear strength with respect to suction, provided the suction range is taken into account.

Several other investigators have also worked in the area of shear strength of unsaturated soils during the past 40 years. Graecen (1960) proposed an empirical approach for estimating the shear strength of unsaturated soils. Sridharan (1968) provided an understanding of undrained shear strength of unsaturated using soil mineralogy concepts. Satija (1978) proposed a statistical analysis approach for shear strength of unsaturated soils. Peterson (1988) extended Hvorslev's theory to explain the shear strength behavior of unsaturated soils. Karube (1988), Toll

(1990), and Wheeler and Sivakumar (1992) have used the concepts of critical state soil mechanics to the shear strength of unsaturated soils. These studies were based on experimental procedures that are costly and time consuming. However, these studies were valuable and have provided a greater understanding with respect to the shear strength of unsaturated soils.

In recent years several investigators have proposed empirical procedures to predict the shear strength function for unsaturated soils by using the soil-water characteristic curve (SWCC) and the saturated shear strength parameters.

2.1 *The relationship between the soil-water characteristic curve (SWCC) and the shear strength of unsaturated soils*

The soil-water characteristic curve (SWCC) defines the relationship between the soil suction and either gravimetric water content, w, or the degree of saturation, S, or the volumetric water content, θ. The SWCC is a conceptual and interpretative tool by which the behavior of unsaturated soils can be understood.

As the soil moves from a saturated state to drier conditions, the distribution of soil, water, and air phases change as the stress state changes. The relationship between these phases take on different forms and influence the engineering behavior of unsaturated soils (Barbour 1999). The wetted area of contact between the soil particles decreases with an increase in the soil suction. There is a relationship between the rate at which shear strength changes in unsaturated conditions to the wetted area of water contact between the soil particles (Vanapalli 1994). In other words, a relationship exists between the SWCC and the shear strength of unsaturated soils (Vanapalli et al. 1996a, Vanapalli et al. 1996b & Fredlund et al. 1996).

2.2 *Equations to predict the shear strength of unsaturated soils using the soil-water characteristic curve and the saturated shear strength parameters*

Lamborn (1986) proposed a shear strength equation for unsaturated soils by extending a micromechanics model based on irreversible thermodynamic to the energy versus volume relationship in a multiphase material (i.e., solids, fluids, and voids). The equation is as follows:

$$\tau = \left[c' + (\sigma_n - u_a)\tan\phi'\right] + (u_a - u_w)\theta_w(\tan\phi') \qquad [3]$$

where:

θ_w = volumetric water content,

The volumetric water content, θ_w, is defined as the ratio of the volume of water to the total volume of the soil. The volumetric water content, θ_w, decreases as suction increases, and it is a nonlinear function of soil suction. However, it should also be noted that the friction angle associated with soil suction, ϕ^b, does not become equal to ϕ' at saturation unless the volumetric water is equal to one.

Vanapalli et al. (1996a) and Fredlund et al. (1996) have proposed a more general, non-linear form for the shear strength of an unsaturated soil as shown below:

$$\tau = \left[c' + (\sigma_n - u_a)\tan\phi'\right] + \left[(u_a - u_w)\left\{\left(\Theta^{\kappa}\right)(\tan\phi')\right\}\right] \qquad [4]$$

where:

κ = fitting parameter used for obtaining a best-fit between the measured and predicted values.

Θ = normalized water content, θ_w/θ_s.

