Model for the prediction of shear strength with respect to soil suction

S.K. Vanapalli, D.G. Fredlund, D.E. Pufahl, and A.W. Clifton

Abstract: Experimental studies on unsaturated soils are generally costly, time-consuming, and difficult to conduct. Shear strength data from the research literature suggests that there is a nonlinear increase in strength as the soil desaturates as a result of an increase in matric suction. Since the shear strength of an unsaturated soil is strongly related to the amount of water in the voids of the soil, and therefore to matric suction, it is postulated that the shear strength of an unsaturated soil should also bear a relationship to the soil-water characteristic curve. This paper describes the relationship between the soil-water characteristic curve and the shear strength of an unsaturated soil with respect to matric suction. An empirical, analytical model is developed to predict the shear strength in terms of soil suction. The formulation makes use of the soil-water characteristic curve and the saturated shear strength parameters. The results of the model developed for predicting the shear strength are compared with experimental results for a glacial till. The shear strength of statically compacted glacial till specimens was measured using a modified direct shear apparatus. Specimens were prepared at three different water contents and densities (i.e., corresponding to dry of optimum, at optimum, and wet of optimum conditions). Various net normal stresses and matric suctions were applied to the specimens. There is a good correlation between the predicted and measured values of shear strength for the unsaturated soil.

Key words: soil-water characteristic curve, shear strength, unsaturated soil, soil suction, matric suction.

Résumé : Les études expérimentales sur les sols partiellement saturés sont généralement dispendieuses, prennent du temps et sont difficiles à réaliser. Les données de résistance au cisaillement dans la littérature suggèrent que, par suite de l'accroissement de la succion matricielle, il y a un accroissement non linéaire de la résistance lorsque le sol se désature. Puisque la résistance au cisaillement d'un sol non saturé est fortement reliée à la quantité d'eau dans les vides du sol, et par conséquent à la succion matricielle, l'on postule que la résistance au cisaillement d'un sol non saturé devrait aussi être en relation avec la courbe caractéristique sol-eau. Cet article décrit la relation entre la courbe caractéristique sol-eau et la résistance au cisaillement d'un sol non saturé en fonction de la succion matricielle. Un modèle empirique analytique est développé pour prédire la résistance au cisaillement par rapport à la succion dans le sol. La formulation utilise la courbe caractéristique sol-eau et les paramètres de la résistance au cisaillement du sol saturé. Les résultats du modèle développé pour prédire la résistance au cisaillement sont comparés avec les résultats expérimentaux pour un till glaciaire. La résistance au cisaillement de spécimens de till glaciaire compactés sous charge statique a été mesurée au moyen d'un appareil de cisaillement direct modifié. Des spécimens ont été préparés à trois différentes teneurs en eau et densités (c'est-à-dire correspondant à des conditions sèches par rapport à l'optimum, à l'optimum, ou au côté mouillé de l'optimum). Différentes contraintes normales net et succions matricielles ont été appliquées aux spécimens. Il y a une bonne corrélation entre les valeurs prédites et mesurées de la résistance au cisaillement pour le sol non saturé.

Mots clés : courbe caractéristique sol-eau, résistance au cisaillement, sol non saturé, succion dans le sol, succion matricielle.

[Traduit par la rédaction]

Introduction

A value for the shear strength of a soil is required in the prediction of the stability of slopes and embankments, the bearing capacity of foundations, and pressures against earth retaining structures. The Mohr-Coulomb theory, using the effective stress state, is commonly used for predicting the shear strength of saturated soils. Even though soils encountered in engineering practice are often unsaturated, slope stability analyses are usually based on the saturated shear strength parameters. Similar design approaches have been adopted for retaining structures, pavements, and other earth structures. These approaches are conservative to varying degrees in that the influence of soil suction is ignored. However, even low suctions can be responsible for maintaining the stability of slopes (Walle and Hachich 1989).

The concept of stress state variables to describe the behavior of unsaturated soils was introduced by Fredlund and Morgenstern (1977). The shear strength of an unsaturated soil, in terms of these stress state variables, was proposed by Fredlund et al. (1978). Elastic-plastic, critical state soil

S.K. Vanapalli, D.G. Fredlund, and D.E. Pufahl. Department of Civil Engineering, University of Saskatchewan, Saskatoon, SK S7N 5A9, Canada.
A.W. Clifton. Clifton Associates, Regina, SK S4N 5Y5, Canada.

mechanics theories have also been proposed using the concept of stress state variables (Karube 1988; Toll 1990; Wheeler and Sivakumar 1995).

Using a phenomenological approach consistent with continuum mechanics, the shear strength behavior of an unsaturated soil can be written in terms of $(\sigma - u_a)$ and $(u_a - u_w)$, with independent soil properties. This approach has been thoroughly studied in laboratory investigations by various researchers (Gan et al. 1988; Abramento and Carvalho 1989; Escario and Juca 1989; Vanapalli 1994).

Although shear strength theories of an unsaturated soil have been formulated and found to be consistent with observed experimental behavior, experimental measurements of shear strength are time-consuming and require costly laboratory facilities. This has to some degree, limited the application of the shear strength theories for unsaturated soils to research and academic areas. To-date, there has been only limited practical application of the unsaturated soils shear strength theory in practice. The success of any theory depends on how readily and successfully it can be applied in engineering practice. It is therefore important to develop a simpler approach for predicting the shear strength of an unsaturated soil for various engineering applications. This would encourage the use of unsaturated shear strength theories in engineering practice.

Some attempts have been made to predict the shear strength of an unsaturated soil using empirical procedures. Escario and Juca (1989), for their experimental data on different soils have found that an ellipse with a 2.5°, reproduced the variation of shear strength with respect to suction reasonably well. Abramento and Carvalho (1989) used a curve-fitting technique for their experimental data using an exponential function that retains the form of the shear strength equation proposed by Fredlund et al. (1978), treating $(\tan \phi^b)$ as a variable with respect to suction. These empirical procedures may or may not be suitable for all types of soils. This paper concentrates on developing an equation for the shear strength of an unsaturated soil using the soil-water characteristic curve and the shear strength parameters of the saturated soil.

Use of the soil-water characteristic curve in predicting the shear strength of an unsaturated soil

Features of a typical soil-water characteristic curve

The soil-water characteristic curve defines the relationship between the amount of water in the soil and soil suction. The amount of water can be a gravimetric water content, w, a volumetric water content, θ, or degree of saturation, S. Typical soil-water characteristic features for the drying and wetting of a soil are shown in Fig. 1.

Volumetric water content, θ, is defined as the ratio of volume of water to the total volume of soil. Relationships can be written between the various volume–mass designations for water content. The relationship between volumetric water content and other variables can be written as

[1] $\theta = \dfrac{Se}{1 + e} = Sn$

where

e is the void ratio; and

Fig. 1. Typical soil-water characteristic curve features for the drying and wetting of a soil.
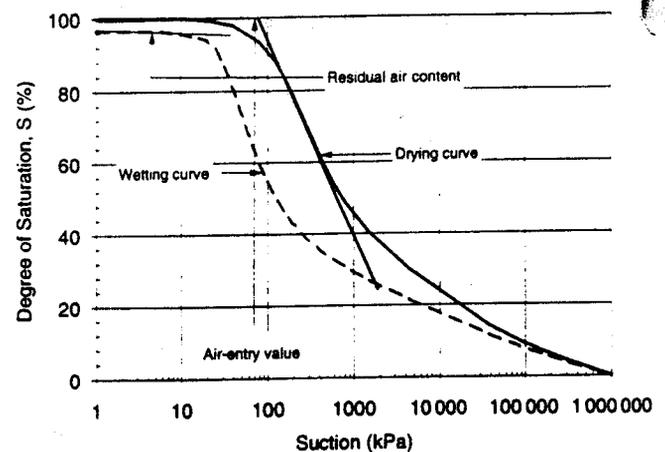
n is the porosity.

The relationship between volumetric water content, θ, and gravimetric water content, w, can be written

[2] $\theta = w\rho_d$

where

ρ_d is the dry density of the soil; and

w is the gravimetric water content.

Some soils undergo significant void ratio changes (i.e., overall volume changes) as a result of changes in soil suction. An equation can also be written for the relationship between changes in the various volume–mass variables (Fredlund and Rahardjo 1993).

[3] $\Delta e = \dfrac{G_s \Delta w - e_i \Delta S}{S_f}$

where

Δw is the change in water content;

e_i is the initial void ratio;

ΔS is the change in degree of saturation; and

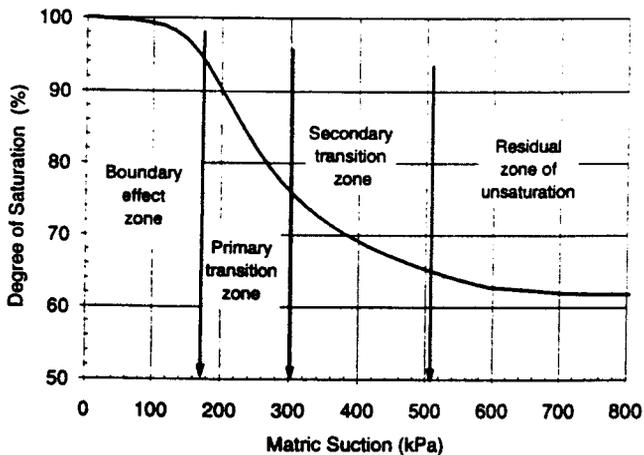S_f is the final degree of change in saturation.

