Université d'Ottawa · University of Ottawa

Faculté de génie	Faculty of Engineering
Génie chimique	Chemical Engineering

CHG 2314
HEAT TRANSFER

Professor: **B. Kruczek** **2005/02/11**

Assignment No. 5

1. Electronic devices in form of very thin disks are mounted flush on a square (10 cm x 10 cm) metal plate of thickness 2 mm and thermal conductivity 12 W/m K. Both sides of the plate are exposed to an environment at 25°C with the combined heat transfer coefficient, h_o = 8 W/m^2 K. Neglecting the contact resistance between the devices and the plate, determine the steady state temperature of the devices when they generate 40 W.

   To decrease the steady state temperature of the devices during their operation, it is proposed to attach 20 aluminum straight rectangular fins (thickness = 1 mm, width = 10 cm, height = L) to the backface of the plate. The fins are to be attached using an epoxy, and as a result a contact resistance, $R''_{t,c}$ = 5 x 10^{-4} m^2 K/W, between the fins and the plate is expected. If the combined heat transfer coefficient for both the finned backface and unfinned face of the plate with the electronic devices is 8 W/m^2 K, determine the required length of the fins for the devices not to exceed 75°C, while dissipating 40 W. For aluminum fins use the thermal conductivity of 160 W/m K.

2. Problem 3.140

3. Problem 4.24. In this problem you will need to evaluate the shape factor graphically.

Due Date: **Feb. 18, 2005 at 4:00 p.m. in the assignment box.**

MARKING SCHEME OF ASSIGNMENT#5

There are three parts for marking this assignment, understanding of problems, using equations and results.

Problem 1 and Total: 10

Understanding of the problem	
Description of Known, unknown, properties, Schematic	1
Part1	
Thermal circuit	1
Equations	
Using correct equations	1
Clearly show your calculations	1
Results	1
Part2	
Thermal circuit	1
Equations	
Using correct equations	2
Clearly show your calculations	1
Results	1
Total:	**10**

Problem 2 Total: 10

Understanding of the problem	
Description of Known, unknown, properties, Schematic	1
Part1	
Results	3
Part2	
Equations	
Using correct equations	1
Clearly show your calculations	1
Results	1
Part 3	
Equations	
Using correct equations	1
Clearly show your calculations	1
Spreadsheet	0.5
Results	0.5
Total:	**10**

Problem 3 Total: 10

Understanding of the problem	
Description of Known, unknown, properties, Schematic	1
Part1	
Thermal circuit	1
Equations	
Using correct equations	1
Clearly show your calculations	0.5
Results	0.5

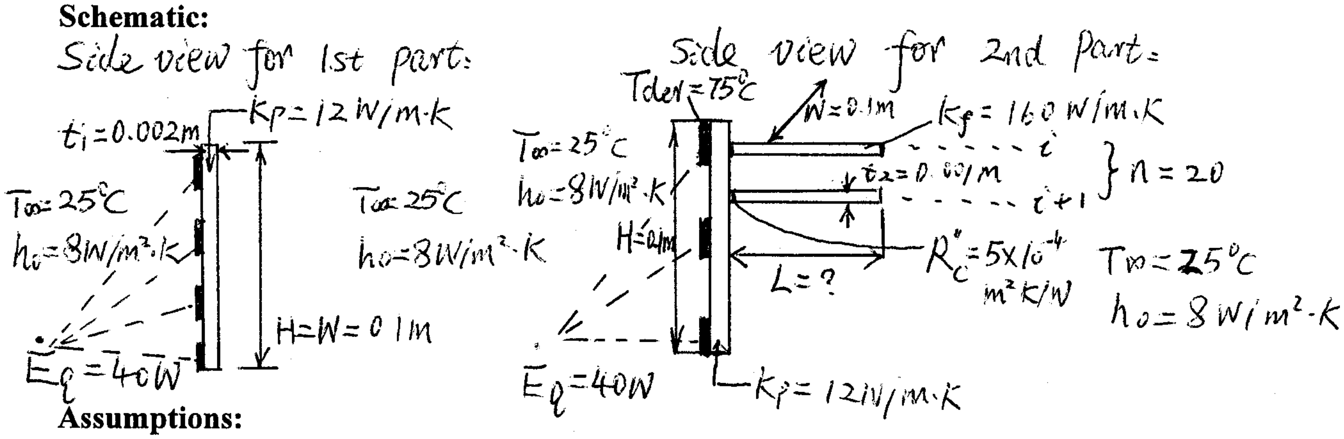
Part2	
Equations	
Using correct equations	1
Clearly show your calculations	0.5
Results	0.5
Part 3 & 4	
Equations	
Using correct equations	1
Clearly show your calculations	1
Results	2
Total:	**10**

