CHG 2314
HEAT TRANSFER

Professor: **B. Kruczek** **2005/01/14**

Assignment No. 1

1. The hot combustion gases of a furnace are separated from the ambient air and its surroundings, which are at $25^{o}C$, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m K and a surface emissivity of 0.7. Under the steady-state conditions an inner surface is at $350^{o}C$. Free convection heat transfer to the air adjoining the outer surface is 20 W/m^2 K. What is the brick outer surface temperature?

   To reduce the risk of burn injuries to operating personnel, the outer surface should be maintained at or below $65^{o}C$. What should be the minimum thickness of the brick wall to accomplish this goal?

2. Problem 1.38.

3. Problem 1.70.

Due Date: **Jan 21, 2005 at 4:00 p.m. in the assignment box.**

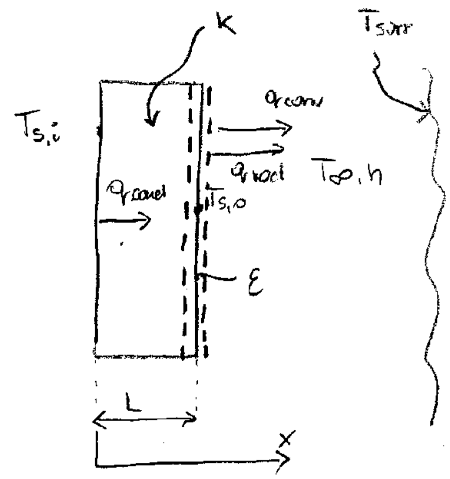
Problem 1

Known: Furnace wall at prescribed inner surface temperature, thickness, thermal conductivity, and surface emissivity and the outer surface exposed to known thermal environment

Unknown: i) Outer surface temperature of the wall ($T_{s,o}$)

ii) Thickness of the wall for the outer surface temp $T_{s,o} \leq 65°C$

Schematic:



$T_{s,i} = 350°C$

$T_{s,o} = ?$ or $65°C$ for (ii)

$T_\infty = T_{surr} = 25°C$ $h = 20\ W/m^2K$

$\varepsilon = 0.7$, $k = 1.2\ W/mK$

Assumptions:
1) 1-D conduction
2) Steady state conditions
3) The wall can be treated as small object in large isothermal surroundings
4) Constant k
5) $T_{s,i}$ is independent of wall thickness

Properties: All required properties are given in the problem statement

Analysis: • We the outer surface of the brick wall as the system for which we will apply the 1st law of Thermodynamics

• Since the surface has no volume, the 1st law of Thermodynamics becomes:

$$\dot{E}_{in} - \dot{E}_{out} = 0 \quad \ldots \quad (1)$$

• The outer surface receives the energy by conduction $\Rightarrow \dot{E}_{in} = q_{cond}$, and looses the energy by convection and radiation $\Rightarrow \dot{E}_{out} = q_{conv} + q_{rad}$

• Substituting appropriate rate equations into Eq(1) leads to

$$kA\frac{(T_{s,i} - T_{s,o})}{L} - hA(T_{s,o} - T_\infty) - \varepsilon\sigma A(T_{s,o}^4 - T_{surr}^4) = 0 \quad \Big| : A \Rightarrow \text{Since } A \text{ is independent of } A$$

$$k\frac{(T_{s,i} - T_{s,o})}{L} - h(T_{s,o} - T_\infty) - \varepsilon\sigma(T_{s,o}^4 - T_{surr}^4) = 0 \quad \ldots \quad (2)$$

$T_{s,o}$ is the only unknown in Eq (2). Substituting numbers into Eq (2):

$$1.2\,\frac{(350+273-T_{s,o})}{0.15} - 20\left(T_{s,o}-25-273\right) + 0.7\cdot 5.67\cdot 10^{-8}\left(T_{s,o}^4 - (273+25)^4\right) = 0$$

where $T_{s,o}$ is in $[K]$. NB: we must use absolute temperatures because of radiation exchange.