The changes in void ratio that occur due to soil suction changes can be taken into consideration by substituting [3] into [1], if the variables in [3] are known. Therefore, it is possible to incorporate the effect of changes in void ratio into the study of the soil-water characteristic curve.

Total suction, ψ, is comprised of both matric and osmotic suction. However, it is primarily the matric suction component, $(u_a - u_w)$, which governs the engineering behavior of unsaturated soils in the lower suction range encountered in most field situations. Laboratory data has indicated that a change in total suction is essentially equivalent to a change in the matric suction in an unsaturated soil where the water contents are less than the residual value (Krahn and Fredlund 1972).

A physical model for explaining the unsaturated shear strength behavior

Different saturation stages can be identified as the desaturation process of a soil takes place. White et al. (1970) have provided the original concepts for the different stages of desaturation, and the authors have modified these ideas

Fig. 2. Degree of saturation – matric suction curve for a hypothetical porous medium.

Fig. 3. Probable variation of water area in different stages of a soil-water characteristic curve.

(a) Boundary effect stage

(b) Primary transition stage

(c) Secondary transition stage

(d) Residual stage of unsaturation

somewhat. The results are shown in Fig. 2. There are three identifiable stages of desaturation: the boundary effect stage, the transition stage (i.e., primary and secondary transition stages), and the residual stage of unsaturation.

Figure 3 illustrates the variation of area of water with desaturation for different stages of the soil-water characteristic curve (Vanapalli 1994). In the boundary effect stage all the soil pores are filled with water (i.e., the water menisci in contact with the soil particles or aggregates are continuous in this stage) (see Fig. 3a). The soil is essentially saturated at this stage, and there is no reduction in the area of water in this stage. Under these conditions, the single stress state, $(\sigma - u_w)$, describes the behavior of the soil. The first point of importance on the soil-water characteristic curve is the air-entry value, $(u_a - u_w)_b$. This value of suction identifies the point at which air enters the largest pores of the soil. The soil starts to desaturate in the transition stage. The water content in the soil reduces significantly with increasing suction in this stage. The amount of water at the soil particle or aggregate contacts reduces as desaturation continues (i.e., the water menisci area in contact with the soil particles or aggregates is not continuous and starts reducing in this stage (Figs. 3b and 3c). Eventually large increases in suction lead to a relatively small change in water content (or degree of saturation). This stage is referred as the residual stage of unsaturation (Fig. 3d). The water content in the soil at the commencement of this stage is generally referred to as the residual water content. The amount of water is small in this stage (i.e., the water menisci is small).

The rate at which shear strength changes in unsaturated conditions appears to be related to the area of water (i.e., the water menisci area in contact with the soil particles or aggregates) (see Fig. 3d). Thus, it is apparent that there should be a relationship between the soil-water characteristic curve and the shear strength of an unsaturated soil. The physical model described in Fig. 3 along with the corresponding desaturation stages shown in Fig. 2 assist in understanding the shear strength behavior of an unsaturated soil.

The typical relationship between the shear strength and the soil-water characteristic curve can be seen by comparing
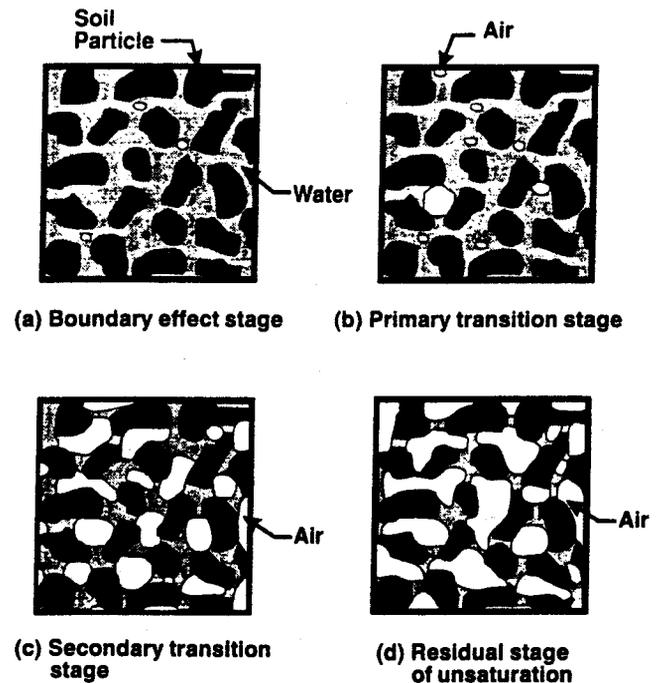
Figs. 4a and 4b. There is a linear increase in shear strength up to the air-entry value. The rate of desaturation with respect to an increase in matric suction, (i.e., $dS/(u_a - u_w)$) is greatest between the air-entry value and the suction corresponding to residual water content conditions. There is a nonlinear increase in shear strength in this region. However, beyond the residual suction conditions, the shear strength of an unsaturated soil may increase, decrease, or remain relatively constant during further desaturation. In some cases, particularly in soils that desaturate relatively fast (e.g., sands and silts), it can be expected that the shear strength will decrease. Generally, it can be expected that there is little water left in soil pores when the soil reaches the residual state. The water content in sands and silts at residual suction conditions can be quite low and may not transmit suction effectively to the soil particle or aggregate contact points. Thus, even large increases in suction will not result in a significant increase in shear strength.

In contrast, clays may not have a well-defined residual state. Even at high values of suction there could still be considerable water (i.e., in the form of adsorbed water) available to transmit suction along the soil particle or aggregate contacts, which contributes towards increases in the shear strength. This phenomenon can occur for a large range of suction values for clays. This subject is further discussed later in this paper.

The residual state occurs at a relatively low suction value for gravels, sands, silts, and their mixtures (i.e., generally between 0 and 200 kPa suction range) and is well defined. The residual state conditions of such soils can be reasonably well predicted from the soil-water characteristic curve plotted on an arithmetic scale. For clays with low plasticity, the residual state will generally be in the range of 500–1500 kPa. However, for intermediate to highly plastic soils, the residual state can be greater than 1500 kPa.

Fig. 4. (*a*) A typical soil-water characteristic curve. (*b*) Shear strength behavior of soil as it relates to the soil-water characteristic curve.
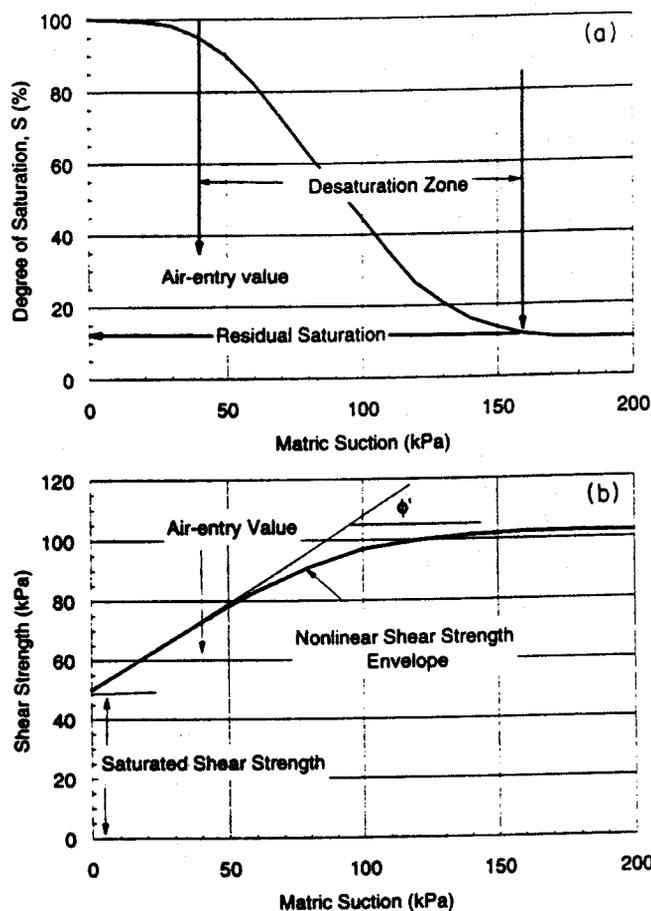
In some cases, (e.g., highly plastic, intact clays) it is difficult to define the residual break in the curve.

The residual water content and its corresponding value of soil suction have been defined in a number of ways in the literature; however, the accepted use of these terms is not always in agreement. Some investigators suggest that a water content corresponding to a suction of about 1500 kPa can be used as the residual water content (van Genuchten 1980). This magnitude of suction, corresponding to residual conditions, is similar to the wilting point of many plants. Water in the liquid phase drains from most of the soil pores when the suction has attained a value of about 1500 kPa. Desaturation beyond residual conditions occurs primarily as a result of vapor movement up to the point where the soil water content is in equilibrium with the vapor pressure of its surroundings.

