CHG 2314
HEAT TRANSFER

<u>Professor:</u> **B. Kruczek** **2005/01/21**

Assignment No. 2

1. Problem 2.6. *Hint:* The sign of the second derivative of a function provides useful information about the shape of the function.
2. Problem 2.19.
3. Problem 2.26.

<u>Due Date:</u> **Jan 28, 2005 at 4:00 p.m. in the assignment box.**

PROBLEM 2.6

KNOWN: Temperature dependence of the thermal conductivity, k(T), for heat transfer through a plane wall.

FIND: Effect of k(T) on temperature distribution, T(x).

ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: From Fourier's law and the form of k(T),

$$q''_x = -k\,\frac{dT}{dx} = -(k_o + aT)\frac{dT}{dx}. \qquad (1)$$

The shape of the temperature distribution may be inferred from knowledge of $d^2T/dx^2 = d(dT/dx)/dx$. Since q''_x is independent of x for the prescribed conditions,

$$\frac{dq''_x}{dx} = -\frac{d}{dx}\left[(k_o + aT)\frac{dT}{dx}\right] = 0$$

$$-(k_o + aT)\frac{d^2T}{dx^2} - a\left[\frac{dT}{dx}\right]^2 = 0.$$

Hence,

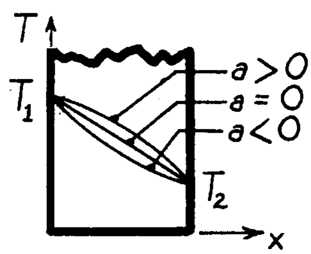
$$\frac{d^2T}{dx^2} = \frac{-a}{k_o + aT}\left[\frac{dT}{dx}\right]^2 \qquad \text{where} \begin{cases} k_o + aT = k > 0 \\ \left[\dfrac{dT}{dx}\right]^2 > 0 \end{cases}$$

from which it follows that for

$a > 0$: $d^2T/dx^2 < 0$

$a = 0$: $d^2T/dx^2 = 0$

$a < 0$: $d^2T/dx^2 > 0$.



COMMENTS: The shape of the distribution could also be inferred from Eq. (1). Since T decreases with increasing x,

$a > 0$: k decreases with increasing x => | dT/dx | increases with increasing x

$a = 0$: $k = k_o$ => dT/dx is constant

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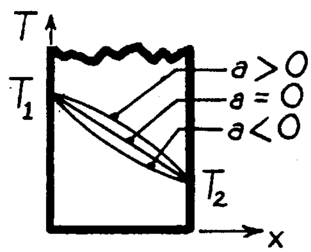

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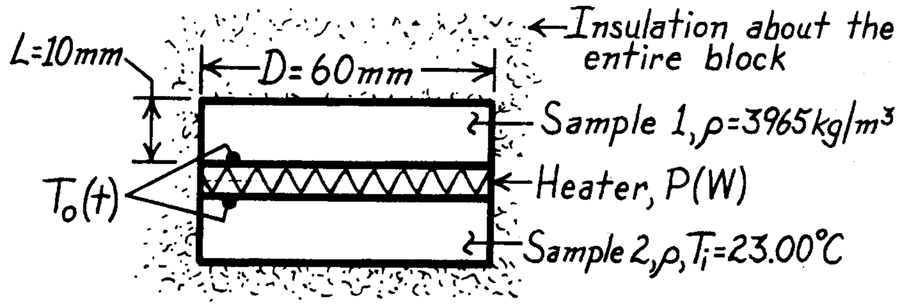
PROBLEM 2.19

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q_o'' for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

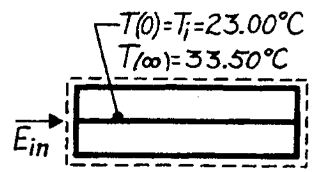
FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:



ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the Case A power condition and the final temperature T_f by Case B. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\ \text{W} \times 120\ \text{s}}{2 \times 3965\ \text{kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\ \text{m}\left[33.50 - 23.00\right]^\circ\text{C}}$$

$$c_p = 765\ \text{J/kg}\cdot\text{K}$$ <

where $M = \rho V = 2\rho(\pi D^2/4)L$ is the mass of both samples. For Case A, the transient thermal response of the heater is given by

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PROBLEM 2.19 (Cont.)