PROBLEM 1

Known: Electronic devices are mounted flush on a plate with or without fins

Find: (a) Temperature of the devices, (b) required length of the fins for the devices not to exceed 75°C.

Schematic:



Assumptions:

(1) Steady state 1-D conduction (neglect 2-D effects near the edges of the plate)

(2) Negligible contact resistance between devices and the plate => this implies that the temperature of the plate at the face where the devices are attached = temperature of devices

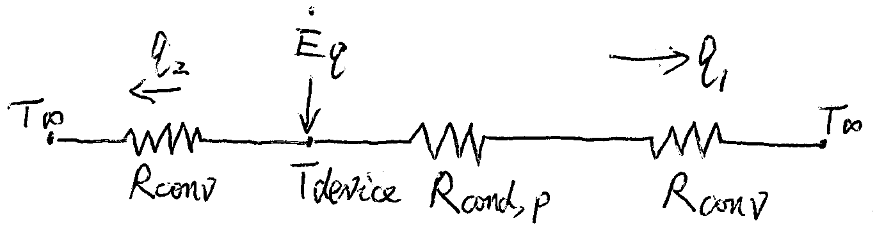
(3) Neglect the heat transfer from the edges of the plate. This assumption is a consequence of the first assumption

(4) Constant properties

Analysis:

Part 1

1) Thermal circuit



where

$$R_{cond,p} = \frac{t_1}{k_p * A_p} = \frac{0.002}{12*(0.1)^2} = 0.0167\frac{k}{w}$$

$$R_{conv} = \frac{1}{h_0 * A_p} = \frac{1}{8*(0.1)^2} = 12.5\frac{k}{w}$$

2) At steady state:

$E_g = q_1 + q_2$

$$40 = \frac{T_{dev} - 25}{0.0167 + 12.5} + \frac{T_{dev} - 25}{12.5}$$

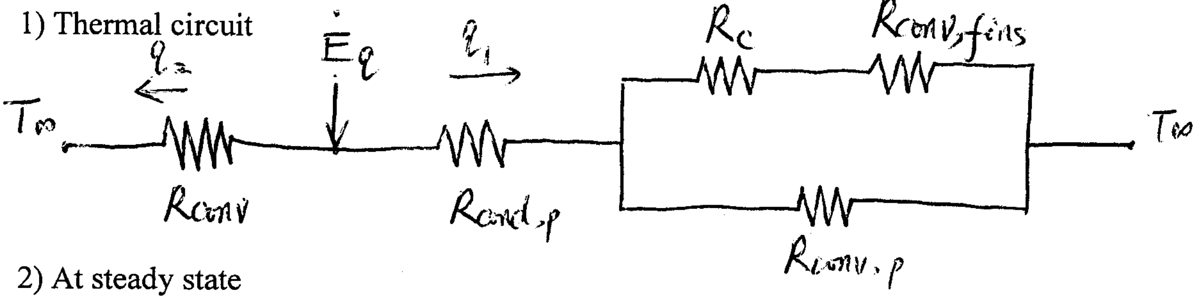
One equation and one unknown T_{dev}

calculating T_{dev}:

$T_{dev} = 275.17°C$

Part 2

1) Thermal circuit



2) At steady state

$E_g=q_1+q_2$

$$=\frac{T_{dev}-T_\infty}{\frac{1}{h_0A_p}}+\frac{T_{dev}-T_\infty}{\frac{t_1}{k_pA_p}+(\frac{1}{1/(h_o*W*H-n*t_2)}+\frac{1}{\frac{R_c}{n*W*t_2}+\frac{1}{h_0*A_f*\eta_f}})^{-1}}$$

$$40=\frac{75-25}{\frac{1}{8*(0.1)^2}}+\frac{75-25}{\frac{0.002}{12*(0.1)^2}+(8*0.1*(0.1-20*0.001)+\frac{1}{\frac{5*10^{-4}}{20*0.1*0.001}+\frac{1}{8*A_f*\eta_f}})^{-1}}$$

$$40=4+\frac{50}{0.0167+(0.064+\frac{1}{0.25+\frac{1}{8*A_f*\eta_f}})^{-1}}$$

where $A_f=2*n*L*W=2*20*0.1*L=4L$

$$\eta f=\frac{\tanh mL}{mL} \text{ where } m=(\frac{h_oP}{k_fA_c})\approx(\frac{2h_0}{k_ft_2})^{1/2}=(\frac{2*8}{160*0.001})^{1/2}=10$$

(If the width of a rectangular fin is much larger than its thickness, w>>t, the perimeter may be approximated as P=2w)

Therefore, the above equation becomes:

$$36=\frac{50}{0.0167+(0.064+\frac{1}{0.25+\frac{1}{8*4L*\frac{\tanh(10L)}{10L}}})^{-1}}=\frac{50}{0.0167+(0.064+\frac{1}{0.25+\frac{1}{3.2*\tanh(10L)}})^{-1}}$$

solving for L

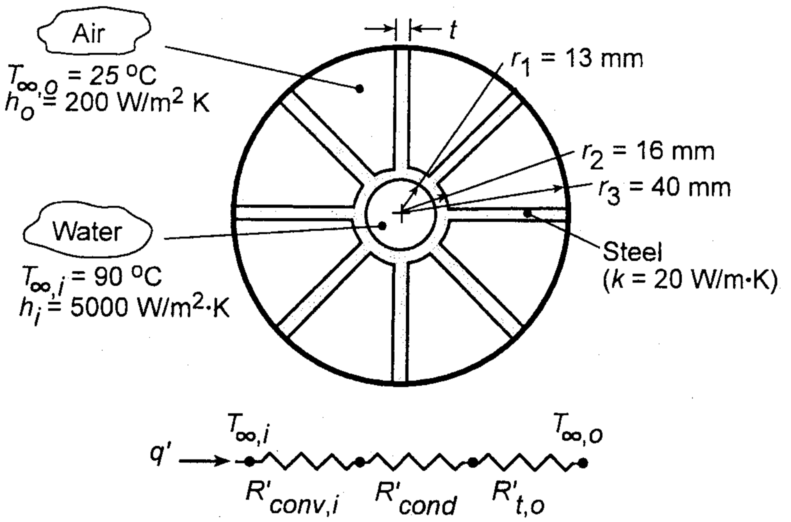
L=0.02545m

PROBLEM 3.140

KNOWN: Geometrical and convection conditions of internally finned, concentric tube air heater.

FIND: (a) Thermal circuit, (b) Heat rate per unit tube length, (c) Effect of changes in fin array.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in radial direction, (3) Constant k, (4) Adiabatic outer surface.

ANALYSIS: (a) For the thermal circuit shown schematically,

$$R'_{conv,i} = (h_i 2\pi r_1)^{-1}, \qquad R'_{cond} = \ln(r_2/r_1)/2\pi k\text{, and} \qquad R'_{t,o} = (\eta_o h_o A'_t)^{-1},$$

where

$$\eta_o = 1 - \frac{NA'_f}{A'_t}(1-\eta_f), \quad A'_f = 2L = 2(r_3 - r_2), \quad A'_t = NA'_f + (2\pi r_2 - Nt)\text{, and} \quad \eta_f = \frac{\tanh mL}{mL}.$$

(b) $$q' = \frac{(T_{\infty,i} - T_{\infty,o})}{R'_{conv,i} + R'_{cond} + R'_{t,o}}$$

