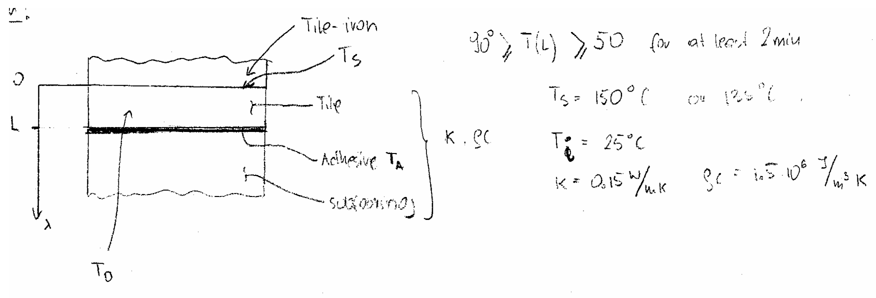
Problem 1
Known:

Surface temperature of a tile iron used to soften adhesive underneath tiles in order to lift tiles.
Thickness and properties of tiles and subflooring

Unknown:

1. Minimum thickness of tiles to meet the requirement for softing of the adhesive while without damaging it
2. The require time to lift the tile of calculated minimum thickness
3. Effect of surface temperature on the energy required to lift the tile

Schematic:



Assumptions:

1. 1-D conduction along x-------tile, adhesive and subflooring represents a system which can be treated as semi-infinite medium
2. No contact resistance between tile-iron and tile so that surface temperature of tile becomes 150°C (or 135°C) instantaneously upon the contact with the tile-iron
3. Adhesive does not impose any resistance to heat transfer or the properties of adhesive are the same as those of the tile and subflooring
4. Constant properties of tile and subflooring

Analysis:
i) and ii)
Due to assumptions 1 and 2, the tile can be treated as semi-infinite medium subjected to sudden change in surface temperature. In addition, because of assumptions 3 and 4 and because $k_{\rho c\ \text{iron plate}} >> k_{\rho c\ tile}$, the temperature response in the system consisting of the tile, adhesive and subflooring is given by eq (5.57)

$$\frac{T - T_s}{T_i - T_s} = erf1\left(\frac{x}{(4\alpha t)^{1/2}} \right)$$

Where T is $T(t,x)$, $\alpha = k/\rho c = 0.15/1.5*10^6 = 1.0*10^{-7}\ m^2/s$

Using the specifications for softening of adhesive without damaging it, we can write the following two equations:

$$\frac{T(t,L)-T_s}{T_i-T_s} = erf1\left(\frac{L}{(4\alpha t)^{1/2}}\right)$$
$$\frac{T(t+2,L)-T_s}{T_i-T_s} = erf1\left(\frac{L}{(4\alpha(t+2))^{1/2}}\right)$$

Where *T(t, L)* = 50°C and *T(t+2, L)* = 90°C

We have a system of two equations and two unknowns *L* and *t*, solving the equations at T_i = 25°C and T_s = 150°C

$$\frac{50-150}{25-150} = 0.8 = erf1\left(\frac{L}{(4\alpha t)^{1/2}}\right)$$
$$\frac{90-150}{25-150} = 0.48 = erf1\left(\frac{L}{(4\alpha(t+2))^{1/2}}\right)$$

From table B2,

$erf1(\eta)$ = 0.8 ➔ η = 0.9063

$erf1(\eta)$ = 0.48 ➔ η = 0.4549

Therefore,

$$\frac{L}{(4\alpha t)^{1/2}} = 0.9063$$
$$\frac{L}{(4\alpha(t+2))^{1/2}} = 0.4549$$

Solving above two equations, we have

***t* = 40.4 s**

***L* = 0.0036 m = 3.6 mm**

Therefore, the required time is ***t + 120 = 160.4 s***

iii)

The total energy per unit area when T_s is 150°C is

$$Q'' = Q/A = \int_0^t q_s dt = \int_0^t \frac{k(T_s-T_i)}{(\pi\alpha t)^{1/2}}dt = \frac{k(T_s-T_i)}{(\pi\alpha)^{1/2}}\cdot 2t^{1/2}$$

For T_s = 150°C, *t* = 160.4 s

$$Q'' = \frac{k(T_s-T_i)}{(\pi\alpha)^{1/2}}\cdot 2t^{1/2} = \frac{0.15\times(150-25)}{(\pi\times1.0\times10^{-7})^{1/2}}\cdot 2\times160.4^{1/2}$$

