Université d'Ottawa · University of Ottawa

Faculté de génie | Faculty of Engineering
Génie chimique | Chemical Engineering

CHG 2314
HEAT TRANSFER

Professor: **B. Kruczek** **2005/04/05**

Assignment No. 10

1. The outer surface of a spherical container of O.D. = 60 cm is maintained at 85°C while exposed to quiescent air at 25°C. Neglecting radiation effects, what is the rate of heat transfer from the container? To minimize burn hazard it is proposed to install a thin-walled spherical surface over the container to create an intervening cavity, to be filled with air at atmospheric pressure. Neglecting radiation effects, what should be the minimum diameter of the outer sphere to ensure that its temperature does not exceed 45°C? What is the reduction in the rate of heat transfer from the container resulting from installation of the outer spherical surface?

2. Fluid A, available at 10°C is to be heated in a one-shell-pass four-tube-pass exchanger, heat using fluid B available at 180°C. When the mass flow rates of fluids A and B are m_A and m_B, the respective outlet temperatures are, $T_{A,out}$ = 100°C and $T_{B,out}$ = 125°C. To increase $T_{A,out}$ it is proposed to double the mass flow rate of fluid B ($2m_B$) while keeping the mass flow rate of fluid A unchanged. If as a result of doubling the mass flow rate of fluid B the overall heat transfer coefficient (U) in the exchanger increases by 20%, what is the new outlet temperature of fluid A.

3. Problem 11.60 a). Do not do part b).

Due Date: **April 12, 2005 at 4:00 p.m. in the assignment box.**

Problem 1 - Solution

Known: Spherical container in quiescent air, whose surface is maintained at $T_s = 85°C$

Find: i) Rate of heat transfer from the container without a cover - q_1

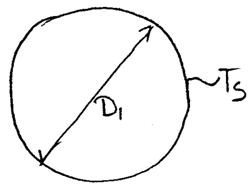
ii) Diameter of the cover (D_2) for the cover temperature $T_{s2} = 45°C$

iii) Rate of heat transfer with cover in place - q_2, and the reduction in q due to cover

Schematic

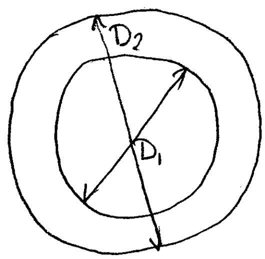
Without cover

quiescent air
$T_\infty = 25°C$
$D_1 = 0.6m$
$T_s = 85°C$
$q_1 = ?$



With cover

quiescent air
$T_\infty = 25°C$
$D_1 = 0.6m$
$D_2 = ?$
$T_{s1} = T_s = 85°C$
$T_{s2} = 45°C$
$q_2 = ?$



Assumptions

1) Steady state conditions
2) Outer surface temperature of the container is not affected by the presence of the cover → $T_{s1} = T_s = 85°C$
3) Negligible resistance to heat transfer though the cover
4) Negligible radiation effects
5) Constant properties

Properties:

1) For external free convection without the cover, $T_f = (T_s + T_\infty)/2 = (25+85)/2 = 55°C = 328\ K$

For air at 328 K: $k = 0.0284\ W/mK$, $\nu = 18.71 \cdot 10^{-6}\ m^2/s$, $Pr = 0.703$

2) For internal free convection in the enclosure, $\bar{T} = (T_{s1} + T_{s2})/2 = (85+45)/2 = 65°C = 338\ K$

For air at 338 K: $k = 0.0291\ W/mK$, $\nu = 19.71 \cdot 10^{-6}\ m^2/s$, $Pr = 0.702$

3) For external free convection from the cover, $T_f = (45+25)/2 = 35°C = 308\ K$

For air at 308 K: $k = 0.0269\ W/mK$, $\nu = 16.69 \cdot 10^{-6}\ m^2/s$, $Pr = 0.706$

Analysis

1) Without cover, we have free external convection from the surface of a sphere, $L_c = D_1 = 0.6\,m$

$$Ra_D = \frac{\beta\,(T_s - T_\infty)\, g\, D_1^3}{\nu^2} \cdot Pr \qquad \text{where } \beta = \frac{1}{T_f} = \frac{1}{328}$$

$$Ra_D = \frac{1/328\,(85-25)\cdot 9.2 \cdot 0.6^3}{(18.71\cdot 10^{-6})^2} \cdot 0.703 = 7.78\cdot 10^8$$