Terminal suction and the soil-water characteristic curve

There appears to be a common value of total suction (i.e., the sum of matric suction and osmotic suction) where all types of soils approach zero water content (see Fig. 1). This suction value corresponds to approximately 1 000 000 kPa. The experimental results of Russam (1958), Croney et al. (1958), Fredlund (1964), Fleureau et al.

(1993), and Vanapalli (1994) on various soils experimentally supports this value. This observed behavior is also supported using thermodynamic principles (Richards 1965). Engineers are generally concerned with the performance of geotechnical structures in the relatively low suction range of 0–500 kPa. Suction values approaching 1 000 000 kPa and the corresponding low water contents are, however, useful when defining flux boundary conditions, and, therefore, it is of value to mathematically define the entire soil-water characteristic curve.

Mathematical representation of the soil-water characteristic curve

There are several empirical equations proposed in the literature to represent the soil-water characteristic curve (Brooks and Corey 1964; McKee and Bumb 1987; van Genuchten 1980). These equations, while suitable for the data at hand, have often been restricted to certain types of soils or to soil-water characteristic curves of a particular shape or to a limited range of suction values.

Fredlund and Xing (1994) provided an analytical basis for mathematically defining the entire soil-water characteristic curve. The equation applies over the entire range of suctions from 0 to 1 000 000 kPa. This relationship is empirical but is derived based on the pore size distribution, assuming that the soil consists of a set of interconnected pores that are randomly distributed. The equation is most commonly written in terms of volumetric water content, θ.

$$[4] \qquad \theta = C(\psi)\left\{ \frac{\theta_s}{\ln\left[\left(e + \left(\frac{\psi}{a}\right)^n\right)\right]^m} \right\}$$

where
θ is volumetric water content;
θ_s is saturated volumetric water content;
a is a suction related to the air-entry value of the soil;
n is a soil parameter related to the slope at the inflection point on the soil-water characteristic curve;
ψ is soil suction;
m is a soil parameter related to the residual water content;
θ_r is volumetric water content at residual conditions;
e is a natural number, 2.71828...; and
$C(\psi)$ is a correction function that forces the soil-water characteristic curve through a suction of 1 000 000 kPa and zero water content.
The correction factor is defined as

$$[5] \qquad C(\psi) = \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{1\,000\,000}{C_r}\right)} \right]$$

where C_r is the suction value corresponding to residual water content, θ_r.

Equation 4 can be written in a normalized form by dividing both sides of the equation by the volumetric water content at saturation:

$$\text{[6]} \quad \Theta = [C(\psi)] \left[\frac{1}{\ln\left(e + \left(\frac{\psi}{a} \right)^n \right)} \right]^m$$

The normalized volumetric water content, Θ, is defined as

$$\text{[7]} \quad \Theta = \frac{\theta}{\theta_s}$$

where

θ is volumetric water content; and

θ_s is volumetric water content at a saturation of 100%. However, the degree of saturation, S, is also equal to the normalized volumetric water content.

$$\text{[8]} \quad \Theta = S$$

Equation 4 or 6 can be used to best-fit soil-water characteristic curve data of any soil for the entire range of suctions. The fitting parameters (i.e., a, n, and m values) must be determined using a nonlinear regression procedure (Fredlund and Xing 1994). An empirical, analytical model is developed both in terms of volumetric water content, θ, and degree of saturation, S, and the saturated shear strength parameters, effective cohesion, c', and effective angle of shearing resistance, ϕ' to predict the variation of shear strength with respect to suction.

The analytical relationship between the soil-water characteristic curve and the shear strength

Linear shear relationship

A linear shear strength equation for an unsaturated soil was proposed by Fredlund et al. (1978):

$$\text{[9]} \quad \tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b$$

where

τ_f is the shear strength of an unsaturated soil;

c' is the effective cohesion of saturated soil;

ϕ' is the effective angle of shearing resistance for a saturated soil;
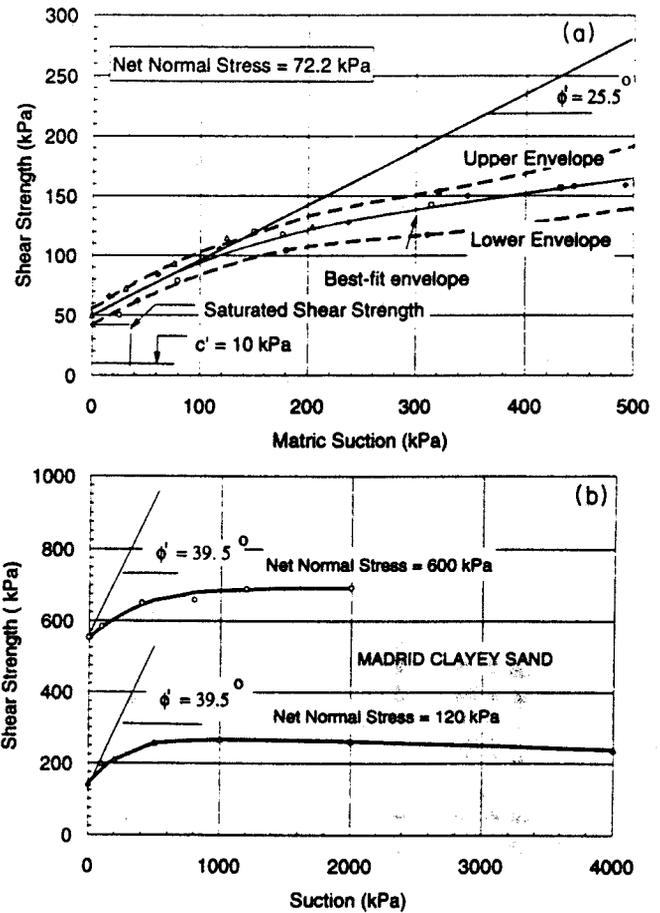
ϕ^b is the angle of shearing resistance with respect to matric suction;

$(\sigma_n - u_a)$ is the net normal stress on the plane of failure at failure; and

$(u_a - u_w)$ is the matric suction of the soil on the plane of failure.

Some experimental results showing the variation in shear strength with respect to matric suction are shown in Fig. 5 (Gan et al. 1988; Escario and Juca 1989). The effective angle of shearing resistance, ϕ', of glacial till tested by Gan et al. 1988 is 25.5°, and that of clayey sand tested by Escario and Juca (1989) is 39.5°. The experimental results show nonlinear shear strength behavior when tests are performed over a wide range of suctions (Fig. 5). Soils that are resistant to desaturation, such as highly plastic clays, for example, can exhibit essentially a linear shear strength behavior over a relatively large range of soil suctions. Rahardjo et al. (1995) reported essentially linear shear strength behavior for a suction range of 0–500 kPa for

Fig. 5. (a) Variation of shear strength with matric suction (modified after Gan et al. 1988). (b) Variation of shear strength with matric suction (modified after Escario and Juca 1989).

a residual clay in Singapore. Equation [4] can be used to describe the shear strength behavior with respect to soil suction provided the suction range is specified. The angle of shearing resistance with respect to soil suction, ϕ^b, is a variable for soils that exhibit nonlinear shear strength behavior.

Role of area of water

At lower values of matric suction (i.e., at high degrees of saturation) the pore-water pressure acts directly to increase the effective stress in contributing to the shear strength. This condition applies until the soil begins to desaturate under an applied matric suction. The rate at which suction contributes towards shear strength can be related to the normalized area of water, a_w. The normalized area of water is assumed to be in direct proportion to the water volume in the soil by applying Greens theorem (Fung 1977). The normalized area of water, a_w, is defined as

$$\text{[10]} \quad a_w = \frac{A_{dw}}{A_{tw}}$$

where

A_{tw} is the total area of water at 100% saturation; and

A_{dw} is the area of water corresponding to any degree of saturation.

The normalized area of water, a_w, is a dimensionless number. This number can be visualized as representing the amount of water in the soil. The value of a_w varies from unity at saturation, to a small value under residual state conditions, and zero when the soil is dry. The normalized volumetric water content, Θ, of the soil also varies in a manner similar to the area of water, a_w, for a large range of suction values.

Due to the similarity in the normalized area of water, a_w, and the normalized volumetric water content, Θ, the following relationship can be written:

$$[11] \qquad a_w = \left(\Theta^\kappa\right)$$

where κ is a fitting parameter. The significance of the fitting parameter, κ, will be discussed later.