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\ \mathrm{s}}{\pi \times 3965\ \mathrm{kg/m^3} \times 765\ \mathrm{J/kg \cdot K}}\left[\frac{2 \times 2653\ \mathrm{W/m^2}}{(24.57\ -\ 23.00)^\circ\mathrm{C}}\right]^2 = 36.0\ \mathrm{W/m \cdot K}$$ <

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2(\pi D^2/4)} = \frac{15\ \mathrm{W}}{2(\pi \times 0.060^2/4)\mathrm{m}^2} = 2653\ \mathrm{W/m^2}.$$

With the following properties now known,

$\rho = 3965\ \mathrm{kg/m^3}$ $\qquad c_p = 765\ \mathrm{J/kg \cdot K}$ $\qquad k = 36\ \mathrm{W/m \cdot K}$

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. <

on class example

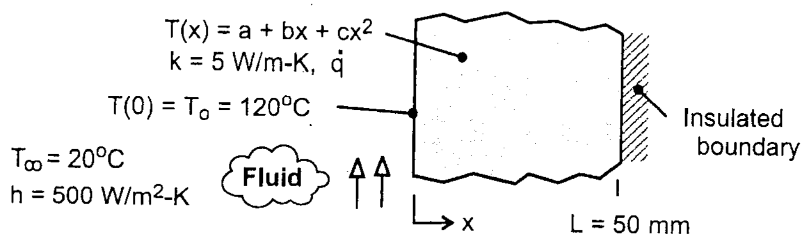
assignment ?

PROBLEM 2.26

KNOWN: Steady-state conduction with uniform internal energy generation in a plane wall; temperature distribution has quadratic form. Surface at x=0 is prescribed and boundary at x = L is insulated.

FIND: (a) Calculate the internal energy generation rate, $\dot{q}$, by applying an overall energy balance to the wall, (b) Determine the coefficients a, b, and c, by applying the boundary conditions to the prescribed form of the temperature distribution; plot the temperature distribution and label as Case 1, (c) Determine new values for a, b, and c for conditions when the convection coefficient is halved, and the generation rate remains unchanged; plot the temperature distribution and label as Case 2; (d) Determine new values for a, b, and c for conditions when the generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ W/m^2 \cdot K$); plot the temperature distribution and label as Case 3.

SCHEMATIC:



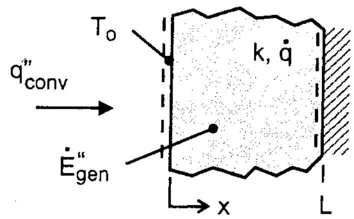
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties and uniform internal generation, and (3) Boundary at x = L is adiabatic.

ANALYSIS: (a) The internal energy generation rate can be calculated from an overall energy balance on the wall as shown in the schematic below.

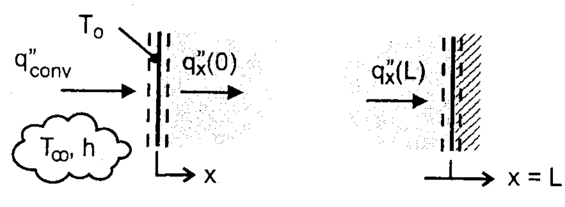
$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \qquad \text{where} \qquad \dot{E}''_{in} = q''_{conv}$$

$$h(T_\infty - T_o) + \dot{q}L = 0 \tag{1}$$

$$\dot{q} = -h(T_\infty - T_o)/L = -500\ W/m^2 \cdot K(20-120)°C/0.050\ m = 1.0 \times 10^6\ W/m^3 \quad <$$



(a) Overall energy balance



(b) Surface energy balances

(b) The coefficients of the temperature distribution, $T(x) = a + bx + cx^2$, can be evaluated by applying the boundary conditions at x = 0 and x = L. See Table 2.1 for representation of the boundary conditions, and the schematic above for the relevant surface energy balances.

Boundary condition at x = 0, convection surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \qquad \text{where} \qquad q''_x(0) = -k\frac{dT}{dx}\bigg)_{x=0}$$

$$h(T_\infty - T_o) - \left[-k(0 + b + 2cx)_{x=0}\right] = 0$$

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$$b = -h(T_\infty - T_o)/k = -500\,W/m^2\cdot K(20-120)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Boundary condition at x = L, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x(L) = 0 \qquad \text{where} \qquad q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L}$$

$$k[0 + b + 2cx]_{x=L} = 0 \qquad (3)$$

$$c = -b/2L = -1.0\times10^4\,K/m/(2\times0.050m) = -1.0\times10^5\,K/m^2 \quad <$$

Since the surface temperature at x = 0 is known, $T(0) = T_o = 120°C$, find

$$T(0) = 120°C = a + b\cdot 0 + c\cdot 0 \qquad \text{or} \qquad a = 120°C \qquad (4) \quad <$$

Using the foregoing coefficients with the expression for T(x) in the Workspace of IHT, the temperature distribution can be determined and is plotted as Case 1 in the graph below.