Substituting the known conditions, it follows that

$$R'_{conv,i} = \left(5000\,\text{W/m}^2\cdot\text{K} \times 2\pi \times 0.013\text{m}\right)^{-1} = 2.45\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{cond} = \ln(0.016\text{m}/0.013\text{m})/2\pi(20\,\text{W/m}\cdot\text{K}) = 1.65\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{t,o} = \left(0.575\times200\,\text{W/m}^2\cdot\text{K}\times0.461\text{m}\right)^{-1} = 18.86\times10^{-3}\,\text{m}\cdot\text{K/W}$$

where $\eta_f = 0.490$. Hence,

$$q' = \frac{(90-25)^\circ\text{C}}{(2.45+1.65+18.86)\times10^{-3}\,\text{m}\cdot\text{K/W}} = 2831\,\text{W/m} \qquad <$$

(c) The small value of η_f suggests that some benefit may be gained by increasing t, as well as by increasing N. With the requirement that $Nt \le 50$ mm, we use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to consider the following range of conditions: t = 2 mm, $12 \le N \le 25$; t = 3 mm, $8 \le N \le 16$; t = 4 mm, $6 \le N \le 12$; t = 5 mm, $5 \le N \le 10$. Calculations based on the foregoing model are plotted as follows.

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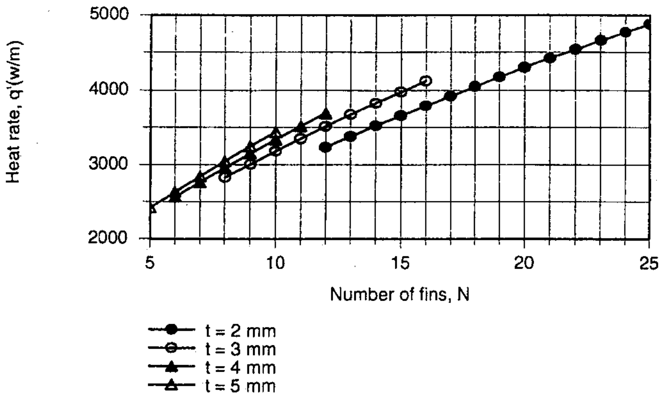
PROBLEM 3.140 (Cont.)



By increasing t from 2 to 5 mm, η_f increases from 0.410 to 0.598. Hence, for fixed N, q' increases with increasing t. However, from the standpoint of maximizing q'_t, it is clearly preferable to use the larger number of thinner fins. Hence, subject to the prescribed constraint, we would choose t = 2 mm and N = 25, for which q' = 4880 W/m.

COMMENTS: (1) The air side resistance makes the dominant contribution to the total resistance, and efforts to increase q' by reducing $R'_{t,o}$ are well directed. (2) A fin thickness any smaller than 2 mm would be difficult to manufacture.