For $Ra_D \leq 10^{11}$ $\quad \overline{Nu}_D = \frac{\bar{h}\,D_1}{k} = 2 + \frac{0.589\,Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}} = 2 + \frac{0.589\,(7.78\cdot 10^8)^{1/4}}{\left[1 + (0.469/0.703)^{9/16}\right]^{4/9}} = 77.81$

$$\therefore\ \bar{h} = \frac{\overline{Nu}_D \cdot k}{D_1} = \frac{77.81 \cdot 0.0284}{0.6} = 3.68\ W/m^2K$$

$$\therefore\ q_1 = \bar{h}\,A\,(T_{s_1} - T_\infty) = 3.68 \cdot \pi (0.6)^2\,(85-25) = \underline{\underline{250\ W}}$$

2) With cover we have free convection in the enclosure and free convection from the outer surface of the cover. Since the system is at steady state

$$q_i = q_o = q_2 \quad \cdots \quad (1)$$

where: $$q_i = \frac{4\pi\, k_{eff}\,(T_{s_1} - T_{s_2})}{\left(\frac{2}{D_1} - \frac{2}{D_2}\right)} = 2\pi\, k_{eff}\,\frac{D_1 D_2}{(D_2 - D_1)}\,(T_{s_1} - T_{s_2}) \quad (2)$$

$$k_{eff} = k \cdot 0.74 \left(\frac{Pr}{0.861 + Pr}\right)^{1/4} Ra_s^{*\,1/4} \quad \cdots \quad (3)$$

$$Ra_s^* = \frac{(D_2 - D_1)}{2\,(D_2 D_1)^4}\,\frac{Ra_L}{\left(D_1^{-7/5} + D_o^{-7/5}\right)^5} \quad \cdots \quad (4)$$

$$Ra_L = \frac{1/T_r\,(T_{s_2} - T_{s_1})\, g\,[(D_2 - D_1)/2]^3}{\nu^2} \cdot Pr = \frac{1/328\,(85-45)\,9.8\,(D_2 - 0.6)}{8\cdot(19.71\cdot 10^{-6})^2} \cdot 0.702$$

$$\therefore\ Ra_L = 2.620\cdot 10^8\,(D_2 - 0.6)^3 \qquad \text{where } L = (D_2 - D_1)/2$$

$\Rightarrow$ sub in Eq. (4): $$Ra_s^* = \frac{(D_2 - 0.6)}{2\,D_2^4\,(0.6)^4}\,\frac{2.620\cdot 10^8\,(D_2 - 0.6)^3}{(0.6^{-7/5} + D_2^{-7/5})^5} = \frac{1.011\cdot 10^9\,(D_2 - 0.4)^4}{D_2^4\,(2.045 + D_2^{-7/5})^5}$$

$\Rightarrow$ Sub in Eq. (3) $$k_{eff} = 0.0291 \cdot 0.74 \left(\frac{0.702}{0.861 + 0.702}\right)^{1/4} \left[\frac{1.011\cdot 10^9\,(D_2 - 0.4)^4}{D_2^4\,(2.045 + D_2^{-7/5})^5}\right]^{1/4}$$

$$= 3.143\,\frac{(D_2 - 0.4)}{D_2\,(2.045 + D_2^{-7/5})^{5/4}}$$

$\Rightarrow$ Sub in Eq. (2) $$q_i = 2\pi \cdot 3.143\,\frac{\cancel{(D_2 - 0.4)} \cdot 40}{\cancel{D_2}\,(2.045 + D_2^{-7/5})^{5/4}} \cdot \frac{0.6 \cdot \cancel{D_2}}{\cancel{(D_2 - 0.4)}} = \frac{474}{(2.045 + D_2^{-7/5})^{5/4}}$$

$$q_o = \bar{h}\,\pi D_2^2\,(T_{s2} - T_\infty) \quad \ldots \quad (5)$$

Similarly to the case without cover, we have free external convection from the sphere, $L_c = D_2$

$$Ra_D = \frac{1/308\,(45-25)\cdot 9.8\cdot D_2^3}{(16.69\cdot 10^{-6})^2}\cdot 0.706 = 1.613\cdot 10^9\cdot D_2^3$$