Solving for $T_{s,o}$ using solver (or by trial and error) gives: $T_{s,o} = 374\ K$ ◀

ii) To determine the thickness of the wall for $T_{s,o} \leq 65°C$ we set $T_{s,o} = 65°C$ in Eq (2) and solve for L. When $T_{s,o}$ is known, Eq (2) can be rearranged to get an explicit expression for L:

$$\Rightarrow L = \frac{k\,(T_{s,i}-T_{s,o})}{h\,(T_{s,o}-T_{\infty}) + \varepsilon\sigma\,(T_{s,o}^4 - T_{sur}^4)}$$

Substituting numbers: $$L = \frac{1.2\,(350-65)}{20\,(65-25) + 0.7\cdot 5.67\cdot 10^{-8}\left((65+273)^4 - (25+273)^4\right)}$$

$\therefore L = 0.340\ m$ ◀

Comments: (EXTRA) In part (i) heat loss $\Rightarrow q''_{cond} = \frac{k}{L}(T_{s,i}-T_{s,o}) = \frac{1.2}{0.15}(350-101) = 1992\ \frac{W}{m^2}$

In part (ii) heat loss $\Rightarrow q''_{cond} = \frac{1.2}{0.34}(350-65) = 1006\ \frac{W}{m^2}$

1) So increasing L reduces the heat loss, but the reduction of heat loss is not directly proportional to $\frac{1}{L}$ because as L increases $T_{s,i} - T_{s,o}$ increases.

2) In reality assumption 5 is not valid, i.e., as L increases and q_{cond} decreases, $T_{s,i}$ might increase. This means that the actual reduction in q_{loss} would be less that what is calculated above.

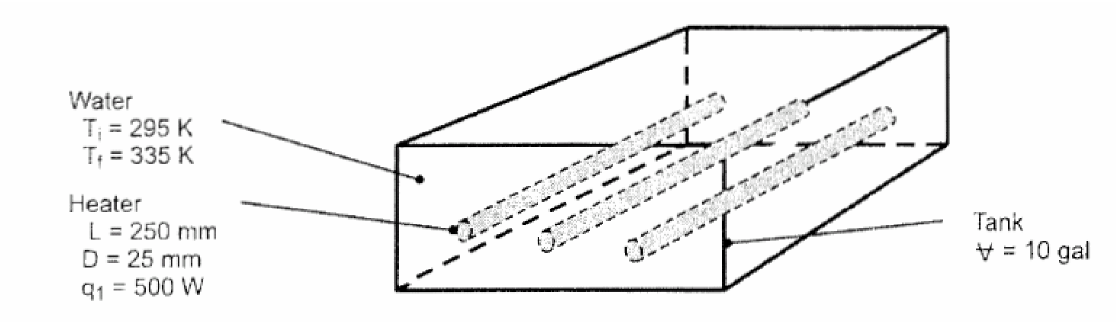
Problem 2
Known:

1. Initial temperature of water and tank volume
2. Power dissipation emissivity
3. Diameter and length of heater
4. Expressions for convection coefficient

Unknown:

1. Time to raise temperature of water to prescribed value
2. Heater temperature after activation
3. Heater temperature if activated in air

Schematic:



Assumptions:

1. Negligible heat loss from tank to surroundings
2. Water is well mixed during heating
3. Negligible changes in thermal energy storage for heaters
4. Constant properties
5. Surroundings afforded by tank wall are large relative to heaters

Analysis:
1. Applying conservation of energy to a closed system (the water), we have

$$\rho Vc\frac{dT}{dt} = 3q1$$

So the time needed to increase the temperature of water from T_i to T_f is:

$$\int_0^t dt = \frac{\rho Vc}{3q1}\int_{T_i}^{T_f} dT$$

$$t = \frac{\rho Vc}{3q1}(T_i - T_f)$$

$$= \frac{900kg/m^3 \times 10gal \times 3.79\times10^{-3}m^3/gal \times 4180J/kg.K}{3\times 500W}(335-295) = 4180s$$

