Université d'Ottawa · University of Ottawa

Faculté de génie Faculty of Engineering
Génie chimique Chemical Engineering

CHG 2314
HEAT TRANSFER

Professor: **B. Kruczek** **2005/03/18**

Assignment No. 8

1. Problem 7.26

2. Problem 7.58

3. Glass balls of diameter D = 0.15 m, which are initially at a uniform temperature T_i = 200°C, are cooled in a stream of cold air at T_∞ =10°C flowing with a constant velocity of 6 m/s. Neglecting radiation effects, estimate the time required for the balls to cool to a final, safe-to-touch temperature of T_f = 50°C.

   Use the following properties for glass: k = 0.88 W/m K, ρ = 2400 kg/m^3, and c_p = 840 J/kg K.

Due Date: **March 29, 2005 at 4:00 p.m. in the assignment box.**

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MARKING SCHEME OF ASSIGNMENT#8

There are three parts for marking this assignment, understanding of problems, using equations and results.

Problem 1 and Total: 10	
Understanding of the problem	
Description of Known, unknown, properties, Schematic	1
Part a	
Equations	
Using correct equations	2
Clearly show your steps	3
Results	1
Part b	
Equations	
Clearly show your calculations	2
Results	1
Total:	**10**

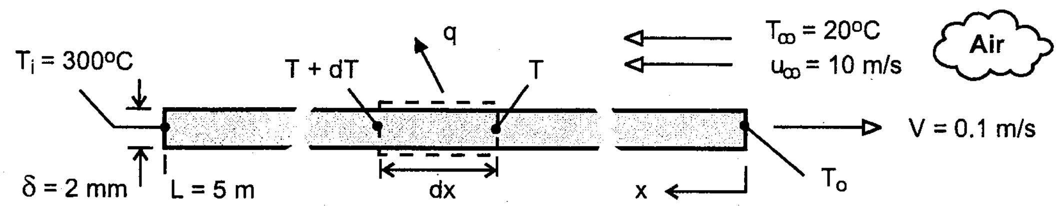
Problem 2 & 3 Total: 20 (10 for each problem)	
Understanding of the problem	
Description of Known, unknown, properties, Schematic	1
Equations	
Using correct equations	4
Clearly show your calculations	4
Results	1
Total:	**10**

PROBLEM 7.26

KNOWN: Velocity, initial temperature, and dimensions of aluminum strip on a production line. Velocity and temperature of air in counter flow over top surface of strip.

FIND: (a) Differential equation governing temperature distribution along the strip and expression for outlet temperature, (b) Value of outlet temperature for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of sheet temperature across its thickness, (2) Negligible effect of conduction along length (x) of sheet, (3) Negligible radiation, (4) Turbulent flow over entire top surface, (5) Negligible effect of sheet velocity on boundary layer development, (6) Negligible heat transfer from bottom surface and sides, (7) Constant properties.

PROPERTIES: *Table A-1*, Aluminum, 2024-T6 $(\overline{T}_{AL} \approx 500\text{K})$: $\rho = 2770\,\text{kg/m}^3$, $c_p = 983\,\text{J/kg}\cdot\text{K}$, k=186 W/m·K. *Table A-4*, Air $(p = 1\,\text{atm},\ T_f \approx 400\text{K})$: $\nu = 26.4\times10^{-6}\,\text{m}^2/\text{s}$, $k = 0.0338\,\text{W/m}\cdot\text{K}$, $Pr = 0.69$

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the sheet moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$. Hence, with *inflow* due to *advection* and outflow due to *advection* and *convection*,

$$\rho V A_c c_p (T + dT) - \rho V A_c c_p T - dq = 0$$

$$+\rho V \delta W c_p dT - h_x (dx \cdot W)(T - T_\infty) = 0$$

$$\frac{dT}{dx} = +\frac{h_x}{\rho V \delta c_p}(T - T_\infty) \qquad (1) <$$

Alternatively, if the control surface is fixed to the sheet, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-h_x (dx \cdot W)(T - T_\infty) = \rho (dx \cdot W \cdot \delta) c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{h_x}{\rho \delta c_p}(T - T_\infty)$$

Dividing the left- and right-hand sides of the equation by dx/dt and dx/dt = - V, respectively, equation (1) is obtained. The equation may be integrated from x = 0 to x = L to obtain

$$\int_{T_o}^{T_i} \frac{dT}{T - T_\infty} = \frac{L}{\rho V \delta c_p}\left[\frac{1}{L}\int_0^L h_x\, dx\right]$$

Continued .....

PROBLEM 7.26 (Cont.)

where $h_x = (k/x)0.0296\,Re_x^{4/5}\,Pr^{1/3}$ and the bracketed term on the right-hand side of the equation reduces to $\bar{h}_L = (k/L)0.037\,Re_L^{4/5}\,Pr^{1/3}$.