The shear strength contribution due to suction, τ_{us}, in terms normalized area of water, a_w, can be mathematically expressed as

$$[12] \qquad \tau_{us} = (u_a - u_w)(a_w \tan\phi')$$

Substituting [11] in [12] results in [13]:

$$[13] \qquad \tau_{us} = (u_a - u_w)\left[\left(\Theta^\kappa\right)(\tan\phi')\right]$$

The incremental shear strength contribution due to suction, $d\tau$, can be obtained by differentiating [13] with respect to suction, $(u_a - u_w)$. The result is

$$[14] \qquad d\tau = d(u_a - u_w)\left[\left(\Theta^\kappa\right)(\tan\phi')\right]$$
$$+ (u_a - u_w)\left[d\left(\Theta^\kappa\right)(\tan\phi')\right]$$

The value of $(\tan\phi^b)$ at any suction is

$$[15] \qquad \tan\phi^b = \frac{d\tau}{d(u_a - u_w)}$$
$$= \left[\left(\Theta^\kappa\right) + (u_a - u_w)\frac{d\left(\Theta^\kappa\right)}{d(u_a - u_w)}\right]\tan\phi'$$

Up to the air-entry value of the soil, Θ is equal to unity and there is no change in the normalized area of contact, a_w. The rate of change of Θ (i.e., $[d(\Theta^\kappa)]/[d(u_a - u_w)]$) equals zero up to the air-entry value of soil. In other words, the value of a_w is unity in the boundary effect stage, as the soil is in a saturated state (see Figs. 2 and 3). Up to the air-entry value, the ϕ^b angle is equal to the effective angle of shearing resistance, ϕ', in [4]. Figure 6a shows the variation of shear strength with respect to suction in this stage with reference to the soil-water characteristic curve in Fig. 2.

The rate of change of Θ, (i.e., $[d(\Theta^\kappa)]/[d(u_a - u_w)]$) is always a negative value for increments of suction beyond the air-entry value (i.e., in the primary and secondary transition stage and residual stage). However, the net shear strength contribution due to suction, $d\tau$, in [15] is positive. This occurs because the shear strength contribution due to a suction change is more effective than the reduction in a_w in this stage. In other words, even though a_w decreases as a result of an increase in suction, the net contribution due to suction is positive, and, hence, there is an increase in the shear strength. Figures 6b and 6c show the variation of

shear strength with respect to suction in the transition stage with reference to the soil-water characteristic curve in Fig. 2.

At high values of suction (i.e., in the residual stage of unsaturation) Θ is extremely small and the value of $[d(\Theta^\kappa)]/[d(u_a - u_w)]$ in this stage is negative. The net summation of

$$\left[\left(\Theta^\kappa\right) + (u_a - u_w)\frac{d\left(\Theta^\kappa\right)}{d(u_a - u_w)}\right]\tan\phi'$$

in [15] may approach negative values. In other words, the net contribution of suction in the residual stage of unsaturation causes a reduction in the shear strength.

Equation [15] satisfies the conceptual behavior of the shear strength of unsaturated soils and provides a theoretical basis for the use of the soil-water characteristic curve to develop a shear strength function.

Shear strength equation and the soil-water characteristic curve

It is proposed that the shear strength of an unsaturated soil at any given value of suction be written as follows:

$$[16] \qquad \tau = [c' + (\sigma_n - u_a)\tan\phi'] + (u_a - u_w)\left[\left(\Theta^\kappa\right)(\tan\phi')\right]$$

The first part of the equation is the saturated shear strength, when the pore-air pressure, u_a, is equal to the pore-water pressure, u_w. This part of the equation is a function of normal stress, as the shear strength parameters c' and ϕ' are constant for a saturated soil. For a particular net normal stress, this value is a constant. The second part of the equation is the shear strength contribution due to suction, which can be predicted using the soil-water characteristic curve.

To obtain a better correlation between predictions and experimental shear strength data, a fitting parameter such as κ is useful. This is similar to the matching factor used by Green and Corey (1971) to match experimental and calculated values for the coefficient of permeability function. The analyses carried out using [16] are referred to as the "first approach" in the paper.

Extending the same philosophical concepts, another equation is proposed in this paper for predicting the shear strength without using the fitting parameter, κ. The equation is given below:

$$[17] \qquad \tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left[(\tan\phi')\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)\right]$$

where θ_r is the residual volumetric water content.
Equation [17] can also be written directly in terms of the degree of saturation:

$$[18] \qquad \tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left[(\tan\phi')\left(\frac{S - S_r}{100 - S_r}\right)\right]$$

where S_r is the residual degree of saturation.

The residual volumetric content, θ_r, or the residual degree of saturation, S_r, can be determined from the soil-water characteristic curve. The results analyzed using [17] are referred to as the "second approach" in the rest of the paper. Sensitivity analyses will be presented for the proposed shear strength equations (i.e., [16] and [17]).

Fig. 6. (*a*) Variation of shear strength with matric suction in the boundary effect stage. (*b*) Variation of shear strength with matric suction in the primary transition stage. (*c*) Variation of shear strength with matric suction in the secondary transition stage. (*d*) Variation of shear strength with matric suction in the residual zone of unsaturation.
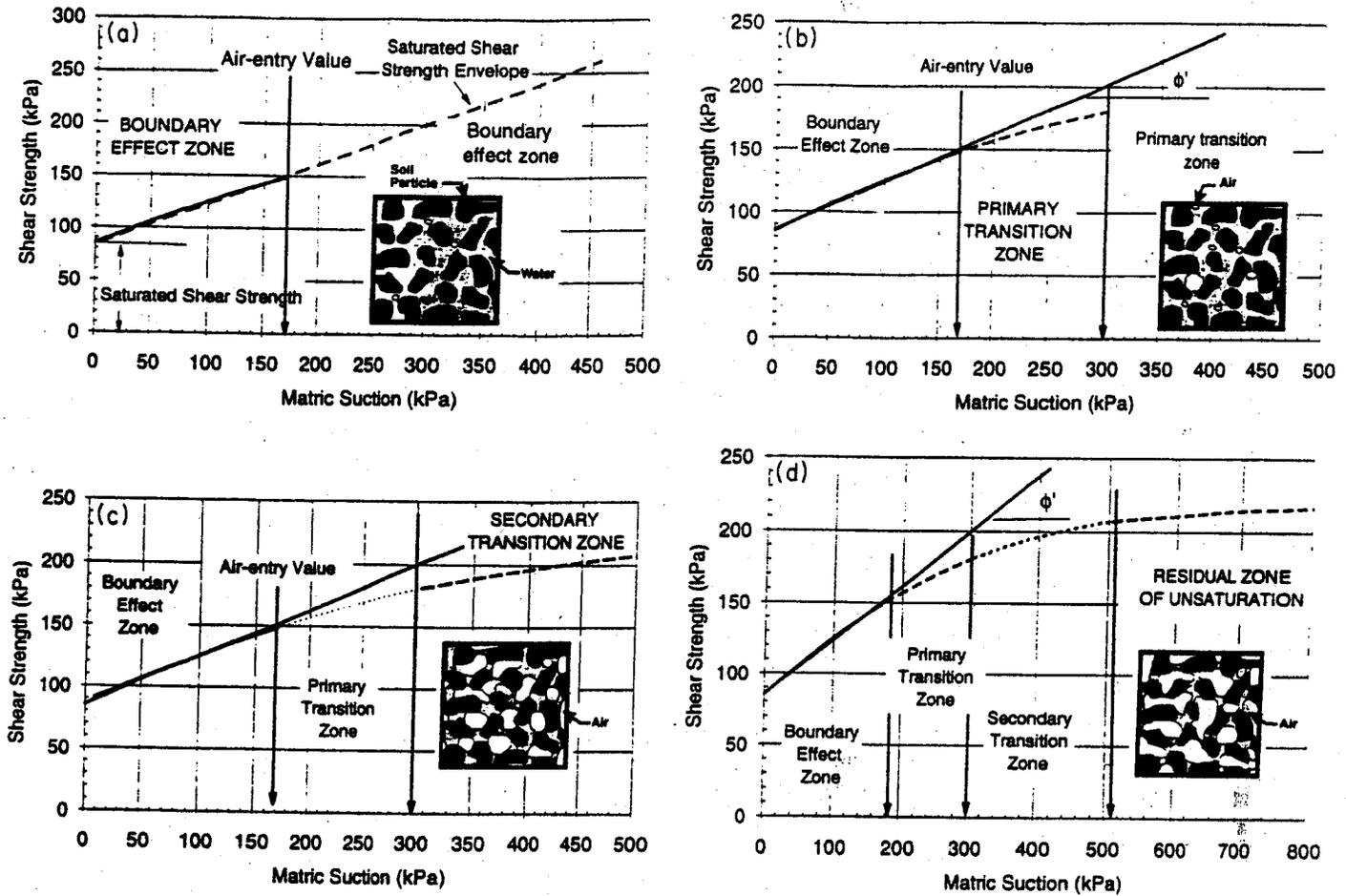
The advantages and disadvantages of each equation are discussed later while presenting the results.

Procedure for predicting the shear strength of unsaturated soils

The procedure for predicting the shear strength of unsaturated soils is as follows:

(1) The three fitting parameters for the soil-water characteristic curve (i.e., *a*, *n*, and *m*) apply for the entire suction range from 0 to 1 000 000 kPa. Therefore, [4] or [6] can be estimated using a nonlinear minimization technique. The initial values used in nonlinear regression analysis for all the soil-water characteristic curves were as follows: $a = 10$, $n = 1$, and $m = 1$.) The objective is to obtain three soil parameters that produce a curve that closely matches the measured values on the soil-water characteristic curve.

