Problem 3.73

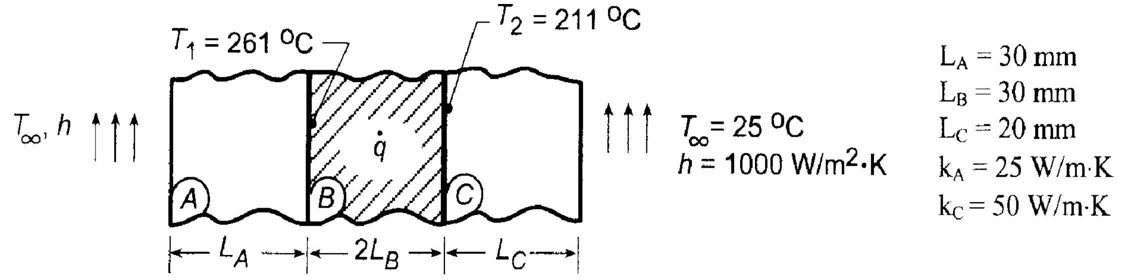
Known:

Composite wall with outer surfaces exposed to convection process while the inner wall experiences uniform heat generation

Unknown:

Volumetric heat generation and thermal conductivity for material B required for special conditions

Schematic:

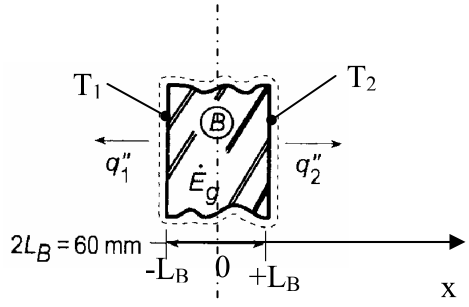


Assumptions:

1. Steady-state, one-dimensional heat transfer
2. Negligible contact resistance at interfaces
3. Uniform generation in B, zero in A and C
4. Constant properties

Analysis:

From an energy balance on wall B,

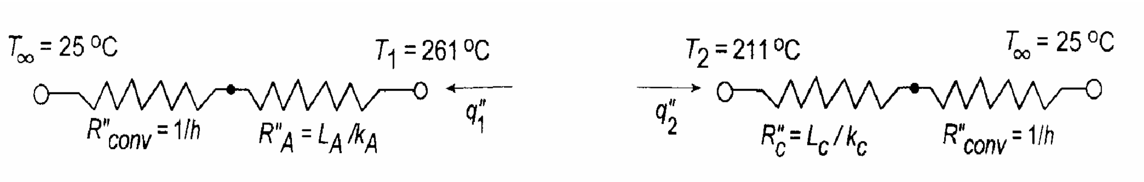


$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$-q_1'' - q_2'' + 2\dot{q}_B L_B = 0$$

$$\dot{q}_B = (q_1'' + q_2'')/2L_B \qquad (1)$$

To determine the heat fluxes, construct thermal circuits for A and C:



$$q_1'' = \frac{(T_1 - T_\infty)}{1/h + L_A/k_A} = \frac{(261-25)}{1/1000 + 0.03/25} = 107{,}273 W/m^2$$

$$q_2'' = \frac{(T_2 - T_\infty)}{L_C/k_C + 1/h} = \frac{(211-25)}{0.02/50 + 1/1000} = 132{,}857 W/m^2$$

Using the values for q_1'' and q_2'' in equation (1), we find:

$$\dot{q}_B = 4.00*10^6 W/m^3$$

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1x + C_2 \qquad q_x''(x) = -k_B\frac{dT}{dx} = -k_B\left[-\frac{\dot{q}_B}{k_B}x + C_1\right]$$

There are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions

$$T(-L_B) = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1L_B + C_2 = T_1 \qquad \text{Where } T_1\text{=261°C}$$

$$T(+L_B) = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1L_B + C_2 = T_2 \qquad \text{Where } T_2\text{=211°C}$$

$$q_x''(-L_B) = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] = -q_1'' \qquad \text{Where } q_1''\text{=107273W/m}^2$$

Solving for these equations simultaneously with $\dot{q}_B = 4.00*10^6 W/m^3$

k_B=15.3W/m.K

Problem 3.92

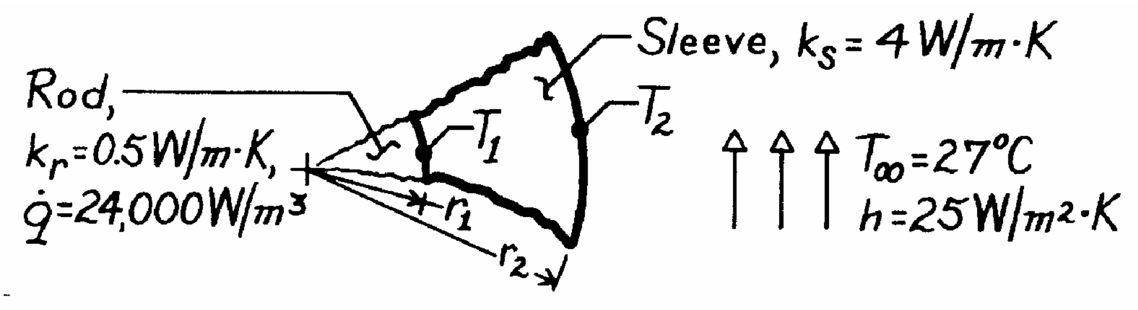
Known:

Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection

Unknown:

1. Temperature at the interface between rod and sleeve and on the outer surface
2. Temperature at center of rod

Schematic:

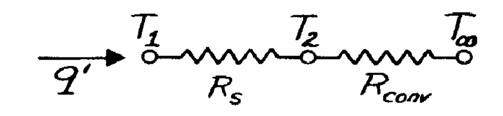


Assumptions:

1. One-dimensional radial conduction in rod and sleeve
2. Steady state conditions
3. Uniform volumetric generation in rod
4. Negligible contact resistance between rod and sleeve
5. Constant properties

Analysis:

a) Construct a thermal circuit for the sleeve:



Where

$$q' = \dot{E}'_{gen} = \dot{q}\pi r_1^2 = 24{,}000 \times \pi \times 0.1^2 = 754.0 W/m$$

$$R_S = \frac{\ln(r_2/r_1)}{2\pi k_S} = \frac{\ln(0.4/0.2)}{2\pi \times 4} = 2.758 \times 10^{-2} m.K/W$$

$$R_{conv} = \frac{1}{h\pi D_2} = \frac{1}{25\pi \times 0.4} = 3.183 \times 10^{-2} m.K/W$$

The rate equation can be written as:

$$q' = \frac{T_1 - T_\infty}{R_S + R_{conv}} = \frac{T_2 - T_\infty}{R_{conv}}$$

$$T_1 = T_\infty + q'(R_S + R_{conv}) = 27 + 754 \times (2.758 \times 10^{-2} + 3.183 \times 10^{-2}) = 71.8\,°\mathrm{C}$$
$$T_2 = T_\infty + q' R_{conv} = 27 + 754 \times 3.183 \times 10^{-2} = 51.0\,°\mathrm{C}$$

b) The temperature at the center of the rod is
For 1-D conduction in cylinder with uniform heat generation, temperature distribution is given by:

$$T(r) = \frac{\dot{q} r_1^2}{4k_r}\left(1 - \left(\frac{r}{r_1}\right)^2\right) + T_1$$

at r=0, we have

$$T(0) = T_0 = \frac{\dot{q} r_1^2}{4k_r} + T_1 = \frac{24{,}000 \times 0.1^2}{4 \times 0.5} + 71.8 = 192\,°\mathrm{C}$$

c) To minimize the temperature in the center, since we have:

$$T(0) = T_0 = \frac{\dot{q} r_1^2}{4k_r} + T_1$$

Where $\dot{q}$, r_1 and k_r are unchanged, so T_1 must be minimized.

Since, $T_1 = T_\infty + q'(R_S + R_{conv})$ and q' is unchanged.

In order to minimize T_1, R_S+R_{conv} must be minimized. Assuming that h is independent of r_2, then R_S+R_{conv} is minimized when $r_2 = r_{cr} = k_s/h = 4/25 = 0.16m$

To minimize T_0, the thickness of the sleeve should be decreased from *0.2-0.1=0.1m* to *0.16-0.1=0.06m*

d) If $h = 15.81 D^{-0.5}$, then the expression for r_2 will change

$$R_{tot} = \frac{\ln(r_2/r_1)}{2\pi k} + \frac{1}{15.81(2r_2)^{-0.5} \cdot 2\pi r_2}$$

$$\frac{dR_{tot}}{dr_2} = \frac{1}{2\pi k r_2} - \frac{0.5}{11.179 r_2^{1.5} \cdot 2\pi} = 0$$

$$\Rightarrow r_2 = r_{cr} = \left(\frac{0.5k}{11.179}\right)^2 = 0.032m$$

Since $r_2 < r_1$, the sleeve should be removed.

For $r_2 = r_1 = 0.1$, $h = 15.81(0.2)^{-0.5}$=35.35W/m$_2$.K

T_1=27+754/(35.35*2π*0.1)=60.9°C

T_0=192-(71.8-60.9)=181.1°C

Problem 3

Known:

Dimensions and properties of pin fin A, dimensions (except length of pin fin B) and properties of pin fin B. Same heat flow rate through the both pin fins, i.e $q_A = q_B$

Unknown:

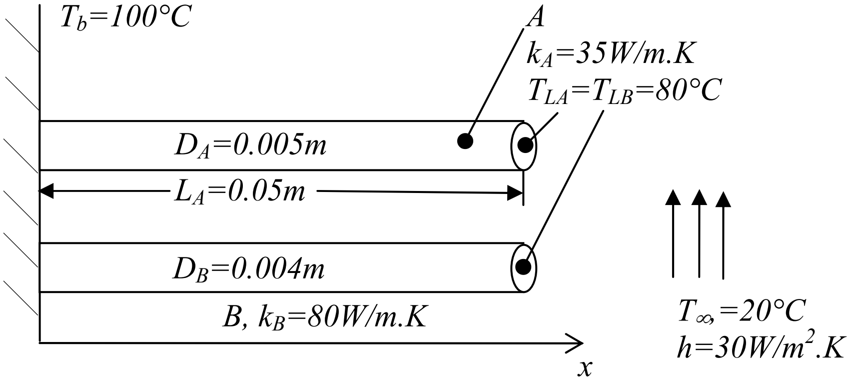
Length of pin fin B if

a) $\left.\dfrac{dT}{dx}\right|_{x=L_A(orL_B)} = 0$

b) $-k_A \left.\dfrac{dT}{dx}\right|_{x=L_A} = h(T_{L_A} - T_\infty)$ and $-k_B \left.\dfrac{dT}{dx}\right|_{x=L_B} = h(T_{L_B} - T_\infty)$

c) $T_{L_A} = T_{L_B} = 80\ °C$

Schematic:



Assumptions:

1. Steady state 1-D conduction along x
2. *h* independent of D; *h* from the side surface is the same as *h* from the tip
3. Constant properties
4. Negligible radiation effects

Analysis

a) Negligible rate of heat transfer from the tip

Heat flow from the fin with "insulated tip" is given by Eq. (3.76):

$$q_f = M \tanh(mL)$$

Where

$$m = \sqrt{\frac{hP}{kA_c}}$$

For pin fin $P = \pi D$ and $A_c = \pi D^2/4$, Therefore $m = \sqrt{\dfrac{4h}{kD}}$, $M = \sqrt{hPkA_c}\theta_b$ Where $\theta_b = T_b - T_\infty$

For pin fin A: L_A =0.05 m D_A =0.005 m k_A =35 W/m K

For pin fin B: L_B = ? D_B =0.004 m k_B =80 W/m K

For same heat transfer rate from both fins:

$$q_{f,A} = M_A \tanh(m_A L_A) = q_{f,B} = M_B \tanh(m_B L_B)$$

Solving for L_B, We have $\boldsymbol{L_B}$**=0.05656m**

Table shows solving results (**This is optional**)

m_A	m_AL_A	M_A	$q_{f,A}$	L_B	m_B
[m^{-1}]	[-]	[W]	[W]	[m]	[m^{-1}]
26.18615	1.309307	1.4389288	1.243378	**0.056561**	19.36492

m_BL_B	M_B	$q_{f,B}$	A_A	A_B	A_A/A_B
[-]	[W]	[W]	[m^2]	[m^2]	
1.095304	1.5566295	1.243445	0.000785	0.00071	1.104996

Temperature of fin tip can be written as following:

$$T_L = T_\infty + \frac{T_b - T_\infty}{\cosh(mL)}$$

$T_{L,A}$ = 60.27°C

$T_{L,B}$ = 68.13 °C

b) Convective heat transfer from both tips
Heat flow from the fin with "convective tip" is given by Eq. (3.72)

$$q_f = M \frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

All parameters are as defined before

Solving for **L$_B$=0.056635m**

Table shows solving results **(This is optional)**

m_A	m_AL_A	$(h/mk)_A$	M_A	$q_{f,A}$	L_B	m_B
[m^{-1}]	[-]	[-]	[W]	[W]	[m]	[m^{-1}]
26.18615	1.309307	0.0327327	1.438929	1.254982	**0.056635**	19.36492

m_BL_B	$(h/mk)_B$	M_B	$q_{f,B}$	A_A	A_B	A_A/A_B
[-]	[-]	[W]	[W]	[m^2]	[m^2]	
1.0967248	0.019365	1.556629	1.254963	0.000785	0.000711	1.103565

Temperature of fin tip,

$$T_L = T_\infty + \frac{T_b - T_\infty}{\cosh(mL) + (h/mk)\sinh(mL)}$$

$T_{L,A}$ =59.15805 ^{o}C $\qquad$ $T_{L,B}$ =67.33974 ^{o}C

Notes: the results for a) and b) are almost identical, because the area of the tip is very small compared to the total surface area.

c) Prescribed temperature of the tip; $T_{L,A} = T_{L,B} = 80^{o}$C; $q_L = 80^{o}$C

Heat flow from the fin of prescribed temperature is given by Eq. (3.78)

$$q_f = M \frac{\cosh(mL) - \theta_L / \theta_b}{\sinh(mL)}$$

Solving for **L_B=0.047188m**

Table shows solving results **(This is optional)**

m_A	$m_A L_A$	$(\theta_L/\theta_b)_A$	M_A	$q_{f,A}$	L_B	m_B
[m^{-1}]	[-]	[-]	[W]	[W]	[m]	[m^{-1}]
26.18615	1.309307	0.75	1.438929	1.036625	0.047188	19.36492

$m_B L_B$	$(\theta_L/\theta_b)_B$	M_B	$q_{f,B}$	A_A	A_B	A_A/A_B
[-]	[-]	[W]	[W]	[m^2]	[m^2]	
0.9137832	0.75	1.556629	1.037446	0.000785	0.000593	1.324502