Université d'Ottawa · University of Ottawa

Faculté de génie
Génie chimique

Faculty of Engineering
Chemical Engineering

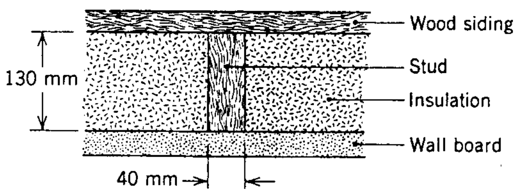
CHG 2314
HEAT TRANSFER

Professor: **B. Kruczek** **2005/01/28**

Assignment No. 3

1. Modified problem 3.15.

   Consider a composite wall that includes an 8-mm-thick hardwood siding, 40 mm by 130 mm hardwood studs on 0.65-m centers with glass fiber insulation (paper faced, 28 kg/m^3), and a 12-mm layer of gypsum (vermiculite) wall board.



   What is the thermal resistance associated with a wall that is 2.5 m high and 6.5 m wide (having 10 studs, each 2.5 m high) assuming that,

   a) surfaces normal to the direction of heat flow are isothermal?
   b) surfaces parallel to the direction of heat flow are adiabatic?

   Which of the calculated values is more realistic and why?

2. Problem 3.40.

   This problem involves solving a nonlinear equation. You may use the software which came along with your textbook, or write your own program using Goal Seek or Solver available in MS Excel.

3. Two identical spherical reactors are insulated with different materials. The first reactor is insulated with a 10 cm layer of fused silica (k_1 = 1.38 W/m K). The inner and outer surface temperatures of the silica insulation are 85°C and 35°C, respectively. The second reactor is covered with an 8 cm thick layer of borosilicate (k_2 = 1.09 W/m K), and the temperature drop across the borosilicate insulation is

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55°C. If the heat losses from both reactors are identical ($q_1 = q_2$), what is the outer diameter of the reactors?

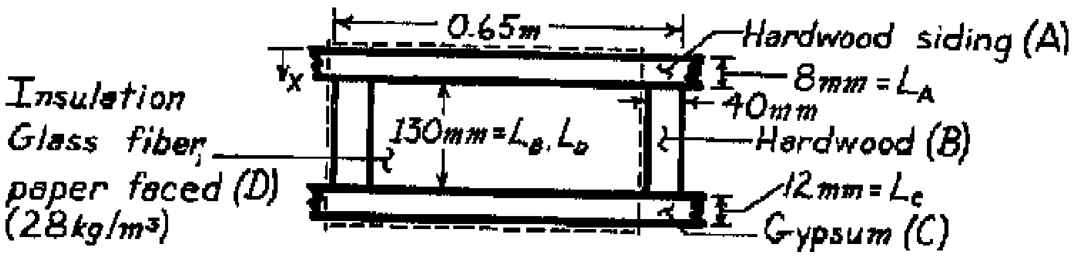
If both reactors are in the same environment ($T_{\infty 1} = T_{\infty 2} = 25°C$) with the same outside heat transfer coefficient ($h_{o1} = h_{o2}$), what are the inner and outer surface temperatures of the borosilicate insulation?

Due Date: **Feb. 4, 2005 at 4:00 p.m. in the assignment box.**

Problem 1
Known:
Dimensions and materials of the composition wall, 10 studs each with 2.5m high

Unknown:
1. Thermal resistance associate with wall when surfaces normal to the direction of heat flow are isothermal
2. Thermal resistance associate with wall when surfaces parallel to the direction of heat flow are adiabatic

Schematic:



Assumptions:
1. Steady state conditions
2. Constant properties
3. Negligible contact resistance
4. Temperature of composite depends only on x (surfaces normal to x are isothermal)

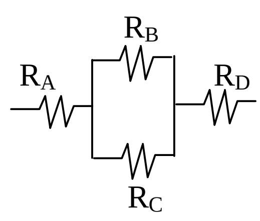
Properties:

k_A=0.094W/m.K
k_B=0.16W/m.K
k_C=0.038W/m.K
k_D=0.17W/m.K

These values can be found from table A3

Analysis:
a) Surfaces normal to the direction of heat flow are isothermal

Thermal circuit of a single unit of the wall:



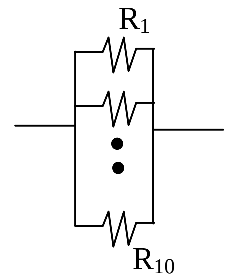
$$R_{element} = R_A + \frac{1}{1/R_B + 1/R_C} + R_D$$
$$= 0.0524 + \frac{1}{1/8.125 + 1/2.243} + 0.0434$$
$$= 1.854 k/W$$