$$\bar{h} = \frac{\overline{Nu}_D\cdot k}{D_2} = \left\{2 + \frac{0.589\,Ra_D^{1/4}}{[1+(0.469/Pr)^{9/16}]^{4/9}}\right\}\frac{k}{D_2} = \left\{2 + \frac{0.589\,(1.613\cdot 10^9 D_2^3)^{1/4}}{[1+(0.469/0.706)^{9/16}]^{4/9}}\right\}\frac{0.0269}{D_2}$$

$$= \left(2 + 91.03\,D_2^{3/4}\right)\frac{0.0269}{D_2}$$

Substituting into Eq. (5)

$$q_o = \left(2 + 91.03\,D_2^{3/4}\right)\frac{0.0269}{\cancel{D_2}}\cdot\pi D_2^{\cancel{2}}\,(45-25) = \left(2 + 91.03\,D_2^{3/4}\right)1.69\,D_2$$

$$\therefore\ q_o = 3.38\,D_2 + 153.86\,D_2^{7/4}$$

Substituting Equations for q_i and q_o into Eq.(1) gives

$$\frac{474}{(2.045 + D_2^{-7/5})^{5/4}} = 3.38\,D_2 + 153.86\,D_2^{7/4} \Rightarrow \text{solving numerically for } D_2 \text{ gives: } \underline{\underline{D_2 = 0.766\,m}}$$

$$\therefore\ q_2 = 3.38\cdot 0.766 + 153.86\cdot 0.766^{7/4} = \underline{\underline{99\ W}}$$

$\therefore$ Reduction in q due to the cover $= \dfrac{q_1 - q_2}{q_1}\cdot 100\% = \dfrac{250-99}{250}\cdot 100\% = \underline{\underline{60.4\%}}$

<u>Notes:</u> Knowing D_2 we can verify applicability of the approach

$$Ra_s^* = \frac{1.011\cdot 10^9\,(0.766-0.6)^4}{0.766^4\,(2.045 + 0.766^{-7/5})^5} = 4261 \Rightarrow \text{O.K. since } 10^2 < Ra_s^* < 10^4$$

$$k_{eff} = 3.143\,\frac{(0.766-0.6)}{0.766\,(2.045 + 0.766^{-7/5})^{5/4}} = 0.142\,\frac{W}{mK} > 0.0291\,\frac{W}{mK} \Rightarrow \text{O.K.}$$

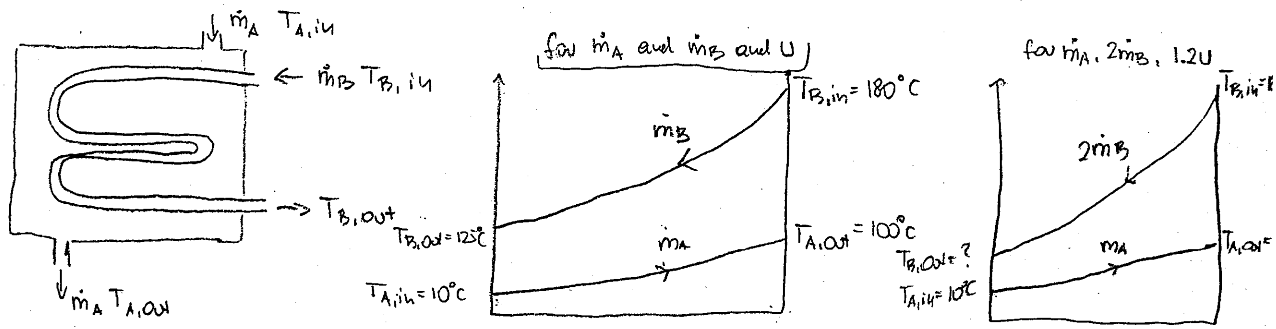
For external free convection from the cover $Ra_D = 1.613\cdot 10^9\,(0.766)^3 = 7.24\cdot 10^8 < 10^{11} \Rightarrow$ O.K.