= **847 kJ**

Changing T_s would change *t*, first we must determine t to bring adhesive to 50°C

$$\frac{50-135}{25-135} = 0.7727 = erf1(\eta)$$ ➔ From table B.2 η = 0.8538 (by interpolation)

$$\frac{0.0036}{(4\times10^{-7}\times t)^{1/2}} = 0.8538$$ ➔ *t = 44.4 s* ➔ t_{total} *= 44.4 + 120 =* ***164.4 s***

Since T_s< 150°C, *T(164.4, 0.0036)* should not exceed 90°C, we can check this temperature as following

$$\frac{T(164.4,0.0036)-135}{25-135} erf1\left(\frac{0.0036}{(4\times10^{-7}\times164.4)^{1/2}}\right) = erf1(0.5561) = 0.5302$$

T(164.4, 0.0036) **= 83.3°C < 90°C**

$$Q'' = \frac{k(T_s - T_i)}{(\pi\alpha)^{1/2}}\cdot 2t^{1/2} = \frac{0.15\times(135-25)}{(\pi\times1.0\times10^{-7})^{1/2}}\cdot 2\times164.4^{1/2}$$

= **755 kJ**

755 kJ < 847 kJ ➔ decreasing surface temperature of the tile-iron would indeed decrease the amount of energy required to lift the tile.

Problem 2

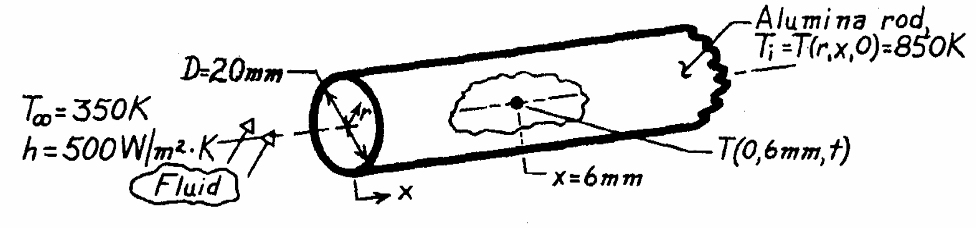
KNOWN:

A long alumina rod, initially at a uniform temperature of 850K, is suddenly exposed to a cooler fluid.

UNKNOWN:

Temperature of the rod after 30s, at an exposed end, *T(0,0.t)*, and at an axial distance 6mm from the end, *T(0, 6mm, t)*.

SCHEMATIC:



ASSUMPTIONS:

(1) Two-dimensional conduction in (*r*,*x*) directions,
(2) Constant properties,
(3) Convection coefficient is same on end and cylindrical surfaces.

PROPERTIES:

Table A-2, Alumina, polycrystalline aluminum oxide (assume

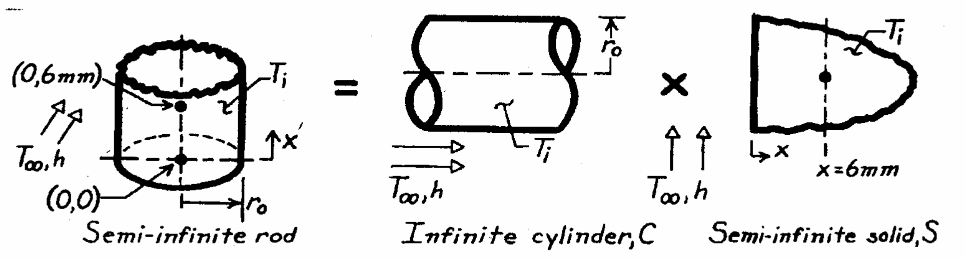
$$\bar{T} \approx (850+600)K/2 = 725K): \ \rho = 3970 \text{ kg/m}^3, c = 1154 \text{ J/kg}\cdot\text{K}, k = 12.4 \text{ W/m}\cdot\text{K}.$$

ANALYSIS:

First, check if system behaves as a lumped capacitance. Find

$$\text{Bi} = \frac{hL_c}{k} = \frac{h(r_o/2)}{k} = \frac{500 \text{ W/m}\cdot\text{K}(0.010\text{m}/2)}{12.4 \text{ W/m}\cdot\text{K}} = 0.202.$$

Since Bi >0.1, rod does not behave as spacewise isothermal object. Hence, treat rod as a semi-infinite cylinder, the multi-dimensional system Case (f), Fig. 5.11.