Problem 3.140 Spreadsheet

R1=	0.013		Ti=	90		k=	20
R2=	0.016		To=	25		hi=	5000
R3=	0.04					ho=	200

Rconv,i=	0.002449	Af=	0.048	W(Unit Length)=	1
Rcond=	0.001652	L=	0.024		

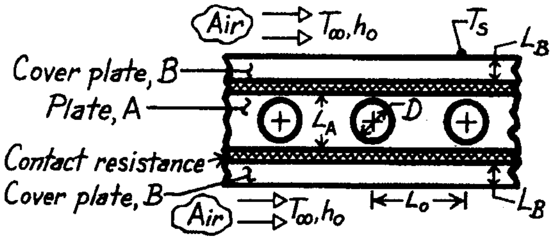
t	N	At	m	ηf	ηo	Rto	q
0.002	12	0.652531	100	0.409865	0.479077	0.015994	3234.618
0.002	13	0.698531	100	0.409865	0.47283	0.015138	3378.508
0.002	14	0.744531	100	0.409865	0.467354	0.014369	3519.152
0.002	15	0.790531	100	0.409865	0.462516	0.013675	3656.66
0.002	16	0.836531	100	0.409865	0.45821	0.013044	3791.135
0.002	17	0.882531	100	0.409865	0.454353	0.012469	3922.676
0.002	18	0.928531	100	0.409865	0.450878	0.011943	4051.378
0.002	19	0.974531	100	0.409865	0.447731	0.011459	4177.332
0.002	20	1.020531	100	0.409865	0.444867	0.011013	4300.626
0.002	21	1.066531	100	0.409865	0.442251	0.010601	4421.342
0.002	22	1.112531	100	0.409865	0.439851	0.010218	4539.561
0.002	23	1.158531	100	0.409865	0.437642	0.009862	4655.36
0.002	24	1.204531	100	0.409865	0.435601	0.009529	4768.811
0.002	25	1.250531	100	0.409865	0.43371	0.009219	4879.986
0.003	8	0.460531	81.64966	0.490438	0.575117	0.018878	2828.689
0.003	9	0.505531	81.64966	0.490438	0.564555	0.017519	3006.456
0.003	10	0.550531	81.64966	0.490438	0.55572	0.016343	3179.434
0.003	11	0.595531	81.64966	0.490438	0.548221	0.015315	3347.815
0.003	12	0.640531	81.64966	0.490438	0.541774	0.014408	3511.779
0.003	13	0.685531	81.64966	0.490438	0.536175	0.013603	3671.498
0.003	14	0.730531	81.64966	0.490438	0.531265	0.012883	3827.135
0.003	15	0.775531	81.64966	0.490438	0.526925	0.012236	3978.845
0.003	16	0.820531	81.64966	0.490438	0.523061	0.01165	4126.773
0.004	6	0.364531	70.71068	0.550978	0.645247	0.021257	2563.267
0.004	7	0.408531	70.71068	0.550978	0.630697	0.019405	2765.21
0.004	8	0.452531	70.71068	0.550978	0.618977	0.01785	2961.107
0.004	9	0.496531	70.71068	0.550978	0.609334	0.016526	3151.225
0.004	10	0.540531	70.71068	0.550978	0.601261	0.015385	3335.815
0.004	11	0.584531	70.71068	0.550978	0.594403	0.014391	3515.116
0.004	12	0.628531	70.71068	0.550978	0.588506	0.013517	3689.352
0.005	5	0.315531	63.24555	0.598415	0.694545	0.022815	2414.897
0.005	6	0.358531	63.24555	0.598415	0.677415	0.020587	2632.894
0.005	7	0.401531	63.24555	0.598415	0.663955	0.018755	2843.93
0.005	8	0.444531	63.24555	0.598415	0.653098	0.017222	3048.333
0.005	9	0.487531	63.24555	0.598415	0.644156	0.015921	3246.411
0.005	10	0.530531	63.24555	0.598415	0.636664	0.014803	3438.452

PROBLEM 4.24

KNOWN: Platen heated by passage of hot fluid in poor thermal contact with cover plates exposed to cooler ambient air.

FIND: (a) Heat rate per unit thickness from each channel, q_i', (b) Surface temperature of cover plate, T_s, (c) q_i' and T_s if lower surface is perfectly insulated, (d) Effect of changing centerline spacing on q_i' and T_s

SCHEMATIC:



D=15 mm $\quad$ L_o=60 mm

L_A=30 mm $\quad$ L_B=7.5 mm

T_i=150°C $\quad$ h_i=1000 W/m^2·K

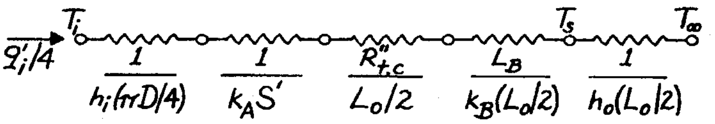
T_∞=25°C $\quad$ h_o=200 W/m^2·K

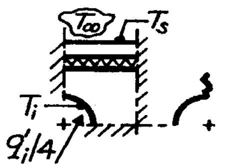
k_A=20 W/m·K $\quad$ k_B=75 W/m·K

$R''_{t,c} = 2.0 \times 10^{-4}\ \mathrm{m^2 \cdot K/W}$

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in platen, but one-dimensional in coverplate, (3) Temperature of interfaces between A and B is uniform, (4) Constant properties.

ANALYSIS: (a) The heat rate per unit thickness from each channel can be determined from the following thermal circuit representing the quarter section shown.