This equation will be referred as University of Saskatchewan, Procedure 1 hereafter in the paper and in the literature. The shear strength contribution due to suction constitutes the second part of [Eq. 4], which is:

$$\tau_{us} = \left[(u_a - u_w)\left\{\left(\Theta^{\kappa}\right)(\tan\phi')\right\}\right] \qquad [5]$$

To use this equation the entire SWCC curve data (i.e., 0 to 1,000,000 kPa) is required along with the saturated shear strength parameters. A best-fit SWCC can then be obtained in terms of a, n, and m parameters using the equation proposed by Fredlund and Xing (1994) which is shown below:

$$\theta_w(\psi) = \theta_s\left[1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{10^6}{h_r}\right)}\right]\left[\frac{1}{\ln\left\{\exp(1) + \left(\frac{\psi}{a}\right)^n\right\}}\right]^m \qquad [6]$$

where:

θ_w = volumetric water content
θ_s = saturated volumetric water content
a = suction related to the air-entry value of the soil
n = soil parameter related to the inflection point on the SWCC
m = soil parameter related to the residual water content
h_r = suction related to the volumetric residual water content, θ_r

Equation [4] is useful to predict the shear strength of unsaturated over the entire range of suction values of 0 to 1,000,000 kPa (i.e., from a fully saturated condition to a total dry condition).

Vanapalli et al. (1996a) proposed another equation for predicting the shear strength of unsaturated soils without using the fitting parameter, κ. The equation is given below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left[\tan\phi'\left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right)\right] \qquad [7]$$

where:

θ_w = volumetric water content
θ_s = saturated volumetric water content
θ_r = residual volumetric water content

Equation [7] can also be written in terms of degree of saturation, S, or gravimetric water content, w, to predict the shear strength yielding same results. To use this equation the residual volumetric water content, θ_r, has to be estimated from the SWCC data. The shear strength of soil may start to decrease beyond the residual state conditions. This equation is referred as University of Saskatchewan, Procedure 2 hereafter in the paper and in the literature (Vanapalli et al. 1996a).

Oberg & Sallfours (1997) proposed [Eq. 8] for predicting the shear strength of primarily non-clayey soils such as sands and silts:

$$\tau = c' + \left[(\sigma - S u_w - (1 - S)u_a)\right]\tan\phi' \qquad [8]$$

The proposed equation is similar to that suggested by Bishop's (1959) except that the χ parameter proposed by Bishop is replaced by the degree of saturation, S, in Eq. [8]. The authors state that the χ -factor proposed by Bishop reflects the fraction of the pore area that is occupied by water (i.e., A_w/A_t) which is approximately equal to the degree of saturation, S.

Equation [8], can be rearranged such that the shear strength can be interpreted in terms of the two independent stress state variables as shown below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)[(\tan\phi')(S)] \quad [9]$$

The use of Eq. [9] requires that the user know the degree of saturation soil at failure. Equation [9] suggests that there is a one to one relationship between the degree of saturation, S, and the area of water contact along the shear plane in the soil. Such an assumption may or not be valid for all types of soils.

The form of Eq. [9] is similar to that used by the Lamborn (1986) (i.e, Eq. 3). While Lamborn (1986) used volumetric water content, θ_w, Oberg and Sallfours (1997) used degree of saturation, S, in their proposed equation for predicting the shear strength of unsaturated soils.

Khallili and Khabbaz (1998) have proposed an equation for interpreting the triaxial shear test results of unsaturated soil specimens as below:

$$q = c'\cos\phi' + (p - u_a)\tan\phi' + (u_a - u_w)[(\lambda')(\sin\phi')] \quad [10]$$

where:

$$q = \frac{\sigma_1 - \sigma_3}{2}$$

$$p = \frac{\sigma_1 + \sigma_3}{2}$$

Equation [10] can be written in a more generalized form as:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)[(\lambda')(\tan\phi')] \quad [11]$$

where:

$$\lambda' = \left\{\frac{(u_a - u_w)}{(u_a - u_w)_b}\right\}^{-0.55}$$

The parameter, λ', is an empirical constant developed from published results in the literature. The proposed equation is said be valid for all types of soils. The required parameters for predicting the shear strength of unsaturated soils is the air-entry value, $(u_a - u_w)_b$ of the soil and the saturated shear strength parameters. Khallili and Khabbaz (1998) suggest that the limitations of Bishop's form of equation can be avoided by using equation [11].

