PROBLEM 8.41

KNOWN: Duct diameter and length. Thermal conductivity of insulation. Gas inlet temperature and velocity and minimum allowable outlet temperature. Temperature and velocity of air in cross flow.

FIND: Minimum allowable insulation thickness.

SCHEMATIC:

Assumptions:
1. Negligible potential and kinetic energy and flow work changes for gas flow through duct.
2. Fully developed flow throughout duct.
3. Negligible duct wall conduction resistance.
4. Negligible effect of insulation thickness on outer convection coefficient and thermal resistance.
5. Properties of gas may be approximated as those of air.

Properties:
- Table A-4, air (p = 1 atm). $T_{m,i} = 1600K$: ($\rho_i = 0.218$ kg/m$^3$).
- $\bar{T}_m = (T_{m,i} + T_{m,o})/2$
- $T_f = 300K$ (assumed): $\nu = 15.89 \times 10^{-6}$ m$^2$/s, $k = 0.0263$ W/m-K, Pr = 0.707.

Analysis:
From Eqs. (8.46a) and (3.19),

$$\frac{T_{m,o} - T_{m,i}}{T_{m,i} - 1350K} = 0.852 = \exp\left(-\frac{UA_s}{m c_p}\right) = \exp\left(-\frac{1}{R_{\text{tot}} m c_p}\right)$$

Hence, with $\dot{m} = (\rho u_m A_c) = 0.218$ kg/m$^3 \times 10^3 m/s \times \pi (1m)^2 / 4 = 1.712$ kg/s,

$$R_{\text{tot}} = \frac{\ln (0.852)}{-[1.712 kg/s \times 1230 J/kg \cdot K \times (-0.160)]^{-1}} = 2.96 \times 10^{-3} K/W$$

The total thermal resistance is

$$R_{\text{tot}} = R_{\text{conv},i} + R_{\text{cond},\text{ins}} + R_{\text{conv},o} = (h_i \pi D_i L)^{-1} + \frac{\ln(D_o / D_i)}{2\pi k_{\text{ins}} L} + (h_o \pi D_o L)^{-1}$$

With $Re_{D,i} = 4 \dot{m} / \pi D_i \mu = (4 \times 1.712 \text{ kg/s}) / \left(\pi \times 1 \text{ m} \times 557 \times 10^{-6} \text{ N s/m}^2\right) = 39.130$, the Dittus-Boelter correlation yields

$$h_i = \left(\frac{k}{D}\right) 0.023 \frac{Re_{D,i}^{4/5} Pr^{1/3}}{1000} \left(\frac{100 \text{ W/m} \cdot \text{K}}{1 \text{ m}}\right) 0.023 (39.130)^{4/5} (0.685)^{1/3} = 9.57 \text{ W/m}^2 \cdot \text{K}$$

The internal resistance is then

$$R_{\text{conv},i} = (h_i \pi D_i L)^{-1} = \left(9.57 \text{ W/m}^2 \cdot \text{K} \times \pi \times 1 \text{ m} \times 100 \text{ m}\right)^{-1} = 3.33 \times 10^{-4} \text{ K/W}$$

With $Re_D = VD_i / \nu = 15 \text{ m/s} \times 1 \text{ m} / 15.89 \times 10^{-6} \text{ m}^2/\text{s} = 9.44 \times 10^5$, the Churchill-Bernstein correlation yields

Continued.....

Students may use alternative correlations. In your solution please calculate $T_{m,o}$ using the alternative correlations.

Students may use alternative correlations along with Table 7.2 or Zimbaukas correlation along with Table 7.4.
PROBLEM 8.41 (Cont.)

\[
h_o = \left( \frac{k}{D} \right) \left[ 0.3 + \frac{0.62 \text{Re}^{1/2} \text{Pr}^{1/3}}{\left[ 1 + (0.4/\text{Pr})^{2/3} \right]^{1/4}} \left[ 1 + \left( \frac{\text{Re} \cdot D}{282,000} \right)^{5/8} \right]^{7/5} \right] = 30.9 \text{ W/m}^2 \cdot \text{K}
\]

\[R_{\text{conv},o} = (h_o \pi D L)^{-1} = \left( 30.9 \text{ W/m}^2 \cdot \text{K} \times \pi \times 1 \text{m} \times 100 \text{m} \right)^{-1} = 1.03 \times 10^{-4} \text{ K/W}
\]