The product solution can be written as

$$\theta^*(r,x,t) = \frac{\theta(r,x,t)}{\theta_i} = \frac{\theta(r,t)}{\theta_i} \times \frac{\theta(x,t)}{\theta_i} = C(r^*,t^*) \times S(x^*,t^*)$$

Infinite cylinder, C(r*,t*). Using the Heisler charts with r* = r = 0 and

$$Bi^{-1} = \left[\frac{h\, r_o}{k}\right]^{-1} = \left[\frac{500\ W/m^2 \cdot K \times 0.01m}{12.4\ W/m \cdot K}\right]^{-1} = 2.48.$$

Evaluate $\alpha = k/\rho c = 2.71 \times 10^{-6} m^2/s$, find $Fo = \alpha\ t/r_o^2 = 2.71\times10^{-6} m^2/s \times 30s/(0.01m)^2$ = 0.812. From the Heisler chart, Fig. D.4, with Bi^{-1} = 2.48 and Fo = 0.812, read C(0,t*) = $\theta(0,t)/\theta_i = 0.61$.

Semi-infinite inediuni, S(x*,t*). Recognize this as Case (3), Fig. 5.7. From Eq. 5.60, note that the LHS needs to be transformed as follows,

$$\frac{T-T_i}{T_\infty - T_i} = 1 - \frac{T-T_\infty}{T_i - T_\infty} \qquad S(x,t) = \frac{T-T_\infty}{T_i - T_\infty}.$$

Thus,

$$S(x,t) = 1 - \left\{ erfc\left[\frac{x}{2(\alpha\, t)^{1/2}}\right] - \left[exp\left[\frac{hx}{k} + \frac{h^2 \alpha\, t}{k^2}\right]\right]\left[erfc\left[\frac{x}{2(\alpha\, t)^{1/2}} + \frac{h(\alpha\, t)^{1/2}}{k}\right]\right]\right\}.$$

Evaluating this expression at the surface (x = 0) and 6mm from the exposed end, find

$$S(0,30s) = 1 - \left\{ erfc(0) - \left[exp\left[0 + \frac{\left(500\ W/m^2 \cdot K\right)^2 2.71\times10^{-6} m^2/s \times 30s}{(12.4\ W/m \cdot K)^2}\right]\right]\right.$$

$$\left.\left[erfc\left[0 + \frac{500\ W/m^2 \cdot K\left(2.71\times10^{-6} m^2/s \times 30s\right)^{1/2}}{12.4\ W/m \cdot K}\right]\right]\right\}$$

$$S(0,30s) = 1 - \{1 - [exp(0.1322)][erfc(0.3636)]\} = 0.693.$$

Note that Table B.2 was used to evaluate the complementary error function, erfc(w).

$$S(6\text{mm},30\text{s}) = 1 - \left\{ \text{erfc}\left[\frac{0.006\text{m}}{2\left(2.71\times10^{-6}\text{m}^2/\text{s}\times30\text{s}\right)^{1/2}} \right] - \right.$$

$$\left. \left[\exp\left[\frac{500\ \text{W/m}^2\cdot\text{K}\times0.006\text{m}}{12.4\ \text{W/m}\cdot\text{K}} + 0.1322 \right] \right] \left[\text{erfc}\left(0.3327+0.3636\right) \right] \right\} = 0.835.$$

The product solution can now be evaluated for each location. At (0,0),

$$\theta^*(0,0,t) = \frac{T(0,0,30\text{s}) - T_\infty}{T_i - T_\infty} = C\left(0,t^*\right)\times S\left(0,t^*\right) = 0.61\times0.693 = 0.423.$$

Hence, $T(0,0,30\text{s}) = T_\infty + 0.423(T_i - T_\infty) = 350\text{K} + 0.423(850-350)\text{K} = 561\text{K}.$ <

At (0,6mm),

$$\theta^*(0,6\text{mm},t) = C\left(0,t^*\right)\times S\left(6\text{mm},t^*\right) = 0.61\times0.835 = 0.509$$

$$T(0,6\text{mm},30\text{s}) = 604\text{K}.$$ <

COMMENTS:

Note that the temperature at which the properties were evaluated was a good estimate.