Hence,

$$\ln\left(\frac{T_i - T_\infty}{T_o - T_\infty}\right) = \frac{L\bar{h}_L}{\rho V \delta c_p}$$

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{L\bar{h}_L}{\rho V \delta c_p}\right) \quad <$$

(b) For the prescribed conditions, $Re_L \approx u_\infty L/\nu = 20\,m/s \times 5m / 26.4\times10^{-6}\,m^2/s = 3.79\times10^6$ and

$$\bar{h}_L = \left(\frac{0.0338\,W/m\cdot K}{5m}\right)0.037\left(3.79\times10^6\right)^{4/5}(0.69)^{1/3} = 40.5\,W/m^2\cdot K$$

$$T_o = 20°C + (280°C)\exp\left(-\frac{5m\times40.5\,W/m^2\cdot K}{2770\,kg/m^3\times0.1\,m/s\times0.002m\times983\,J/kg\cdot K}\right) = 213°C \quad <$$

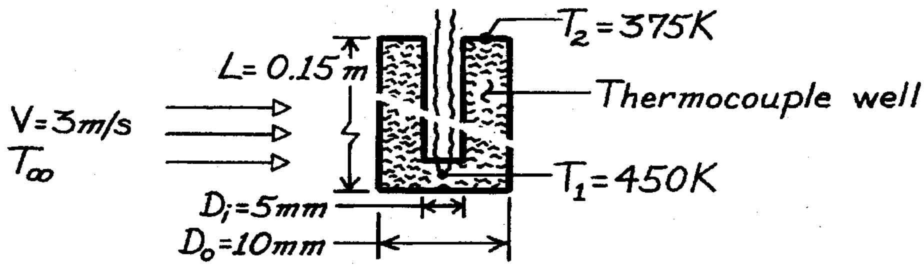
COMMENTS: (1) With $T_o = 213°C$, $\bar{T}_{Al} = 530K$ and $T_f = 411K$ are close to values used to determine the material properties, and iteration is not needed. (2) For a representative emissivity of $\varepsilon = 0.2$ and $T_{sur} = T_\infty$, the maximum value of the radiation coefficient is $h_r = \varepsilon\sigma\left(T_i + T_{sur}\right)\left(T_i^2 + T_{sur}^2\right) = 4.1\,W/m^2\cdot K << \bar{h}_L$. Hence, the assumption of negligible radiation is appropriate.

PROBLEM 7.58

KNOWN: Dimensions and thermal conductivity of a thermocouple well. Temperatures at well tip and base. Air velocity.

FIND: Air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction along well, (4) Uniform convection coefficient, (5) Negligible radiation.

PROPERTIES: Steel (given): k = 35 W/m·K; Air (given): $\rho = 0.774\ \text{kg/m}^3$, $\mu = 251 \times 10^{-7}\ \text{N·s/m}^2$, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: Applying Equation 3.70 at the well tip (x = L), where $T = T_1$,

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \left[\cosh mL + \left(\bar{h}/mk\right)\sinh mL\right]^{-1}$$

⟶ Students may assume negligible heat transfer from the tip

$$m = \left(\bar{h}P/kA_c\right)^{1/2} \qquad P = \pi D_o = \pi(0.010\ \text{m}) = 0.0314\ \text{m}$$

$$A_c = (\pi/4)\left(D_o^2 - D_i^2\right) = (\pi/4)\left(0.010^2 - 0.005^2\right)\text{m}^2 = 5.89\times10^{-5}\ \text{m}^2.$$

With $\quad Re_D = \dfrac{\rho VD}{\mu} = \dfrac{0.744\ \text{kg/m}^3(3\text{m/s})\ 0.01\ \text{m}}{251\times10^{-7}\ \text{N}\cdot\text{s/m}^2} = 925$

⟶ students may use Hilpert or Churchill Bernstein correlation

C = 0.51, m = 0.5, n = 0.37 and the Zhukauskas correlation yields

$$\overline{Nu}_D = 0.51 Re_D^{0.5} Pr^{0.37}\left(Pr/Pr_s\right)^{1/4} \approx 0.51(925)^{0.5}(0.686)^{0.37} \times 1 = 13.5$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D_o} = 13.5\frac{0.0373\ \text{W/m}\cdot\text{K}}{0.01\ \text{m}} = 50.4\ \text{W/m}^2\cdot\text{K}.$$

Hence

$$m = \left[\frac{\left(50.4\ \text{W/m}^2\cdot\text{K}\right)0.0314\ \text{m}}{(35\ \text{W/m}\cdot\text{K})5.89\times10^{-5}\ \text{m}^2}\right]^{1/2} = 27.7\ \text{m}^{-1} \qquad mL = \left(27.7\ \text{m}^{-1}\right)0.15\ \text{m} = 4.15.$$