Hence, from Eq. (1)

\[
\frac{\ln \left( \frac{D_o}{D_i} \right)}{2\pi k_{\text{ins}} L} = \left( 2.96 \times 10^{-3} - 3.33 \times 10^{-4} - 1.03 \times 10^{-4} \right) \text{ K/W} = 2.52 \times 10^{-3} \text{ K/W}
\]

\[D_o = D_i \exp \left( 2\pi k_{\text{ins}} L \times 2.52 \times 10^{-3} \text{ K/W} \right) = 1 \text{m} \times \exp \left( 1.58 \times 10^{-2} \text{ K/W} \times 0.125 \text{ W/m} \cdot \text{K} \times 100 \text{m} \right) = 1.22 \text{m}
\]

Hence, the minimum insulation thickness is

\[t_{\text{min}} = \left( D_o - D_i \right) / 2 = 0.11 \text{m}\]

**COMMENTS:** With \(D_o = 1.22\text{m},\) use of \(D_i = 1\text{m}\) to evaluate the outer convection coefficient and thermal resistance is a reasonable approximation. However, improved accuracy may be obtained by using the calculated value of \(D_o\) to determine conditions at the outer surface and iterating on the solution.
Problem 2 - Solution

Known: Dimensions of air conditioning duct, inlet air temperature and external conditions

Find: Outlet air temperature and effect of practical simplifying assumptions

Assumptions: 1) Steady state conditions
2) Negligible kinetic energy, potential energy, and fluid work changes for air inside the duct
3) Negligible radiation effects
4) Ambient air is quiescent
5) Negligible influence effects
6) Resistance to conduction through the walls of the duct (thin walls)

Schematic

Properties i) For inside heat transfer coefficient $h_i$, the properties are to be evaluated at $5^\circ C$ as requested in problem statement

For air at $5^\circ C = 278K$:
\[ k = 24.54 \times 10^{-3} \text{ W} / \text{m} \cdot \text{K} \]
\[ \mu = 1730 \text{ N} \cdot \text{m} / \text{kg} \cdot \text{s} \]
\[ \beta = 1.25 \times 10^5 \text{ kg/m}^3 \cdot \text{s} \]
\[ \gamma = 0.713 \]
\[ \alpha = 1003 \text{ J} / \text{K} \cdot \text{kg} \]

ii) For outside heat transfer coefficient $h_o$, the properties should be evaluated at film temperature $T_f = \frac{T_o + T_i}{2} = 295.5 \approx 300K$
\[ k = 26.3 \times 10^{-3} \text{ W} / \text{m} \cdot \text{K} \]
\[ \mu = 15.83 \times 10^{-6} \text{ N} \cdot \text{m} / \text{kg} \cdot \text{s} \]
\[ \gamma = 0.707 \]
\[ \alpha = 4.18 \text{ J} / \text{K} \cdot \text{kg} \]

Analysis:
The temperature of air along the duct changes according to
\[ \frac{T_o - T_{o,0}}{T_{o,0} - T_{i,0}} = \exp \left( -\frac{qL}{m_o \cdot \alpha} \right) \quad (1) \]
where \( \bar{J} \) \( \Rightarrow \frac{1}{U} = \frac{1}{\bar{h}_i} = \frac{1}{\bar{h}_0} \) \( \Rightarrow \) because negligible heat transfer to convection through walls of duct and since wall is thin \( \bar{A}_i \approx A_0 \)

**Determination of \( \bar{h}_i \)**

- Case: Forced flow through a noncircular duct \( \Rightarrow \) \( L_c = D_0 = \frac{4A_c}{P} = \frac{4H \cdot W}{28.72W} = \frac{4H^2}{4H} = H = 0.2 \) m
  
  \( U_0 \), \( H \cdot W = 0.2 \) m

  \( \bar{h}_{\text{eq}} = \frac{U_m D_0 \rho}{\mu} \) \( \Rightarrow \frac{U_m}{\mu} \) \( = \frac{\frac{m}{\bar{h} \cdot \bar{h}^2}}{M} \) \( \Rightarrow \frac{m}{\bar{H} M} = \frac{0.06}{0.2 \cdot 175.6 \cdot 10^{-7}} = 11.520 \Rightarrow \) turbulent flow