Where:

$$R_A = \frac{L_A}{k_A A_A} = \frac{0.008}{0.094 * 2.5 * 0.65} = 0.0524 k/W$$

$$R_B = \frac{L_B}{k_B A_B} = \frac{0.13}{0.16*2.5*0.04} = 8.125k/W$$
$$R_C = \frac{L_C}{k_C A_C} = \frac{0.13}{0.038*2.5*0.61} = 2.243k/W$$
$$R_D = \frac{L_D}{k_D A_D} = \frac{0.012}{0.17*2.5*0.65} = 0.0434k/W$$

Thermal circuit of the entire wall



Where:

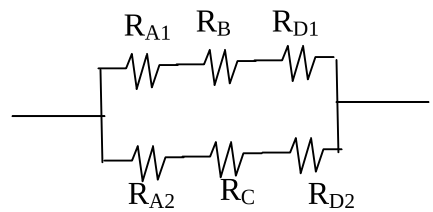
R_1=R_2=…R_{10}=1.854k/W

$$\frac{1}{R_{total}} = \sum_{i=1}^{10} 1/R_i = 10/1.854$$

R_{total}=0.1854k/W

b) Surfaces parallel to the direction of heat flow are adiabatic

Thermal circuit of a single element



Where:

$$R_{A1} = \frac{0.008}{0.094*2.5*0.04} = 0.851k/W$$
$$R_{A2} = \frac{0.008}{0.094*2.5*0.61} = 0.056k/W$$
$$R_{D1} = \frac{0.012}{0.17*2.5*0.04} = 0.706k/W$$
$$R_{D2} = \frac{0.012}{0.17*2.5*0.61} = 0.046k/W$$

$$\frac{1}{R_{element}} = \frac{1}{R_{A1}+R_B+R_{D1}} + \frac{1}{R_{A2}+R_C+R_{D2}}$$
$$= \frac{1}{9.682} + \frac{1}{2.345}$$
$$= 1.888k/W$$

R_{total}=10/$R_{element}$ = 0.1888k/W

Comments:

Forcing 2-D conduction into 1-D conduction problem, which is the case for both thermal circuits, overestimates the heat flow. Thus underestimates the resistance. This means that the arrangement yielding the larger resistance is more realistic.

Since R_{tot} for (b) = 0.1888K/W > R_{tot} for (a) = 0.1854K/W, the approximation by the thermal circuit (b), which assumes surfaces parallel to the direction of heat flow are adiabatic, is more realistic.

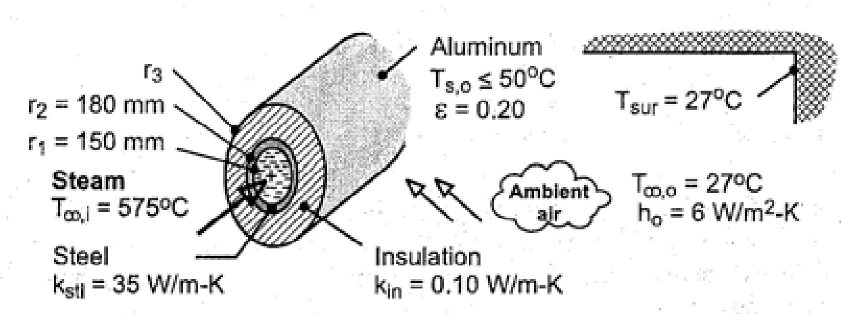
Problem 2
Known:

- Diameter, wall thickness thermal conductivity of steel tubes
- Temperature of stream flowing through the tubes
- Thermal conductivity of insulation and emissivity of aluminum sheath
- Temperature of ambient air and surroundings
- Convection coefficient at outer surface and maximum allowable surface temperature

Unknown:

1. Minimum required insulation thickness and corresponding heat loss per unit length
2. Effect of insulation thickness on outer surface temperature and heat loss

Schematic:



Assumptions:

1. Steady state
2. One-dimensional radial conduction
3. Negligible contact resistances at the material interfaces
4. Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$)
5. Negligible conduction resistance for aluminum sheath
6. Constant properties
7. Small object in large surroundings

Analysis:

a) Applying energy balance at the outer surface for a unit length of pipe:

$q' = q'_{conv,o} + q'_{rad}$ (1) (Here, *q'* is heat flow per unit length, W/m)

Where:

$q'_{conv,o} = 2\pi r_3 h_o (T_{s,o} - T_{\infty,o})$

$q'_{rad} = 2\pi r_3 \varepsilon\sigma (T_{s,o}^4 - T_{sur}^4)$

$$q' = \frac{T_{s,i} - T_{s,o}}{R'_{cond,st} + R'_{cond,ins}}$$

$R'_{cond,st} = \ln(r_2 / r_1) / 2\pi k_{st}$

$R'_{cond,ins} = \ln(r_3 / r_2) / 2\pi k_{ins}$

Substituting above into equation (1)

$$\frac{(T_{s,i} - T_{s,o})}{\frac{\ln(r_2/r_1)}{2\pi k_{st}} + \frac{\ln(r_3/r_2)}{2\pi k_{ins}}} = 2\pi r_3\left[h_o(T_{s,o} - T_{\infty,o}) + \varepsilon\sigma(T_{s,o}^4 - T_{sur}^4)\right]$$

$$\frac{2\pi(848-323)}{\frac{\ln(0.18/0.15)}{35} + \frac{\ln(r_3/0.18)}{0.1}} = 2\pi r_3\left[6*(323-300) + 0.2*5.67*10^{-8}(323^4 - 300^4)\right]$$

Solving for r_3

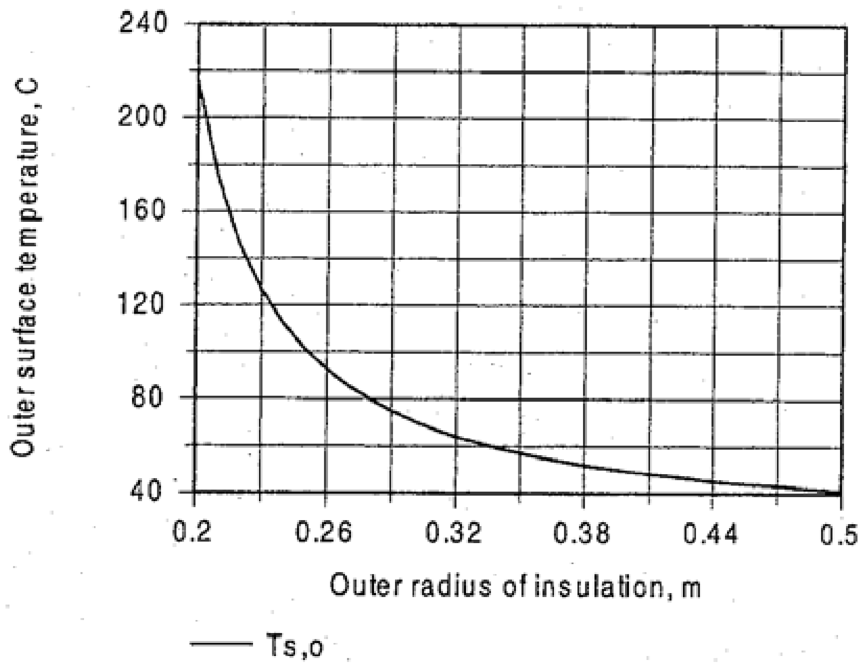
r_3=0.394m

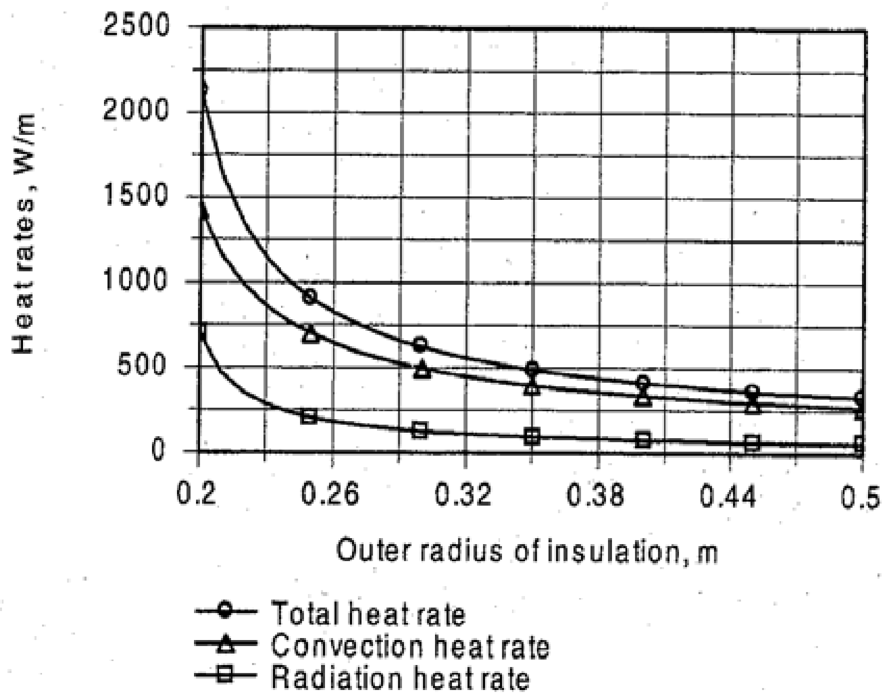
Then thickness of insulation $t_{ins} = r_3 - r_2 = 0.214$m

The heat rate is then

$$q' = \frac{2\pi(848-323)}{\frac{\ln(0.18/0.15)}{35} + \frac{\ln(0.394/0.18)}{0.1}} = 420W/m$$

b) The effects of r_3 on $T_{s,o}$ and q' are shown below.