(2) The residual state of the soil (i.e., S_r and ψ_r) is estimated from the soil-water characteristic curve.

(3) The required input parameters for obtaining the shear strength function are the strength parameters, c' and ϕ', and the soil-water characteristic curve. The shear strength function can generally be predicted using either [16] or [17] for the suction range of interest (i.e., up to 1000 kPa).

Shear strength behavior of soils beyond the residual state

The nonlinear increase in shear strength occurs from the air-entry value of the soil to its residual condition. The shear strength of soils such as sands and silts and certain clays drops beyond a particular value of suction. This drop in shear strength may be assumed to start near the residual value of suction. Similarly, some soils can exhibit a relatively constant shear strength for a large range of suction values. The general nature of the soil-water characteristic curve gives some indication of the behavior of shear strength after the residual state.

The shear strength of an unsaturated soil may remain relatively constant, increase or decrease beyond the residual state. Presently, there are no data available to show the soil-water characteristic curves and experimental shear strength data to support and explain the shear strength behavior beyond the residual state. However, there is experimental evidence available in the literature to show that the shear strength decreases (Donald 1956; Escario and Juca 1989). A decrease in shear strength is possible at low suction values for soils such as sands and gravels that desaturate quite rapidly. Donald (1956) showed results that

indicated a drop in shear strength after a suction value of 10–15 kPa for four different sands tested. For soils that have the highest percentage of fines, the shear strength drops only at larger values of suction. Escario and Juca (1989) have observed increases in shear strength up to suction values of 1000 kPa for a clayey sand. However, for suctions from 1000 to 4000 kPa, a drop in shear strength was observed (see Fig. 5b).

For certain soils like highly plastic clays, there may be no defined residual stage of unsaturation. Such soils show increases in shear strength even at relatively high values of suction. Escario and Juca (1989) report increases in shear strength for a Guadalix red clay at suction values as high as 10 000 kPa. The residual state for highly plastic soils seems to occur only as the water content in the soil approaches zero. It may be assumed that the shear strength for such soils increases up to the point where the degree of saturation approaches a zero value or an extremely high suction. For example, the dry strength of highly plastic clays is high and continues to rise to a dry condition (e.g., adobe bricks).

Parameters influencing the shear strength behavior of unsaturated soils

The soil structure and stress history that may occur in the field should be reasonably simulated in the laboratory while developing the soil-water characteristic curve. Conventional, soil-water characteristic curves are developed using a pressure plate apparatus without any loading applied to the specimen. A method for developing soil-water characteristic curves under differing stress conditions for fine-grained soils using the pressure plate apparatus is provided by Vanapalli (1994). This procedure is briefly explained later in the paper.

Another parameter that has to be considered in the shear strength prediction is the influence of suction on the angle of shearing resistance, ϕ' Vanapalli (1994) showed results where ϕ' was independent of suction for a glacial till tested at various densities and initial water contents for a range of suctions from 0 to 500 kPa. Karube (1988) reported similar results for a kaolonite. Drumright (1989) has reported that ϕ' was slightly influenced by suction. Escario and Juca (1989), however, have found that ϕ' was independent of suction for Madrid clayey sand (i.e., soil having a w_L of 32%, I_p of 15%, with clay, silt, and sand contents of 17%, 31%, and 46%, respectively) but not for Guadalix red clay (i.e., soil having a w_L of 33%, I_p of 13.6%, with clay, silt, and sand contents of 86%, 11%, and 3%, respectively) when tested for a large range of suction values (0 to 10 000 kPa). For all practical purposes, it would appear that ϕ' can be assumed to be constant for a suction range of 0–500 kPa. This is generally the range of practical interest for geotechnical and geoenvironmental engineers.

Comparison of predicted unsaturated shear strength of a glacial till with the experimental results

The unsaturated shear strength behavior of a statically compacted glacial till was studied at three different water

contents and densities. These conditions represented the optimum, dry and wet of optimum conditions. Varying net normal stresses and varying matric suction values were applied to the specimens and the strength behavior was experimentally studied using a modified direct shear apparatus.

Soil-water characteristic curves were developed for the suction range of 0 – 300 000 kPa using a pressure plate apparatus and osmotic desiccators with specimens preconsolidated to an equivalent net normal stresses used in the experimental program. The experimental results of unsaturated shear strength are compared with the predicted shear strengths.

Soil and the testing program

A glacial till obtained from Indian Head, Saskatchewan, was used for the study. The soil used for the entire testing program was obtained in a single batch to ensure uniformity of the specimens. The soil was air dried for several days, pulverized using a rubber mallet, and passed through a 2 mm sieve. The Atterberg limits of the soil showed a liquid limit, w_L, of 35.5% and the plastic limit, w_P, of 16.8%. The fractions of sand, silt, and clay were 28%, 42%, and 30%, respectively. The AASHTO (American Association of State Highway and Transportation Officials) standard compacted density was 1.80 Mg/m^3, which corresponded to an optimum water content of 16.3%.

Precalculated amounts of distilled water were sprayed on to several layers of the soil and left overnight in covered plastic bags and stored in a humidity controlled room. The soil was then thoroughly hand mixed. To avoid soil-water clods the mixed soil was again passed through 2 mm sieve. This soil–water mixture was then placed in plastic bags and kept in a moist room for at least 48 hours.

A comprehensive experimental program was conducted, using statically compacted specimens of the prepared soil. Three initial water contents were selected representing dry of optimum conditions (i.e., initial water content 13%, and a dry density, γ_d, of 1.73 Mg/m^3), optimum conditions (i.e., initial water content 16.3%, and a dry density, γ_d, of 1.80 Mg/m^3), and wet of optimum conditions (i.e., initial water content 19.2%, and a dry density, γ_d, of 1.77 Mg/m^3). The densities and water contents used in the research program were selected from the soil compaction curve.

Statically compacted specimens 100 mm diameter and 21 mm height were prepared using a constant volume mold at the prescribed densities and water contents. The specimens used for the direct shear tests and for developing the soil-water characteristic curves were obtained from the 100 mm diameter specimens.

Saturated shear strength tests in direct shear

Effective shear strength parameters were determined both under single stage and multistage testing with the soil in a saturated state. The residual shear strength of the soil was also studied. Single stage and residual shear strength testing was conducted using a conventional direct shear appara. Multistage testing was conducted using a modified di. shear apparatus. The same apparatus was used for unsaturated shear strength testing. All tests were conducted using a conventional testing procedure for direct shear tests.

Unsaturated shear strength tests using modified direct shear equipment

The unsaturated shear strength of the statically compacted specimens was determined by using modified direct shear equipment designed by Gan and Fredlund (1988) for multistage testing. Multistage testing overcomes two main problems generally encountered in testing unsaturated soils. Firstly, the time required for testing is reduced making it feasible to test unsaturated soils of a low coefficient of permeability in a reasonable time. Secondly, the same specimen can be tested under a relatively large range of matric suctions. This procedure helps avoid the variability caused by nonuniformity of different specimens and results in using fewer specimens for testing purposes. Design details of modified direct shear equipment and the procedure of testing using multistage loading are detailed by Gan and Fredlund (1988).

A matric suction range of 0–500 kPa was selected for testing based on the air-entry value of the ceramic in the modified direct shear testing apparatus. The glacial till used in this study desaturates reasonably well from a degree of saturation of 100% to around 60% over the matric suction range of 0–500 kPa. Since the nonlinear variation of shear strength is dependent on desaturation, there should be a wide variation in shear strength over the testing range used in this study.

Soil-water characteristic curves

Soil-water characteristic curves cannot be measured while applying an external load when using a conventional pressure plate apparatus. An indirect method was used to allow a known level of stress to be applied to the specimens. First, a series of consolidation and swelling curves were measured for the soil using conventional one-dimensional consolidation test procedures. The consolidation and swelling characteristics of the soil (i.e., compression index, C_c, and the swelling index, C_s) were then used to estimate the initial stress state and void ratio of the soil to be used in the pressure plate tests. More details on preparing "preconsolidated" specimens used for developing soil-water characteristic curves are available in Vanapalli (1994).

Soil-water characteristic curves were developed using a pressure plate apparatus on specimens, which had been preconsolidated to the net normal stresses used in the direct shear test for determining the unsaturated shear strength. The preconsolidated specimens were placed in a consolidation ring 63.5 mm in diameter. The ring plus the soil were placed on the ceramic disk of the pressure plate with a 5 N surcharge load to ensure a good contact between the specimen and the ceramic disk. The airtight chamber of the pressure plate apparatus was then pressurized to a desired matric suction.

The initial volume and mass of the preconsolidated specimens were measured. As the suction was applied, water drained from the specimen. The mass of the specimen was calculated after the specimen attained equilibrium conditions under the applied matric suction. Equilibrium was assumed to have been attained when water no longer discharged from the pressure plate. Approximately 5–7 days were required to attain equilibration under each applied

matric suction. After equilibrium was attained, the test assembly was dismantled and the specimens were weighed. This procedure was repeated at every desired matric suction.