  - Since the flow is turbulent, one can use correlations for circular pipes. Dittus-Boelter equation may be used since \( T_b \) is not significantly different from \( T_w \). With \( \bar{S} > \bar{T}_w \)

    \( n = 0.4 \). Neglecting entrance effects: \( \bar{N}_{\text{in}} = \bar{N}_{\text{in}} \)

    \( \bar{N}_{\text{in}} = 0.023 L^0.45 \rho \cdot 0.6 \cdot 0.023 \cdot (11.520)^{4/5} \cdot (0.75) \cdot 0.6 \cdot 35.66 = \frac{\bar{h}_i D_0}{k} \)

    \( \bar{h}_i = 35.66 \cdot 24.54 \cdot 10^{-3} \cdot 0.2 \) \( \Rightarrow \frac{138 \mu^2 / k}{W} \)  

**Determination of \( \bar{h}_0 \)**

\( \bar{h}_0 \) is not the same for bottom, top and side surfaces

\( \bar{h}_0 = \left( \bar{h}_0 - 2 \bar{h}_l - 2 \bar{h}_w \right) / (2 \bar{h}_l + 2 \bar{h}_w) \) \( \Rightarrow \) since \( H = L \)

\( \bar{h}_0 = \frac{1}{2} \bar{h}_l + \frac{1}{4} \bar{T}_w - \frac{1}{4} \bar{h}_w \)  

- For side surface, since the duct is horizontal, no more free convection over vertical plate with \( L_c = H = 0.2 \) m

  \( \bar{R}_{\text{a, lateral}} = \frac{g \beta (T_w - T_0) L^3}{\nu^2} \cdot \bar{R}_{\text{v, lateral}} = \frac{9.8 \cdot 0.0388 (35 - 10) \cdot 0.2^3}{(15.83 \cdot 10^{-6})^2} \) \( \Rightarrow \bar{R}_{\text{a, lateral}} = 1.855 \cdot 10^7 \)

  For \( \bar{R}_{\text{a, lateral}} < 10^3 \), Eq. (3.27) may be used

  \( \bar{N}_{\text{in}} = 0.68 + \frac{0.670 \bar{R}_{\text{a, lateral}}^{1/4}}{\left[ 1 + (0.432 \bar{R}_{\text{a, lateral}})^{3/8} \right]^{1/4}} \cdot \frac{0.69}{0.670 (1.855 \cdot 10^7)^{1/4}} \) \( \Rightarrow \bar{N}_{\text{in}} = 34.42 \)

  \( \bar{h}_l = 34.42 \cdot 26.3 \cdot 10^{-3} \cdot 0.2 \) \( \Rightarrow \frac{44.53 \mu^2 / k}{W} \)
For the top surface since the chord is horizontal, we have natural convection from the top surface at cold plate for which Eq. (2.32) is applicable.

\[
\overline{Nu}_L = 0.27 \cdot \frac{Ra_L^{1/4}}{\overline{Le}^{1/2}} \quad \text{where} \quad \overline{Le} \cdot \frac{As}{\overline{Le}} = \frac{L_c}{2} \cdot \frac{21.21}{2.4 + 2.02} = 0.0876
\]

With different \( Le \) we need to recalculate \( Ra_L \).

\[
Ra_L = \frac{3.8 \cdot 0.00328 \left( 35 \cdot 10^4 \right) \left( 0.0876 \right)^3}{(15.23 \cdot 10^8)^2} \cdot 0.707 = 2.15 \cdot 10^6
\]

\[
\overline{Nu}_L = 0.27 \cdot \left( 2.15 \cdot 10^6 \right)^{1/4} = 10.37 = \frac{\overline{h}_L \cdot L_c}{k} \Rightarrow \overline{h}_L = \frac{10.37 \cdot 0.0876}{0.0876} = 11.7 \frac{W}{m^2 \cdot K}
\]

For the bottom surface since the chord is horizontal, we have free convection from the top surface of cooled plate \( L_c \) and thus \( Ra_L \) are the same as above and Eq. (3.30) is applicable.

\[
\overline{Nu}_L = 0.54 \cdot \frac{Ra_L^{1/4}}{\overline{Le}^{1/2}} = 20.08 = \frac{\overline{h}_b \cdot L_c}{k} \Rightarrow \overline{h}_b = \frac{20.08 \cdot 0.0876}{0.0876} = 2.73 \frac{W}{m^2 \cdot K}
\]