Problem 2 - Solution

Known: one-shell-pass four-tube-pass exchanger to transfer heat from fluid B to fluid A

Unknown: Outlet temperature of stream A ($T_{A,out}$) when mass flow rate of fluid B is doubled and the overall heat transfer coefficient increases 20%

Schematic:



Assumptions:

1) Steady state process
2) No heat loss to surroundings
3) The same fluid properties in both runs
4) for $2\dot{m}_B$, $U = 1.2U$ for $\dot{m}_B$

Properties: No additional properties required

Analysis: Because of Assumption 4 R_c for $\dot{m}_B$ can be determined from R_c for $\dot{m}_B$

- Total energy balance for $\dot{m}_B$

$$\dot{m}_B c_{pB}(T_{B,in} - T_{B,out}) = \dot{m}_A c_{pA}(T_{A,out} - T_{A,in}) \iff C_B(T_{B,in} - T_{B,out}) = C_A(T_{A,out} - T_{A,in})$$

Substituting numbers: $C_A(100-10) = C_B(180-125) \Rightarrow \frac{C_A}{C_B} = \frac{55}{90} = 0.611 = R_c$ and $C_A = C_{min}$

- Determination of NTU for $\dot{m}_B$. From Table ~~8.3b~~ 11.4 for one-shell-pass four-tube-pass exchanger:

$$NTU = -(1+C_r^2)^{-1/2} \ln\left(\frac{E-1}{E+1}\right) \quad \text{where} \quad E = \frac{2/\varepsilon - (1+C_r)}{(1+C_r^2)^{1/2}} \quad \text{and} \quad \varepsilon = \frac{T_{A,out} - T_{A,in}}{T_{B,in} - T_{A,in}} = \frac{100-10}{180-10} = 0.529$$

$$\therefore E = \frac{2/0.529 - (1+0.611)}{(1+0.611^2)^{1/2}} = 1.859 \Rightarrow NTU = -(1+0.611^2)^{-1/2} \ln\left(\frac{1.859-1}{1.859+1}\right) = \underline{1.026}$$

- C_r when flow of B is doubled: for $2\dot{m}_B \Rightarrow 2C_B \Rightarrow C_r = \frac{1}{2}\, 0.611$ since C_A is unchanged
  $C_r = 0.3055$

- NTU when flow of B is doubled: $NTU = \frac{UA}{C_{min}} = \frac{UA}{C_A} = 1.2 \cdot 1.026 = 1.2312$ since U increase 20% while C_A remains the same

- ε for $2\dot{m}_B$, for $NTU = 1.2312$ and $C_r = 0.3055$: $\varepsilon = \varepsilon_1$ (Eq. 11.31a ~~Here 8~~ in Table ~~8.3a~~ 11.3)

$$\varepsilon = 2\left\{1 + C_r + (1+C_r^2)^{1/2}\,\frac{1+\exp[-NTU(1+C_r^2)^{1/2}]}{1-\exp[-NTU(1+C_r^2)^{1/2}]}\right\}^{-1} = 2\left\{1 + 0.3055 + (1+0.3055^2)^{1/2}\,\frac{1+\exp[-1.2312(1+0.3055^2)^{1/2}]}{1-\exp[-1.2312(1+0.3055^2)^{1/2}]}\right\}^{-1}$$

$$\therefore \varepsilon = 0.635 = \frac{T_{A,out} - 10}{180-10} \Rightarrow T_{A,out} = \underline{118\,^\circ C}$$ ans

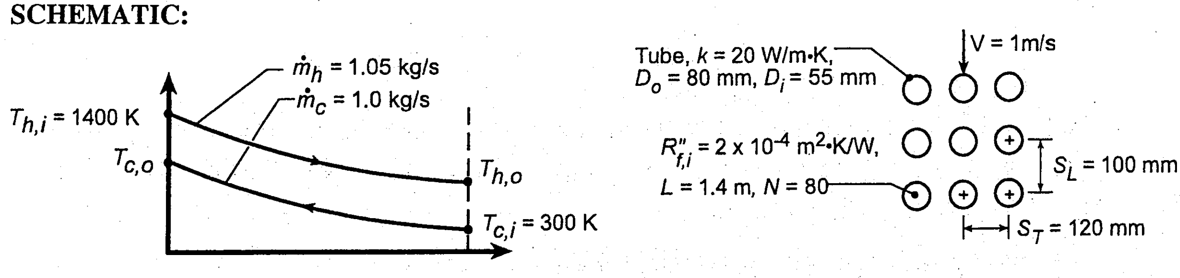
Problem 3

PROBLEM 11.60

KNOWN: Dimensions, configuration and material of a single-pass, cross-flow heat exchanger. Inlet conditions of inner and outer flow. Fouling factor of inner surface.