Soil-water characteristic curves were determined for matric suction ranging from 0 to 1500 kPa. Approximately half the specimen was used for the measurements of the water content. The water contents at various matric suctions were determined from back-calculations. Calculations for the degree of saturation in this study were made from the initial void ratio of the specimen using the volume–mass relationships. Since statically compacted specimens are relatively stiff and resistant to shrinkage, the change in void ratio with increasing suction was not considered to be significant. The suggested procedure facilitates testing of several specimens simultaneously in the conventional pressure plate apparatus.

To attain the suction versus water content relationship beyond 1500 kPa (i.e., 4500 – 300 000 kPa), an osmotic desiccator was used. Small pieces of a specimen (i.e., about 5 g) obtained from a larger specimen after the completion of the pressure plate tests were used to complete the soil-water characteristic curves by placing them in glass desiccators containing salt solutions. The salt solutions in the glass desiccators controlled the relative humidity (or vapor pressure) in the specimen. Five aqueous solutions were selected that covered the range of suction values from 4500 to 300 000 kPa.

Test results

Saturated shear strength results

The saturated shear strength parameters were calculated for all three different initial conditions (i.e., an initial water content of 13% and a dry density, γ_d, of 1.73 Mg/m^3, an initial water content of 16.3% and a dry density, γ_d, of 1.80 Mg/m^3, and an initial water content of 19.2% and a dry density, γ_d, of 1.77 Mg/m^3). The saturated shear strength parameters were approximately the same for all the tests conducted in spite of different initial conditions. Lee and Haley (1958) and Gibbs and Hilf (1953) reported similar results when testing soils with different initial conditions (i.e., water content and dry densities).

A series of residual strength tests was conducted on the saturated specimens at optimum conditions (i.e., initial water content equal to 16.3% and the dry density, γ_d, equal to 1.80 Mg/m^3). Only one series of the tests was conducted because the analysis of the single-stage tests showed that different initial conditions of the soil will not significantly influence the saturated shear strength parameters. The results obtained from the residual shear testing show that there is essentially no drop in strength once the soil has attained a peak strength. Under continued reversals there was still no significant loss of strength. As a result, the soil can be said to be suitable for multistage testing.

A series of multistage tests were conducted on saturated specimens at optimum condition (i.e., initial water content of 16.3% and a dry density, γ_d, of 1.80 Mg/m^3). These tests were conducted using modified direct shear equipment. A strain rate of 2.7 mm/day was used for shearing. A similar strain rate was used for testing the unsaturated specimens using the modified direct shear equipment.

Fig. 7. Results of saturated shear strength tests under different procedures.
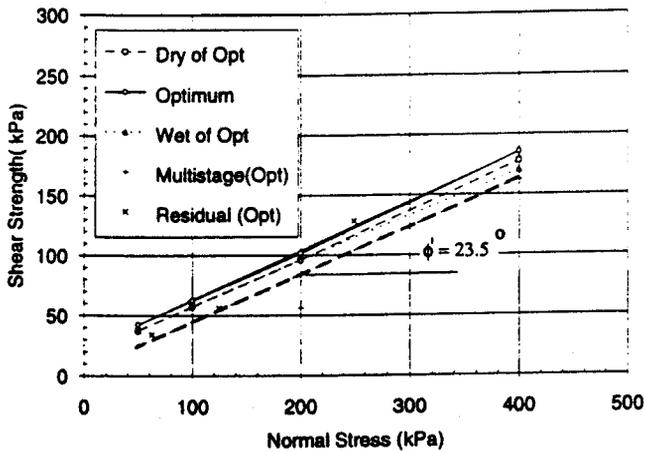

Fig. 8. Soil-water characteristic curves of preconsolidated specimens using pressure plate.

Table 1. Strength parameters of saturated till from different types of direct shear tests.

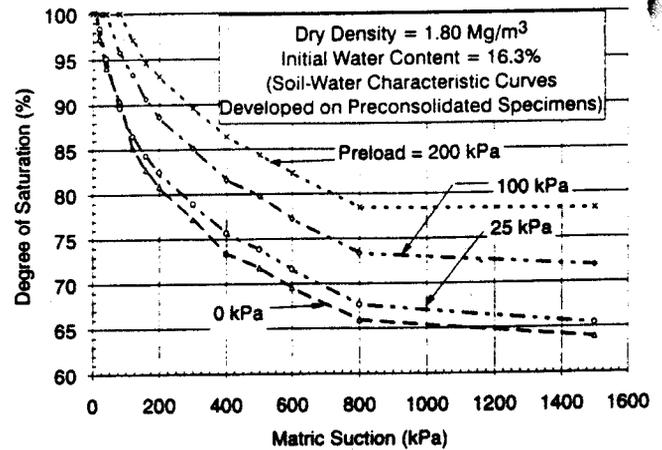| Type of direct shear test	c' (kPa)	ϕ' (°)	Strain rate (mm/day)
Single stage	15	22.5	18
Residual	8	23	18
Multistage	4	23	2.7

Fig. 9. Soil-water characteristic curves of preconsolidated specimens using pressure plate and desiccators.

Shear strength envelopes obtained from the three different types of tests are shown in Fig. 7. The shear strength parameters along with the strain rates are summarized in Table 1.

Three types of tests conducted show that there is no significant variation in the angle of shearing resistance, ϕ', when using the three different types of tests. The variation in cohesion from 15 kPa in single-stage testing to 4 kPa in multistage testing may be due to the variation in strain rate, different pieces of direct shear equipment used, type of testing procedures, and the possible variation in soil properties.

The test that produced the lowest effective cohesion value under drained shearing conditions is assumed to be most accurate (i.e., 4 kPa). Ruddock (1966) postulated that for multistage testing, failure to achieve complete dissipation of pore pressure will be reflected in an increase in the measured cohesion. However, the angle of shearing resistance, ϕ', would not be significantly affected. It is possible that by the time the frictional component of strength is fully mobilized, the actual cohesion present in the measured sheared strength may be small. Gan (1986) tested the same till used for this study and showed that at peak shear strength mobilized, the cohesion component was negligible.

As the unsaturated shear strength tests of the present study were conducted using multistage testing adopting a strain rate of 2.7 mm/day, an effective angle of shear resistance, ϕ', equal to 23° and an effective cohesion, c', equal to 0 kPa would be most reasonable.
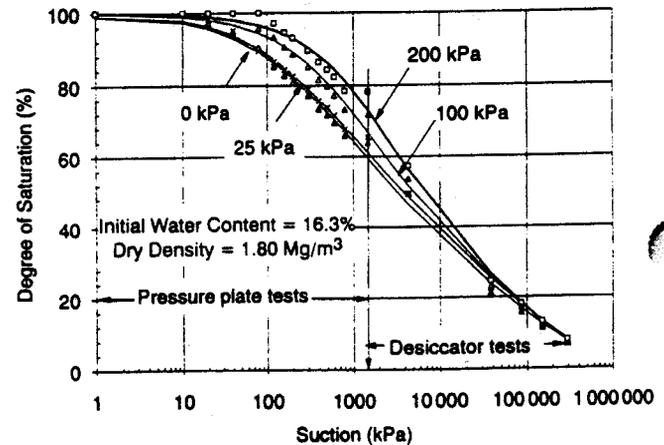
Pressure plate and desiccator test results

Figures 8 and 9 show typical soil-water characteristic curves developed for the specimens with preconsolidated normal stresses of 0, 25, 100, and 200 kPa. These soil-water characteristic curves were developed on specimens with an initial water content of 16.3% and a dry density of 1.80 Mg/m³, reflecting the optimum conditions.

The soil-water characteristic curves are plotted on a linear scale in Fig. 8 with degree of saturation as the ordinate and matric suction as the abscissa. The soil-water characteristic curves for the same preconsolidated specimens for the range of 0 – 300 000 kPa suction are shown on a semilogarithmic scale in Fig. 9. It can be seen that the degree of saturation continues to reduce up to a suction of 300 000 kPa.

Similar trends were observed for the specimens at wet and dry of optimum conditions. However, the specimens that were prepared dry of optimum desaturated considerably faster.

The soil-water characteristic curve data can be defined as a mathematical expression [4] or [6] for suctions ranging from 0 to 1 000 000 kPa. The constants a, n, and m were

Table 2. Input parameters to predict the shear strength of unsaturated till.

Preload stress (kPa)	a	n	m	κ	ψ_r
Dry of optimum initial conditions					
25	34.1	0.80	0.57	2.2	3000
100	71.4	0.66	0.54	2.5	3000
200	125.2	0.81	0.45	2.8	3000
Optimum initial conditions					
25	140.3	0.77	0.57	2.4	3000
100	406.7	0.81	0.60	2.5	3000
200	812.7	0.84	0.61	4.8	3000
Wet of optimum initial conditions					
25	482.8	0.88	0.71	2.4	3000
100	473.1	0.91	0.59	2.4	3000
200	239.5	1.15	0.40	2.4	3000

computed using the program CFVIEW developed by Xing (1993). The soil-water characteristic curve data obtained from the pressure plate apparatus and the desiccator tests are summarized in Table 2.