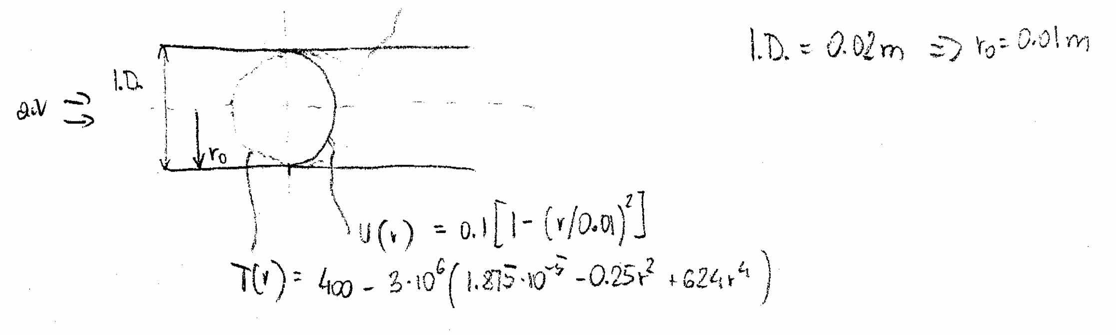
Problem 3
KNOWN:

Velocity and temperature profile in a cylindrical tube for the flow of air

UNKNOWN:

R_{eD}, T_m h, N_{vD} and q''

SCHEMATIC:



PROPERTIES:

k, ρ, u of air at T_m

ASSUMPTIONS:

(1) Steady state conditions
(2) Incompressibility of air

ANALYSIS:

Determination of u_m (mean velocity) for the provided velocity profile

$$u_m = \frac{\int_0^{r_o} u 2\pi r dr}{\pi r_o^2} = \frac{\int_0^{r_o} 0.1\left[1-\left(\frac{r}{0.01}\right)^2\right]2\pi r dr}{\pi r_o^2}$$

$$= \frac{0.2\pi\left[\frac{r^2}{2}-\frac{r^4}{4(0.01)^2}\right]}{\pi r_o^2}\Bigg|_0^{r_o} = \frac{\frac{1}{2}0.2 r_o^2\left[1-\frac{r_o 2}{2(0.01)^2}\right]}{r_o^2}$$

= **0.05 m/s**

Determination of T_m

$$T_m = \frac{\int_0^{r_o} \rho u T 2\pi r dr}{\int_0^{r_o} \rho u 2\pi r dr} = \frac{\int_0^{r_o} uTrdr}{\int_0^{r_o} urdr}$$

$$= \frac{\int_0^{r_o} 0.1\left[1-\left(\frac{r}{0.01}\right)^2\right]\left[400 - 3\cdot 10^6 (1.875\times 10^{-5} - 0.25r^2 + 624r^4\right] rdr}{\int_0^{r_o} 0.1\left[1-\left(\frac{r}{0.01}\right)^2\right] rdr}$$

$$= \left.\frac{17.1875{r_o}^2 - 6.71875\times 10^4 {r_o}^4 - 1.5625\times 10^8 {r_o}^6 + 23.4375\times 10^{10} {r_o}^8}{0.05{r_o}^2 - 250r_o 4}\right|_0^{0.01}$$

=**365.6 K**

Properties of air at 365.6 K

k = 0.0310 W/m.K

$\rho = 0.968$ kg/m^3

$\mu = 21.12 \cdot 10^{-6}$ kg/m.s

Determination of R_{eD}

$$R_{eD} = \frac{u_m D \rho}{\mu} = \frac{0.05\times 0.02\times 0.968}{21.12\times 10^{-6}}$$

= **45.8** **Laminar flow**

Determination of h

According to general definition

$$h = \frac{k\left(\frac{\partial T}{\partial r}\right)\Big|_{r = r_o}}{T_s - T_b}$$

Where $T_s = T(r = r_o = 0.01m)$

$$T_s = 400 - 3\cdot 10^6 (1.875\times 10^{-5} - 0.25\times 0.01^2 + 624\times 0.01^4)$$

= 400 K

Evaluation of $\left.\left(\frac{\partial T}{\partial r}\right)\right|_{r = r_o}$

$$\frac{\partial T}{\partial r} = 3 \cdot 10^6 (2 \times 0.25r + 4 \times 624r^3)\Big|_{r = 0.01}$$

=**7512 K/m**

$$h = \frac{k\left.\left(\frac{\partial T}{\partial r}\right)\right|_{r = r_o}}{T_s - T_b} = \frac{0.0310 \times 7512}{400 - 365.6}$$

= **6.77 W/m^2.K**

Determination of local N_{vD}

$$N_{vD} = \frac{hD}{k} = \frac{6.77 \times 0.02}{0.0310}$$

= **4.368 ≈ 4.364**

$$q'' = h(T_s - T_m) = 6.77(400 - 365.6)$$

= **232.9 W/m^2**