Substituting calculated \( \overline{h}_b \), \( \overline{h}_L \) and \( \overline{h}_0 \) into Eq. (30),

\[
\overline{h}_0 = \frac{1}{2} \left[ 4.53 + \frac{1}{4} \cdot 2.73 + \frac{1}{4} \cdot 5.57 \right] = 4.36 \frac{W}{m^2 \cdot K}
\]

\[
\Rightarrow \text{Now substituting} \quad \overline{h}_0 \quad \text{and} \quad \overline{h}_0 \quad \text{into Eq. (2),}
\]

\[
\frac{1}{U} = \frac{1}{4.38} + \frac{1}{4.36} \Rightarrow U = 2.18 \frac{W}{m^2 \cdot K}
\]

\[
\Rightarrow \text{Rearranging Eq. (1) and substituting numerical values,}
\]

\[
T_{m,2} = T_{m,1} - \left( T_{m,1} - T_{m,2} \right) \exp \left( -\frac{7\overline{h}}{\overline{h}_0} \cdot \frac{U}{E} \right) = 35 - (35 - 5) \exp \left( \frac{4 \cdot 0.2 \cdot 8}{0.04 \cdot 2.18} \right)
\]

\[
= 35 - 30 \exp \left( -0.036 \right) = 6.0 \degree C
\]

\[
\Rightarrow \text{Final answer}
\]

Comments:
1) Since \( T_{m,2} \) is close \( T_{m,1} \) evaluating properties at \( T_{m,1} \) was not a bad idea.
2) In this problem \( L/D_4 = 8/0.2 = 40 \Rightarrow \text{radiation effects might not be negligible.} \)
   \( \text{The radiation effects would lead to some increase in} \overline{h}_L \text{, and thus higher} \overline{T}_{m,2} \)
3) Since \( \overline{h}_0 \) \& \( \overline{h}_0 \), \( \overline{T}_3 \) will be approximately \( (\overline{T}_m \cdot \overline{T}_f) / 2 = 20 \degree C \) rather than \( 10 \degree C \)
   \( \Rightarrow \) Higher \( \overline{T}_3 \) would lead to decrease (very slight) of \( \overline{h}_0 \) thus lower \( T_{m,2} \)
Problem 3 - Solution

Known: Water in an isolated tank being heated by a coil inside the tank in which saturated steam cools.

Find: Time required to heat water from 300 to 360 K, and the amount of steam condensed.

Schematic:

![Diagram of the system showing water and steam at different conditions and a coil for heat transfer.]

Water \( T_{in} = 300 \text{ K} \), \( V = 80L = 80 \times 10^{-3} \text{ m}^3 \), \( T_{sat} = 360 \text{ K} \)

Assumptions:

1) Thermal resistances of the tube wall and on the condensing side (inside the tube) are negligible.
2) Tank is perfectly insulated so that all heat released by the condensing steam is used to heat water inside the tank.
3) The water inside the tank can be treated as a quiescent medium, which changes its temperature from 300 to 360.
4) Heat transfer between the coil and water can be modeled as natural convection over horizontal cylinders.
5) Constant properties.

Properties:

For natural convection over horizontal cylinders, two properties should be evaluated at film temperature \( T_f = \frac{(T_{sat} + T_{in})}{2} \)

Because of Assumption 1, \( T_{sat} = T_{sat, steam} \). From Table A6 for saturated steam at 1.1 atm, \( T_{sat} = 374.5 \text{ K} \). Also, heat of vaporization \( h_g = 24.32 \times 10^3 \text{ J/kg} \).

The changes from 300 to 360 \( \Rightarrow \) \( T_{sat} = 323 \text{ K} \)

\( T_f = \frac{(330 + 374.5)}{2} = 352.2 \text{ K} \)

For water at 352.2 K:

\[ K = \frac{0.609}{4} \text{ m}^2/\text{K} , \quad \Phi = 2.22 , \quad \frac{1}{\dot{m}} = 1028 \text{ m}^3/\text{kg}, \quad \dot{m} = 972.8 \text{ kg/h} \]
Analysis

- The system in this problem is water in the tank. This is a closed system.