FIND: (a) Percent fuel savings for prescribed conditions, (b) Effect of UA on air outlet temperature and fuel savings.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings and potential and kinetic energy changes, (2) Air properties are those of atmospheric air at 300 K, (3) Gas properties are those of atmospheric air at 1400 K, (4) Tube wall temperature may be approximated as 800 K for treating variable property effects.

Note: Exact configuration is not given, ie N_T and N_L are not known. Students may assume any N_L and N_T as long as $N_L \cdot N_T = 80$

PROPERTIES: *Table A.4*, Air (1 atm, T = 300 K): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $c_p = 1007$ J/kg·K, k = 0.0263 W/m·K, Pr = 0.707; (T = 1400 K): $\mu = 530 \times 10^{-7}$ kg/s·m, $c_p = 1207$ J/kg·K, k = 0.091 W/m·K, Pr = 0.703; (T = 800 K): $\mu = 370 \times 10^{-7}$ kg/s·m, Pr = 0.709.

ANALYSIS: (a) With capacity rates of $C_c = \dot{m}_c c_{p,c} = 1\ \text{kg/s} \times 1007\ \text{J/kg·K} = 1007\ \text{W/K} = C_{min}$ and $C_h = \dot{m}_h c_{p,h} = 1.05\ \text{kg/s} \times 1207\ \text{J/kg·K} = 1267\ \text{W/K} = C_{max}$, $C_{min}/C_{max} = 0.795$. The overall coefficient is

$$\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{(2\pi kL)N} + \frac{1}{h_o A_o}.$$

For flow through a single tube,

$$Re_D = \frac{4\dot{m}_h}{N\pi D_i \mu} = \frac{4 \times 1.05\,\text{kg/s}}{80\pi(0.055\,\text{m})530 \times 10^{-7}\,\text{kg/s}\cdot\text{m}} = 5733.$$

Students may alternatively use Gliński's correlation $Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$ where f for smooth pipes is given by: $f = (0.79 \ln Re_D - 1.64)^{-2}$

Assuming fully developed turbulent flow throughout and using the Sieder-Tate correlation,

$$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3}(\mu/\mu_s)^{0.14} = 0.027(5733)^{4/5}(0.703)^{1/3}(530/370)^{0.14} = 25.6$$

$$h_i = Nu_D k/D_i = 25.6(0.091\,\text{W/m·K})/0.055\,\text{m} = 42.4\,\text{W/m}^2\cdot\text{K}.$$

For flow over the tube bank,

$$V_{max} = [S_T/(S_T - D_o)]V = [0.12\,\text{m}/(0.12 - 0.08)\,\text{m}]1\,\text{m/s} = 3\,\text{m/s}$$

$$Re_{D,max} = \frac{V_{max} D_o}{\nu} = \frac{3\,\text{m/s}(0.08\,\text{m})}{15.89 \times 10^{-6}\,\text{m}^2/\text{s}} = 15{,}100$$

From the Zhukauskas correlation for a tube bank, → Note: This correlation implies that $N_L \geq 20$ (e.g $N_L = 20, N_T = 4$) and the constants 0.63 and 0.27 came from Table 7.7. If $N_L < 20$, this equation requires a correction factor C_2 to be determined from Table 7.8.

$$\overline{Nu}_D = 0.27(15{,}100)^{0.63}(0.707)^{0.36}(0.707/0.709)^{1/4} = 102.3$$

$$\bar{h}_o = \overline{Nu}_D (k/D_o) = 102.3(0.0263\,\text{W/m·K})/0.08\,\text{m} = 33.6\,\text{W/m}^2\cdot\text{K}.$$

Hence, based on the inner surface, the *overall coefficient* is

Continued...

Alternatively, students may determine $\overline{Nu}_D$ from $\overline{Nu}_D = C_1 Re_{D,max}^m$ which is applicable for $N_L \geq 10$ and constants C_1 and m are determined based on $Re_{D,max}$ from Table 7.5.