2. After the heater was activated, the heat rate from each heater removed by convection is:
$q1 = Aq1" = Ah(T_s - T)$
Where:
A is the surface area of each tube in the water, = πDL

h is convection coefficient, $=370(T_s-T)^{1/3}$

Hence,

$$T_s = T + \left(\frac{q1}{370\pi DL}\right)^{3/4} = T + \left(\frac{500W}{370W/m^2.K^{4/3} \times \pi \times 0.025m \times 0.25m}\right)^{3/4}$$

$T_s = T + 24$

With water temperatures of T_i=295K and T_f=335K, the surface temperatures of tube will be $T_{s,i}$=319K and $T_{s,f}$=359K respectively.

3. The heat rate in air (include convection and radiation) is

$$q1 = \pi DL\left[0.70(T_s - T_\infty)^{4/3} + \varepsilon\sigma(T_s^4 - T_{sur}^4)\right]$$

Where

$T_\infty = T_{sur}$

D, L is the diameter and length of the tube

Substituting the values of q1, D, L, $T_\infty = T_{sur}$ and ε

T_s=830K

Problem 3

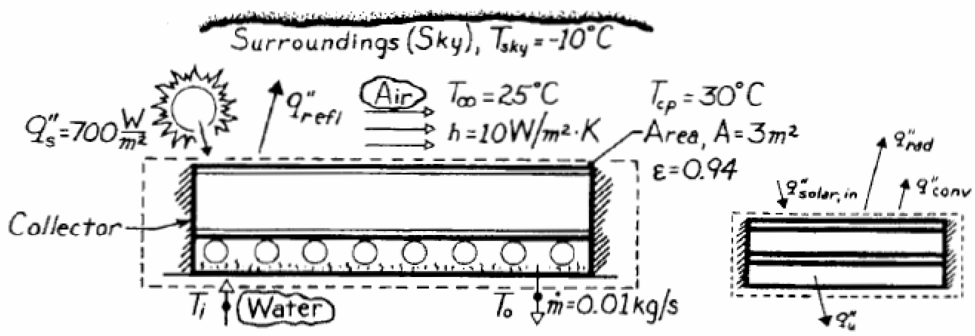
Known:

Solar collector designed to heat water operating under prescribed solar irradiation and loss conditions

Unknown:

1. Useful heat q_u''
2. Temperature rise of the water flow
3. Collector efficiency

Schematic:



Assumptions:

1. Steady-state conditions
2. No heat losses out sides or back of collector
3. Collector area is small compared to sky surroundings

Properties: water at 300K, c_p = 4179J/kg.K

Analysis:

1. Defining the collector as the control volume, we have

$q''_{solar} - q''_{rad} - q''_{conv} - q''_{u} = 0$

$q''_{u} = q''_{solar} - q''_{rad} - q''_{conv}$

Where:

$q''_{solar} = 0.9q_s''$

$q''_{rad} = \varepsilon\sigma(T_{cp}^4 - T_{sky}^4)$

$q''_{conv} = h(T_s - T_\infty)$

Hence,

$$q_u'' = 0.9q_s'' - \varepsilon\sigma(T_{cp}^4 - T_{sky}^4) - h(T_s - T_\infty)$$

$$= 0.9 \times 700\frac{W}{m^2} - 0.94 \times 5.67 \times 10^{-8}\frac{W}{m^2 \cdot K^4}\left(303^4 - 263^4\right)K^4 - 10\frac{W}{m^2 \cdot K}(30 - 25)$$

$$= 386\frac{W}{m^2}$$

2. Defining a control volume about the water tubing, the useful heat causes a temperature rise of the flowing water.

$$q_u''A = \dot{m}c_p(T_i - T_o)$$

$$(T_i - T_o) = \frac{386W/m^2 \times 3m^2}{0.01kg/s \times 4179J/kg.K} = 27.7$$

3. The efficiency is

$$\eta = \frac{q_u''}{q_s''} \times 100 = 55\%$$