Energy balance on water:

\[ E_{in} + E_g - E_{out} = E_{si} \Rightarrow \dot{Q}_D \text{ Ac} (T_s - T_0) = \dot{Q}_{cp} \frac{\Delta t}{\pi} \]

where \( T_0 \) is variable depending on \( t \)

\[ \text{Ac} = \pi L \quad \text{where} \quad T = \pi D_r \quad L = \pi D_2 \text{ n} \]

\[ \text{Ac} = (\pi 0.01) (\pi 0.4) 12 = 0.476 \text{ m}^2 \]

- Evaluation of \( \dot{Q}_D \Rightarrow \text{For free convection on horizontal tube} \quad \text{Le} = D_r 0.01 n \)

\[ \text{Ra}_D = \text{Gr}_D \text{ Pr} = \frac{\rho \beta (T_s - T_0) D^4}{\nu^3} \text{ Pr} \]

\( T_0 \) changes so \( \text{Ra}_D \) changes but we will use \( T_0 \)

\[ \text{Ra}_D = \frac{9.8 \cdot 836.6 \cdot 10^5 (374.5 - 280) \cdot 0.01^3}{(35510^6/372.8)^2} \times 2.22 \approx 1.62 \cdot 10^6 \]

\( \text{Nu}_D \) may be determined using Eq. (8.33) along with Table 1.1 or Eq. (9.36)

\[ \Rightarrow \text{Eq. (9.33)} \quad \text{Nu}_D = C \text{ Ra}_D^n \quad \text{where for} \quad \text{Ra}_D = 4.62 \cdot 10^6 \quad C = 0.480 \quad n = 0.250 \]

\[ \text{Nu}_D = 0.480 \left(4.62 \cdot 10^6\right)^{0.25} = 22.26 \]

\[ \Rightarrow \text{Eq. (9.34)} \quad \frac{\text{Nu}_D}{D} = \left\{ 0.60 + \frac{0.387 \text{ Ra}_D^{1/4}}{[1 + (0.555/\text{Pr})^{3/4} \text{ Ra}_D^{1/4}]} \right\}^2 \]

\[ = \left\{ 0.60 + \frac{0.387 \cdot 4.62 \cdot 10^6^{1/4}}{[1 + (0.555/20)^{3/4} \cdot 4.62 \cdot 10^6^{1/4}]} \right\}^2 \]

\[ \therefore \text{Nu}_D = 25.65 \] \( \text{we will use this value} \)

\[ h_D = \frac{\text{Nu}_D \cdot K}{D} = \frac{25.65 \cdot 0.669}{0.01} = 1716 \text{ W/m}^2 \text{K} \]

- In order to solve Eq. (1) we need initial condition: \( T_0 (t=0) = T_{01}, \quad T_1 (t > 0) \)

\[ \frac{dT}{T_s - T_0} = \frac{\text{h}_D \text{ Ac} \text{ t}}{SV_{cp}} \quad \Rightarrow \quad \ln (T_s - T_0) = \frac{h_D \text{ Ac}}{SV_{cp}} t + C \]

Substituting initial condition: \( C = \ln (T_s - T_{01}) \Rightarrow \ln \left(\frac{T_s - T_0}{T_s - T_{01}}\right) = -\frac{h_D \text{ Ac}}{SV_{cp}} t \)
\[ t = - \frac{1}{h_d} \ln \left( \frac{T_s - T_{oo}}{T_s - T_{oi}} \right) \frac{A_{coil}}{A_{vep}} \]

\[ \frac{1}{h_d} = 1.016 \times 10^{-2} \, \text{m}^3 \text{kg}^{-1} \text{K}^{-1} \]
\[ \rho = 384.2 \, \text{kg} \text{m}^{-3} \]
\[ C_p = 4184 \, \text{J} \text{kg}^{-1} \text{K}^{-1} \]

\[ t = - \frac{1}{h_d} \ln \left( \frac{374.5 - 360}{374.5 - 360} \right) \left/ \left( \frac{1716 \times 0.474}{384.2 \times 80 \times 0.4184} \right) \right. \]
\[ = 663.5 \, \text{s} \]

Note:
1. In principle, \( h_d \) is not constant, it changes with time so that the value determined above based on \( h_d \) at \( T_{oo} \) might be inaccurate.
2. This is a transient heat problem and since heat losses from the tower are negligible, one could use LTEM approach, for water, which would yield identical equation for \( t \).