PROBLEM 11.60 (Cont.)

$$\frac{1}{U_i} = \frac{1}{h_i} + R''_{f,i} + \frac{D_i \ln(D_o/D_i)}{2k} + \frac{D_i}{D_o h_o}$$

$$\frac{1}{U_i} = \left(0.0236 + 0.0002 + \frac{0.055 \ln(0.08/0.055)}{40} + \frac{0.055}{0.08 \times 33.6}\right) m^2 \cdot K/W$$

$$U_i = \left[(0.0236 + 0.0002 + 0.0005 + 0.0246) m^2 \cdot K/W\right]^{-1} = 22.3\, W/m^2 \cdot K.$$

Hence, $(UA)_i = U_i N \pi D_i L = 22.3\, W/m^2 \cdot K \times 80\pi (0.055\, m) 1.4\, m = 432\, W/K$. The number of transfer units is then NTU = UA/C_{min} = 432 W/K/1007 W/K = 0.429, and with $C_{mixed}/C_{unmixed} = C_c/C_h = C_{min}/C_{max}$ = 0.795, Fig. 11.19 yields $\varepsilon \approx 0.3$ or, from Eq. 11.35 a,

$$\varepsilon = 1 - \exp\left(-C_r^{-1}\{1 - \exp[-C_r \cdot NTU]\}\right) = 0.305.$$

→ Note, there is a mistake in the textbook, students may use $\varepsilon = \left(\frac{1}{C_r}\right)\left(1 - \exp\{-C_r [1 - \exp(-NTU)]\}\right)$

Hence, with

$$q_{max} = C_{min}(T_{h,i} - T_{c,i}) = 1007\, W/K (1100\, K) = 1.11 \times 10^6\, W$$

$$q = \varepsilon q_{max} = 0.305 \times 1.11 \times 10^6\, W = 337{,}800\, W$$

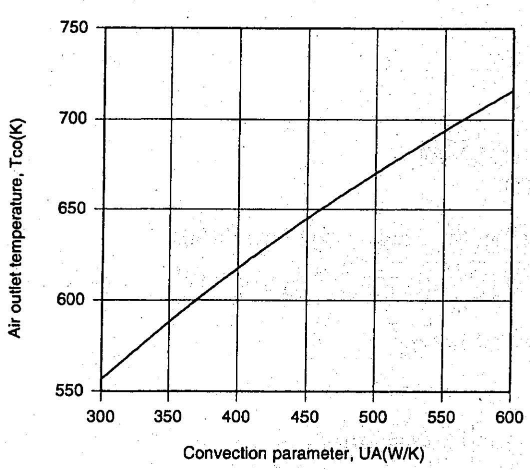
$$T_{c,o} = T_{c,i} + q/C_{min} = 300\, K + (337{,}800\, W/1007\, W/K) = 635\, K.$$

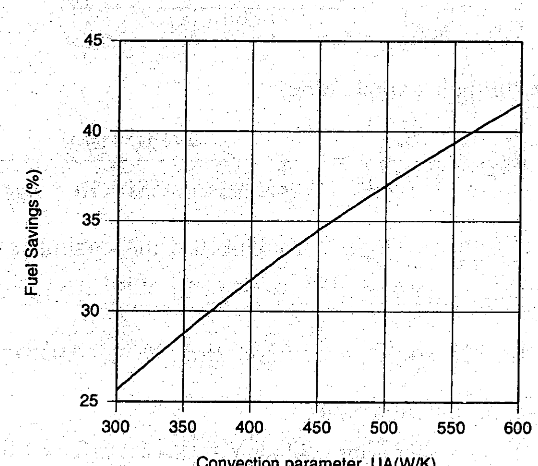
Hence,

$$\%\text{ fuel savings} \equiv FS = (\Delta T_c/10\, K) \times 1\% = (335\, K/10\, K) \times 1\% = 33.5\%$$ <

(b) Using the Heat Exchangers Toolpad of IHT to perform the parametric calculations, the following results are obtained.

NOT REQUIRED





Significant benefits are derived by increasing UA, with values of $T_{c,o}$ = 716 K and FS = 41.6% obtained for UA = 600 W/K. The major contributions to the total resistance are made by the inner and outer convection resistances. These contributions could be reduced by using extended surfaces on both the inner and outer surfaces.

COMMENTS: For part (a), properties of the flue gas should be evaluated at $(T_{h,i} + T_{h,o})/2$ and the calculations repeated. The Colburn equation yields

$$Nu_D = 0.023 Re_D^{4/5} Pr^{1/3} = 20.8$$

which is 19% less than the result of the Sieder-Tate correlation.