With

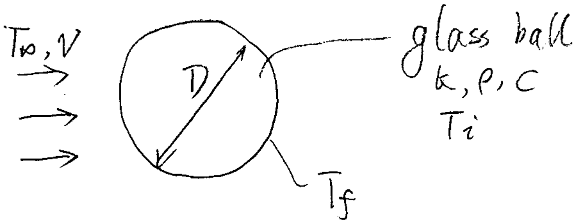
$$\left(\bar{h}/mk\right) = \left(50.4\ \text{W/m}^2\cdot\text{K}\right)/\left(27.7\ \text{m}^{-1}\right)(35\ \text{W/m}\cdot\text{K}) = 0.0519$$

find $\quad \dfrac{T_1 - T_\infty}{T_2 - T_\infty} = \left[32.62 + (0.0519)32.61\right]^{-1} = 0.0291 \qquad T_\infty = 452.2\ \text{K}. \qquad <$

COMMENTS: Heat conduction along the wall to the base at 375 K is balanced by convection from the air.

Problem 3

Known: Size, properties, initial and final temperature of glass balls; Cooling condition.
Unknown: The time required to cool down glass balls to the final safe to touch temp (T_f=)
Schematic:



$T_\infty = 10°C$ $V = 6 m/s$
$T_i = 200°C$ $T_f = 50°C$
$t = ?$
$K_g = 0.88 W/m\cdot K$ $\rho_g = 2400 kg/m^3$
$C_{pg} = 840 J/kg\cdot K$ $D = 0.15 m$

Assumptions:
1) Unsteady state 1-D conduction within the ball
2) Negligible radiation effects
3) Constant properties
4) Forced convection over a sphere

Properties
In case of force convection over a sphere, the properties should be evaluated at free steam temperature (T_∞). Calculated heat transfer coefficient must be convected using $(\mu_\infty / \mu_s)^{1/4}$. μ_s viscosity at surface temperature, T_s changes from 200°C to 50°C, so that $\overline{T_s} = \frac{200+50}{2} = 125°C = 398K$ should be used for evaluation of μ_s
For air at 10°C (283K): K=0.0249W/m.k, $\mu_\infty=17.6*10^{-6}$kg/m.s, v=14.38m2/s, Pr=0.711
For air at 125°C (398K): $\mu_s=23.0*10^{-6}$kg/m.s

Analysis:
It is possible that LTCM is applicable. Therefore we have to start from determining Bi, which requires h
Forced convection over a sphere
$Re_D=VD/v=6*0.15/19.38*10^{-6}=6.26*10^4$
For $Re_D=6.26*10^4$:

$$\overline{Nu_D} = 2 + (0.4 * \mathrm{Re}_D^{1/2} + 0.06 \mathrm{Re}_D^{2/3}) \mathrm{Pr}^{0.4} (\mu_\infty / \mu_s)^{1/4}$$

$$= 2 + (0.4*(6.26*10^4)^{1/2} + 0.06*(6.26*10^4)^{2/3})0.711^{0.4}(17.6/23)^{1/4} = 161$$

So $\overline{h_D} = \frac{161*0.0249}{0.15} 26.7$

For LTCM: $Bi = \frac{L_c \overline{h_D}}{k_g}$ where L_c=D/6

$$Bi = \frac{0.15/6*26.7}{0.88} = 0.759 > 0.1$$ => LTCM is not applicable

Recalculation of Bi for L_c=D/2
Bi=3*0.759=2.28
Assume that single term approximation is applicable (Fo>0.2)

For sphere: $\theta^* = C_1 \exp(-\xi_1^{\,2} Fo)\dfrac{1}{\xi_1 r^*}\sin(\xi_1 r^*)$ r* at the surface is 1.0

$$\theta^* = \frac{Tf - T\infty}{Ti - T\infty} = \frac{50-10}{200-10} = 0.2105 = C_1 \exp(-\xi_1^{\,2} Fo)\frac{1}{\xi_1 r^*}\sin(\xi_1 r^*)$$

where C_1 and ξ_1 are the tabulated values

For a sphere with Bi=2.28, extrapolation from Table 5.1 gives C_1=1.519, ξ_1=2.102

Substituting numbers:

$$0.2105 = 1.519\exp(-2.102^2 Fo)\frac{1}{2.102}\sin(2.102)$$

=> Fo=0.2456

Since Fo>0.2 then indeed single term approximation is applicable

$\text{Fo} = \dfrac{\alpha}{L_c^2}t$ => t=FoLc^2/ α where $\alpha = \dfrac{k_g}{\rho_g Cp_g} = \dfrac{0.88}{2400*840} = 4.365*10^{-7}$

So t=0.2456*(0.075)^2/4.365*10^{-7}=3165s=52min45s