Beyond $r_3 \approx 0.40$m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

Problem 3

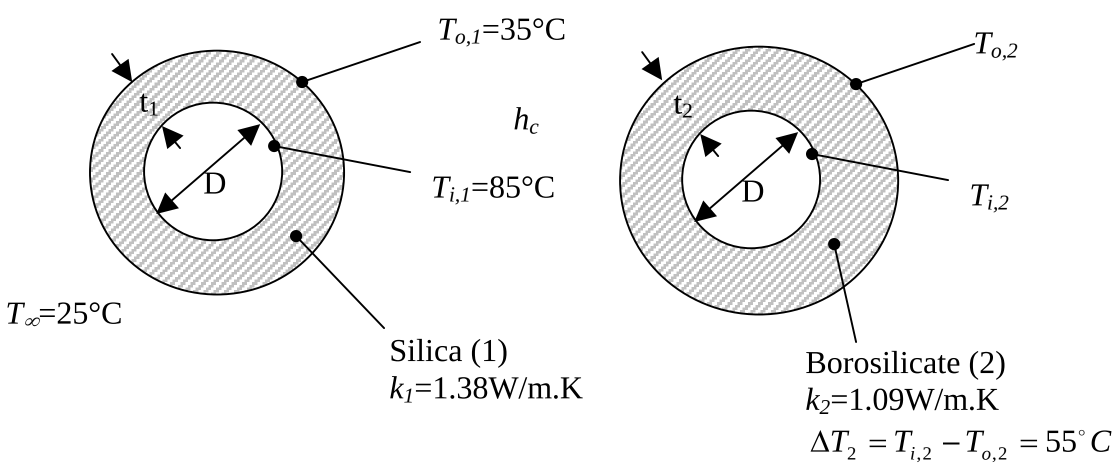
Known:

- Thermal conductivity and thickness of two insulating materials
- Temperature drop across the insulating materials

Unknown:

1. Outer diameter of the reactors
2. Inner and outer surface temperatures of the borosilicate insulation

Schematic:



Assumptions:

1. Steady state conditions
2. 1-D conduction (in radial direction)
3. Constant thermal conductivities
4. Same external environment for the insulated reactors
5. Same heat loses q_1=q_2
6. The same outside heat transfer coefficients, $h_{o1}=h_{o2}=h_c$

Analysis:

a) The heat rate across spherical shell is given by:

$$q = \frac{4\pi k(T_1 - T_2)}{\frac{1}{r_1} - \frac{1}{r_2}} \qquad (1)$$

Due to the same heat loss of both reactors

$$q_1 = q_2 \Rightarrow \frac{4\pi k_1 \Delta T_1}{\frac{1}{r_1} - \frac{1}{r_1 + t_1}} = \frac{4\pi k_2 \Delta T_2}{\frac{1}{r_1} - \frac{1}{r_1 + t_2}} \qquad (2)$$

Rearrange (2), we have

$$r_1 = \frac{t_1 t_2 (k_2 \Delta T_2 - k_1 \Delta T_1)}{k_1 \Delta T_1 t_2 - k_2 \Delta T_2 t_1}$$

$$= \frac{0.1 * 0.02(1.03 * 55 - 1.38 * 50)}{1.38 * 50 * 0.08 - 1.08 * 55 * 0.1}$$

=**0.152m**

D = 2r_1 = 0.304m

b) At steady state, $q_{conv,1} = q_{conv,2}$

$$h_c 4\pi (r_1 + t_1)^2 (T_{o,1} - T_\infty) = h_c 4\pi (r_1 + t_2)^2 (T_{o,2} - T_\infty)$$

Both side of above equation can be divided by h_c and the only unknown is $T_{o,2}$, so,

$$(0.152 + 0.1)^2 (35 - 25) = (0.152 + 0.08)^2 (T_{o,2} - 25)$$

$T_{o,2}$ = **36.8°C**

$T_{i,2} = \Delta T + T_{o,2} = 55 + 36.8 =$ **91.8°C**