(c) Consider Case 2 when the convection coefficient is halved, $h_2 = h/2 = 250\ W/m^2\cdot K$, $\dot{q} = 1\times10^6\ W/m^3$ and other parameters remain unchanged except that $T_o \neq 120°C$. We can determine a, b, and c for the temperature distribution expression by repeating the analyses of parts (a) and (b).

Overall energy balance on the wall, see Eqs. (1,4)

$$a = T_o = \dot{q}L/h + T_\infty = 1\times10^6\,W/m^3\times0.050m/250\,W/m^2\cdot K + 20°C = 220°C \quad <$$

Surface energy balance at x = 0, see Eq. (2)

$$b = -h(T_\infty - T_o)/k = -250\,W/m^2\cdot K(20-220)°C/5\,W/m\cdot K = 1.0\times10^4\,K/m \quad <$$

Surface energy balance at x = L, see Eq. (3)

$$c = -b/2L = -1.0\times10^4\,K/m/(2\times0.050m) = -1.0\times10^5\,K/m^2 \quad <$$

The new temperature distribution, $T_2(x)$, is plotted as Case 2 below.

(d) Consider Case 3 when the internal energy volumetric generation rate is doubled, $\dot{q}_3 = 2\dot{q} = 2\times10^6\,W/m^3$, $h = 500\ W/m^2\cdot K$, and other parameters remain unchanged except that $T_o \neq 120°C$. Following the same analysis as part (c), the coefficients for the new temperature distribution, T (x), are

$$a = 220°C \qquad b = 2\times10^4\,K/m \qquad c = -2\times10^5\,K/m^2 \quad <$$

and the distribution is plotted as Case 3 below.

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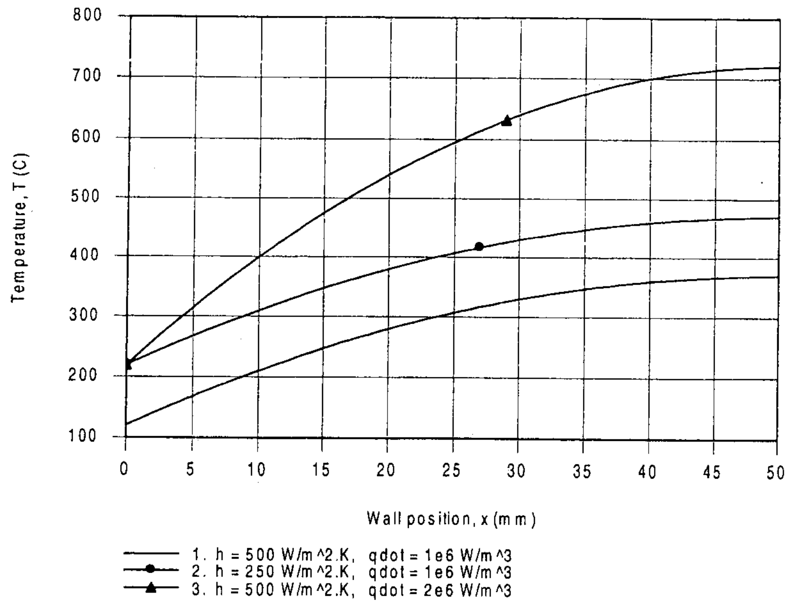

COMMENTS: Note the following features in the family of temperature distributions plotted above. The temperature gradients at x = L are zero since the boundary is insulated (adiabatic) for all cases. The shapes of the distributions are all quadratic, with the maximum temperatures at the insulated boundary.

By halving the convection coefficient for Case 2, we expect the surface temperature T_o to increase relative to the Case 1 value, since the same heat flux is removed from the wall $(\dot{q}L)$ but the convection resistance has increased.

By doubling the generation rate for Case 3, we expect the surface temperature T_o to increase relative to the Case 1 value, since double the amount of heat flux is removed from the wall $(2\dot{q}L)$.

Can you explain why T_o is the same for Cases 2 and 3, yet the insulated boundary temperatures are quite different? Can you explain the relative magnitudes of T(L) for the three cases?