Prediction of shear strength for unsaturated soils using the soil-water characteristic curve and the saturated shear strength parameters

The test results for dry of optimum specimens (series D, as defined in Table 3) tested with a net normal stress of 25 kPa are analyzed and discussed using [16] (i.e., first approach) and [17] (i.e., second approach). The saturated shear strength parameters are $c' = 0$ and $\phi' = 23°$. Soil-water characteristic curve data measured using the pressure plate apparatus and desiccators were best-fit for the entire suction range of 0 to 1 000 000 kPa using [4] or [6]. The constants a, n, and m for the data from the nonlinear regression analysis were 34.1, 0.80, and 0.57, respectively.

The variation of shear strength with respect to suction using [16] for different values of κ (i.e., 1, 2, 2.2, and 3) are shown along with the experimental results in Fig. 10. It can be seen that there is a good correlation between the experimental values for the range of suction tested (i.e., 0–500 kPa) and the predicted shear strength when using a value of $\kappa = 2.2$.

In the second approach (i.e., using [17]), a fitting parameter "κ" is not required. However, residual conditions have to be identified from the soil-water characteristic curve data. Figure 11 shows the variation of volumetric water content with respect to suction. As discussed earlier it is difficult to clearly define the residual conditions for fine-grained soils. The shear strength predicted using [17] for various selected residual suction values (i.e., 1500, 3000, and 5000 kPa) is shown in Fig. 12. Experimental results are also shown in this figure. There is good agreement between the experimental results and the predicted

Fig. 10. Variation of shear strength with matric suction using various values of κ.
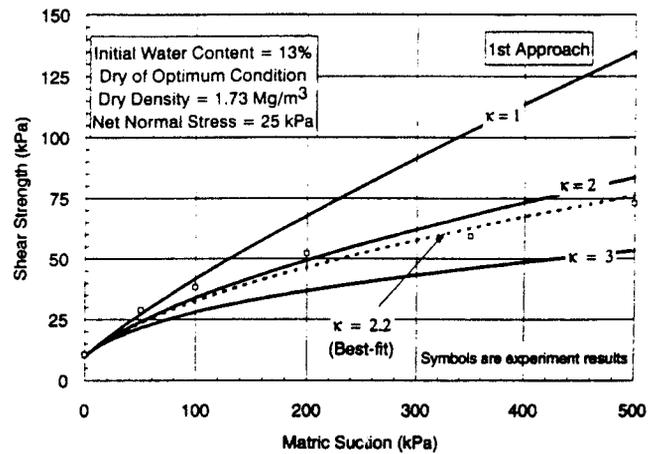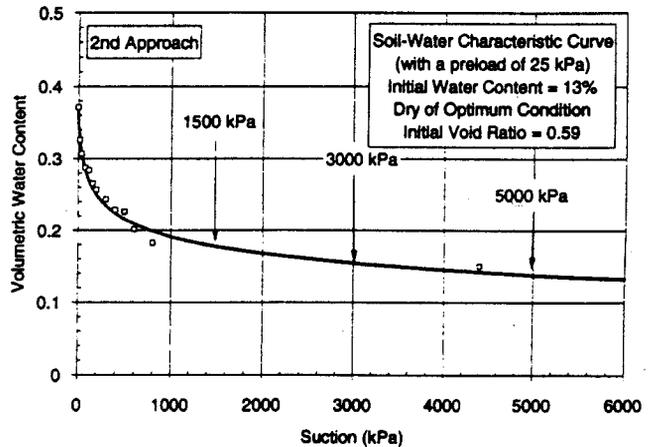

Fig. 11. Variation of volumetric water content versus suction for a specimen at dry of optimum conditions.
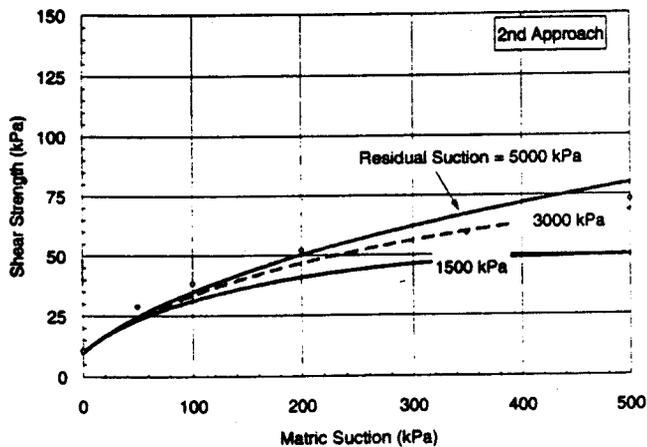

shear strength values for a selected residual suction value of 3000 kPa. The specimens prepared at dry of optimum conditions are more sensitive to suction values towards desaturation in comparison to specimens with optimum and wet of optimum conditions. It was observed that 3000 kPa was a reasonable value for residual suction in the prediction shear strength using [17] for all series of tests conducted in this research program. For many soils, the residual suction value can be estimated reasonably well, and good predictions are possible using the second approach.

The limitation of this approach is that the shear strength drops to zero when suction values approach the residual suction value. There are no experimental results available to confirm by way of experimental data the shear strength at higher suction values for the soil tested. However, both the approaches predict the unsaturated shear strength for the suction range tested (i.e., 0–500 kPa) (see series D results in Fig. 14). The analysis of the remainder of the test results was conducted such that both of the approaches (i.e., first and second) give the same results and are close to the experimental shear strength values.

Table 3. Multistage unsaturated shear strengths in direct shear.

Spl. No.	Series	Water content relative to optimum	Initial water content (%)	Dry density, γ_d (Mg/m^3)	$(\sigma - u_a)$ (kPa)	Final water content (%)
1	Series A1	Optimum	16.6	1.79	75	13.69
2	Series A2	Optimum	16.3	1.80	75	13.41
3	Series B	Optimum	16.3	1.81	200	13.39
4	Series C	Optimum	16.0	1.78	25	13.51
5	Series D	Dry of optimum	13.0	1.73	25	13.60
6	Series E	Dry of optimum	13.3	1.73	100	14.05
7	Series F1	Dry of optimum	12.8	1.74	200	14.05
8	Series F2	Dry of optimum	13.0	1.73	200	14.60
9	Series G	Wet of optimum	18.8	1.77	25	17.07
10	Series H	Wet of optimum	19.1	1.76	100	14.72
11	Series I	Wet of optimum	19.2	1.78	200	13.67

Fig. 12. Variation of shear strength with respect to suction using different residual suction values.

Fig. 13. Variation of shear strength with suction under different net normal stresses for specimens at optimum water content condition.

Unsaturated shear strength results

The unsaturated shear strength behavior of the till specimens prepared at different initial conditions are shown in Table 3.

The variation of shear stress with matric suction for the specimens tested with the initial conditions representing optimum water contents under a net normal stresses of 25, 75, and 200 kPa (i.e., results of series C, A1 and A2, and B) are shown in Fig. 13. All the experimental points are shown with symbols. The continuous curves in Fig. 13 are the predicted shear strength envelopes using the soil-water characteristic curve and the effective shear strength parameters using both the approaches. The κ value used for the first approach and the residual suction value used for the second approach for all the series of tests are summarized in Table 2. There is good correlation between the experimental data on shear strength and the predicted values.

The nonlinear behavior can be observed from all the strength envelopes. Shear strength envelopes representing the adopted value of $\phi' = 23°$, starting at a zero matric suction value are plotted. The ordinate values represent the saturated shear strength values for the respective net normal stresses (i.e., 25, 75, and 200 kPa). The shear strength envelopes obtained from the results of series D, E, F1, and F2 representing the dry of optimum conditions are shown in Fig. 14. Similarly, the results of series G, H, and I representing the wet of optimum initial conditions are shown in Fig. 15. There is a good correlation between the predicted and measured shear strength values.

Limitations of the proposed method

Obtaining a reliable soil-water characteristic curve is important when using the proposed method. The pressure plate apparatuses presently available can be used to measure the soil-water characteristic curves without a confining pressure being applied to the soil specimen. From an engineering point of view, the soil always has confining stress in situ. As such, a soil-water characteristic curve should be measured simulating the field loading condition. Pressure plates should be designed such that soil-water characteristic curves can be measured with specified loading conditions on the specimens. Unfortunately the special advantage of

Fig. 14. Variation of shear strength with matric suction under different net normal stresses for specimens at dry of optimum water content condition.