The value for the shape factor is $S' = 1.06$ as determined from the flux plot shown on the next page. Hence, the heat rate is

$$q_i' = 4(T_i - T_\infty)/R'_{tot} \qquad (1)$$

$$R'_{tot} = [1/1000\ \mathrm{W/m^2 \cdot K}(\pi 0.015\mathrm{m}/4) + 1/20\ \mathrm{W/m \cdot K} \times 1.06$$
$$+ 2.0 \times 10^{-4}\mathrm{m^2 \cdot K/W}(0.060\mathrm{m}/2) + 0.0075\mathrm{m}/75\ \mathrm{W/m \cdot K}(0.060\mathrm{m}/2)$$
$$+ 1/200\ \mathrm{W/m^2 \cdot K}(0.060\mathrm{m}/2)]$$

$$R'_{tot} = [0.085 + 0.047 + 0.0067 + 0.0033 + 0.1667]\mathrm{m \cdot K/W}$$
$$R'_{tot} = 0.309\ \mathrm{m \cdot K/W}$$

$$q_i' = 4(150 - 25)\mathrm{K}/0.309\ \mathrm{m \cdot K/W} = 1.62\ \mathrm{kW/m}. \qquad <$$

(b) The surface temperature of the cover plate also follows from the thermal circuit as

$$q_i'/4 = \frac{T_s - T_\infty}{1/h_o(L_o/2)} \qquad (2)$$

Continued .....

PROBLEM 4.24 (Cont.)

$$T_s = T_\infty + \frac{q_i'}{4}\frac{1}{h_o\left(L_o/2\right)} = 25^\circ\text{C} + \frac{1.62\text{ kW}}{4} \times 0.167\text{ m}\cdot\text{K/W}$$

$$T_s = 25^\circ\text{C} + 67.6^\circ\text{C} \approx 93^\circ\text{C}.$$ <

(c,d) The effect of the centerline spacing on q_i' and T_s can be understood by examining the relative magnitudes of the thermal resistances. The dominant resistance is that due to the ambient air convection process which is inversely related to the spacing L_o. Hence, from Eq. (1), the heat rate will increase nearly linearly with an increase in L_o,

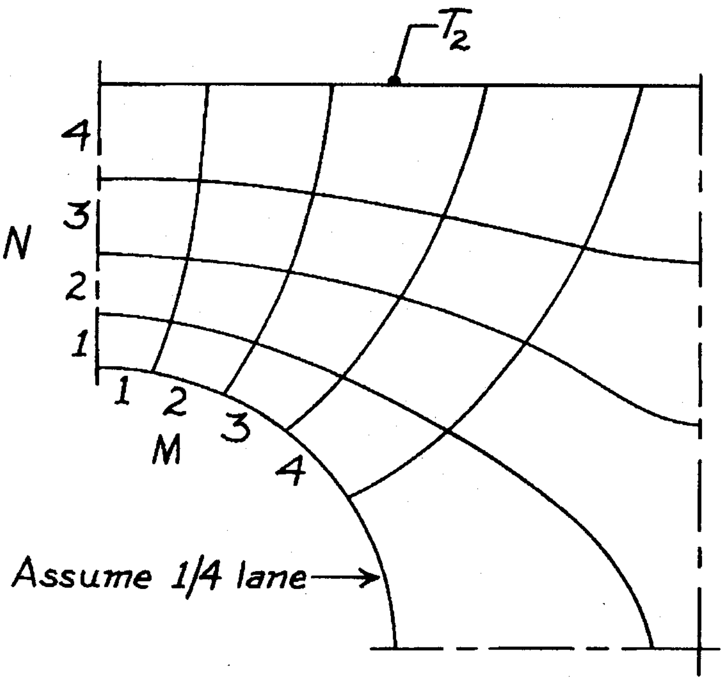
$$q_i' \sim \frac{1}{R_{tot}'} \approx \frac{1}{1/h_o\left(L_o/2\right)} \sim L_o.$$

From Eq. (2), find

$$\Delta T = T_s - T_\infty = \frac{q_i'}{4}\frac{1}{h_o\left(L_o/2\right)} \sim q_i' \cdot L_o^{-1} \sim L_o \cdot L_o^{-1} \approx 1.$$

Hence we conclude that ΔT will not increase with a change in L_o. Does this seem reasonable? What effect does L_o have on Assumptions (2) and (3)?

If the lower surface were insulated, the heat rate would be decreased nearly by half. This follows again from the fact that the overall resistance is dominated by the surface convection process. The temperature difference, T_s - T_∞, would only increase slightly.



$S' = S/l = M/N = 4.25/4 = 1.06$