Practicing engineers are most interested in the shear strength behavior in the transition zone. The transition zone lies between the air-entry value and the residual zone of saturation (Vanapalli et al. 1996a). The variation of the SWCC behavior in the transition zone is linear on a semi-logarithmic plot (i.e., variation of degree of saturation, S, or volumetric water content, θ, or gravimetric water content, w versus logarithm of soil suction). Bao et al. (1998) suggest another equation for predicting the unsaturated shear strength taking into account of the linear variation of the SWCC in the transition zone as below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)[\xi - \zeta\log(u_a - u_w)]\tan\phi' \quad [12]$$

where:

$$\xi = \frac{\log(u_a - u_w)}{\log(u_a - u_w)_r - \log(u_a - u_w)_b}$$

$$\zeta = \frac{1}{\log(u_a - u_w)_r - \log(u_a - u_w)_b}$$

The parameter, ξ, represents the slope and the parameter, ζ, represents the intercept of the linear part of the SWCC curve (i.e., on the abscissa) respectively. Both the parameters, ξ, and ζ are related to the pore size distribution. The form and philosophy of the equation is similar to that of U. of S, Procedure 2 (Vanapalli et al. 1996a).

2.3 *Mathematical formulations and other procedures to predict the shear strength of unsaturated soils*

Several mathematical formulations such as curve fitting techniques have also been used by investigators to predict the shear strength of unsaturated soils. Abramento & Carvalho (1989) used an exponential function that retains the form of the shear strength equation proposed by Fredlund et al. (1978). Escario & Juca (1990) extended an empirical formation using a 2.5 degree elliptical curve for three different soils tested for large range of suction values (i.e., 0 to 10,000 kPa and beyond). The soil properties of the three soils used by Escario & Juca (1990) are : Madrid grey clay (w_L = 71%, I_p = 35%), Guadalix Red silty clay (w_L = 33%, I_p = 13.6%) and Madrid clay sand (w_L = 32%, I_p = 15%).

Shen & Yu (1996) have proposed hyperbolic functions to predict the shear strength of unsaturated soils. The proposed equations are shown below:

$$\tau = \frac{\psi}{1 + \psi d}\tan\phi' \quad [13]$$

where:
ψ = soil suction
d = fitting coefficient

$$\tau = \psi\frac{1}{\cot\alpha + \dfrac{\psi}{\beta}}\tan\phi' \quad [14]$$

where:
α = initial angle of shearing resistance with respect to suction
β = ultimate value of shear strength contribution from suction

Using equations [13] and [14] the shear strength of unsaturated soils can be predicted for a large range of suction values based on experimental results for a small range of suction values. These equations are based on the assumption that the variation of shear strength with respect to suction can be represented by a hyperbolic function.

Xu (1997) has proposed another equation using a fractal dimension fitting model as:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + k^n(u_a - u_w)^{mn+1}\tan\phi' \quad [15]$$

where:
k = fitting coefficient
m & n = parameters related to fractal dimensions

Relationships can be developed between the fitting coefficients and soil properties if a large data base is available for equations such as [13] and [15]. Such relationships will then prove to be more useful for practicing engineers.

Lu (1992) has proposed an equation to predict the shear strength of expansive soils that are in an unsaturated state as shown below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + P_s \tan\phi' \quad [16]$$

where:

P_s = swelling pressure which is a function of soil suction

The form of the equation proposed by Lu (1992) is similar to that of Bishop (1959). In essence, the suction component of shear strength has been transferred to the total stress plane.

3 SUMMARY AND CONCLUSIONS

The empirical equations discussed in this paper to predict the shear strength of unsaturated soils using the SWCC and the saturated shear strength parameters are practical and simple in form to use. Other mathematical and curve fitting formulations developed by various investigators are also useful. Interesting research in the area of shear strength of unsaturated soils is in progress in several countries. Recent studies by investigators such as Fredlund et al. (1997) have shown SWCC behavior can be predicted from the grain size analysis results for several types of soils. Thus, there are simpler tools now available to put the shear strength theories of unsaturated soil mechanics into practice.

The analyses of test results are not undertaken in this paper due to space limitations.

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