Fig. 15. Variation of shear strength with matric suction under different net normal stresses for specimens at wet of optimum water content condition.
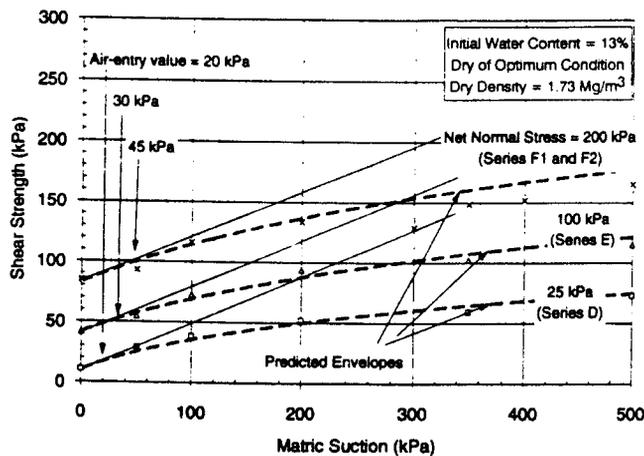
conducting tests simultaneously on several specimens using a single pressure plate would be lost. For each loading condition, a separate pressure plate would have to be used. To overcome this problem, an indirect method was proposed for determining the soil-water characteristic curves for different loading conditions. More details of this procedure are available in Vanapalli (1994). The proposed method should be suitable for soils such as tills and clays.

Summary

An empirical, analytical model is developed and two approaches are provided for predicting the shear strength of an unsaturated soil. This model extends the theory proposed by Fredlund et al. (1978). The models use the soil-water characteristic curve and the effective shear strength parameters of a soil. The soil-water characteristic curve is defined in terms of the soil constants a, n, and m using [4] or [6] for the entire range of suction (i.e., from 0 to 1 000 000 kPa). The shear strength variation with respect to suction can be predicted using the first approach (i.e., using [16]) or the second approach (i.e., using [17]).

The shear strength prediction is dependent the fitting parameter κ using the first approach. For the second approach, the residual conditions can be estimated from the soil-water characteristic curve. The fitting parameters for κ varied between 2.2 and 2.8 for the soil tested under different net normal stresses and initial water content conditions. A fitting parameter, $\kappa = 2.5$ is suggested for the Indian Head till tested for this research program. The authors had to use $\kappa = 4.8$ to obtain a good correlation between the experimental results and predict the values of shear strength for one series of results (i.e., series B). There is good comparison between the experimental results and predicted values using a residual suction of 3000 kPa for all the series of tests (i.e., for the range 0–500 kPa matric suction) for the glacial till tested.

Acknowledgments
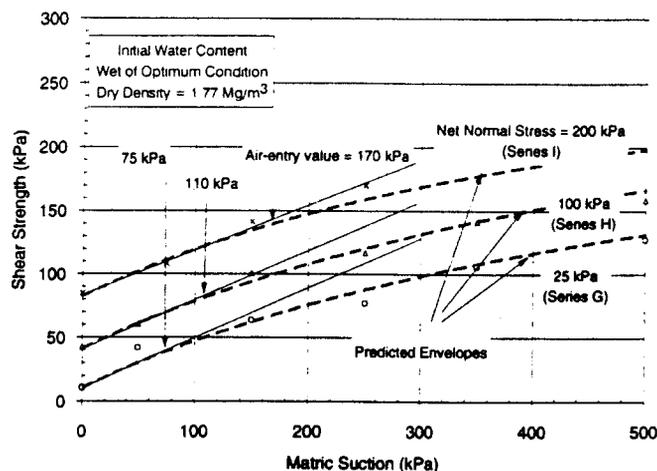
References

Abramento, M., and Carvalho, C.S. 1989. Geotechnical parameters for the study of natural slopes instabilization at Serra do Mar-Brazilian Southeast. Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio de Janeiro, Vol. 3, pp. 1599–1602.

Brooks, R.H., and Corey, A.T. 1964. Hydraulic properties of porous media. Colorado State University Hydrology Paper, Fort Collins, No. 3, Vol. 27, March.

Croney, D., Coleman, J.D., and Black, W.P.M. 1958. The movement and distribution of water in soil in relation to highway design and performance. Road Research Laboratory, Research Note 3209, pp. 1–14.

Donald, I.B. 1956. Shear strength measurements in unsaturated non-cohesive soils with negative pore pressures. Proceedings of the 2nd Australia – New Zealand Conference on Soil Mechanics and Foundation Engineering, pp. 200–204.

Drumright, E.E. 1989. The contribution of matric suction to the shear strength of unsaturated soils. Ph.D. thesis, Colorado State University, Fort Collins.

Escario, V., and Juca, J. 1989. Strength and deformation of partly saturated soils, Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio de Janeiro, Vol. 3, pp. 43–46.

Fleureau, J.-M., Kheirbek-Saoud, S., Soemitro, R., and Taibi, S. 1993. Behavior of clayey soils on drying–wetting paths. Canadian Geotechnical Journal, 30: 287–296.

Fredlund, D.G. 1964. Comparison of soil suction and one-dimensional consolidation characteristics of a highly plastic clay. M.Sc. thesis, Department of Civil Engineering, University of Alberta, Edmonton.

Fredlund, D.G., and Morgenstern, N.R. 1977. Stress state variables for unsaturated soils, Journal of the Geotechnical Engineering Division, ASCE, 103(GT5): 447–466.

Fredlund, D.G., and Rahardjo, H. 1993. Soil mechanics for unsaturated soils. Wiley Publications, New York.

Fredlund, D.G., and Xing, A. 1994. Equations for the soil-water characteristic curve. Canadian Geotechnical Journal. 31: 521–532.

Fredlund, D.G., Morgenstern, N.R., and Widger, R.A. 1978. The shear strength of unsaturated soils. Canadian Geotechnical Journal, **15**: 313–321.

Fung, Y.C. 1977. A first course in continuum mechanics, Prentice-Hall, Inc., Englewood Cliffs, N.J.

Gan, J.K.M. 1986. Direct shear strength testing of unsaturated soils, M.Sc. thesis, Department of Civil Engineering, University of Saskatchewan, Saskatoon.

Gan, J.K.M., and Fredlund, D.G. 1988. Multistage direct shear testing of unsaturated soils. Geotechnical Testing Journal, ASTM, **11**(2): 132–138.

Gan, J.K.M., Fredlund, D.G., and Rahardjo, H. 1988. Determination of the shear strength parameters of an unsaturated soil using the direct shear test. Canadian Geotechnical Journal, **25**: 500–510.

Gibbs, H.J., and Hilf, J.W. 1953. Discussion on Leonards, G.A. (1953). Strengh characteristics of compacted clays. Proceedings of the American Society of Civil Engineers, Separated No. 360, **79**: 1–35.

Green, R.E., and Corey, J.C. 1971. Calculation of hydraulic conductivity: A further evaluation of some predictive methods, Soil Science Society of America Proceedings, **35**: 3–8.

Karube, D. 1988. New concept of effective stress in unsaturated soil and its proving test. *In* Advanced triaxial testing of soil and rock. American Society for Testing and Materials, Philadelphia, Special Technical Publication No. 977, pp. 539–552.

Krahn, J., and Fredlund, D.G. 1972. On total, matric and osmotic suction. Journal of Soil Science, **114**: 339–348.

Lee, K.L., and Haley, S.C. 1968 Strength of compacted clay at high pressure, Journal of the Soil Mechanics and Foundations Engineering Division, ASCE, **94**(SM6): 1303–1329.

McKee, C.R., and Bumb, A.C. 1987. Flow-testing coalbed methane production wells in the presence of water and gas. SPE Formation Evaluation, **Dec**: 599–608.

Rahardjo, H., Lim, T.T., Chang, M.F., and Fredlund, D.G. 1995. Shear-strength characteristics of a residual soil, **32**: 60–77.

Richards, B.G. 1965. Measurement of the free energy of moisture by the psychrometric technique using thermistors. *In* Moisture Equilibria and Moisture Changes in Soils Beneath Covered Areas. A Symposium in Print, Sydney, Australia. *Edited by* G.D. Aitchison. Butterworths & Co. Ltd., pp. 35–46.

Ruddock, E.C. 1966. Correspondence: The engineering properties of residual soils. Géotechnique, **16**(1): 78–81.

Russam, K. 1958. An investigation into the soil moisture conditions under roads in Trinidad, B.W.I. Géotechnique, **8**: 55–71.

Toll, D.G. 1990. A framework for unsaturated soils behavior. Géotechnique, **40**(1): 31–44.

Vanapalli, S.K. 1994. Simple test procedures and their interpretation in evaluating the shear strength of unsaturated soils. Ph.D. thesis, University of Saskatchewan, Saskatoon.

van Genuchten, M.Th. 1980. A closed-form equation of predicting the hydraulic conductivity of unsaturated soils. Soil Science Society of American Journal, **44**: 892–898.

Walle, C.M., and Hachich, W. 1989. Rain-induced landslides in south eastern Brazil. Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering, Rio de Janeiro, Vol. 3, pp. 1639–1642.

Wheeler, S.J., and Sivakumar, V. 1995. An elastic-plastic critical state framework for unsaturated soil. Géotechnique, **45**: 35–53.

White, N.F., Duke, H.R. Sunada, D.K., and Corey, A.T. 1970. Physics of desaturation in porous materials, Journal of the Irrigation and Drainage Division, ASCE, **96**(IR2): 165–191.

Xing, A. 1993. CFVIEW—A curve fitting utility designed to fit soil-water characteristic curve data. Civil Engineering Department University of Saskatchewan, Saskatoon.