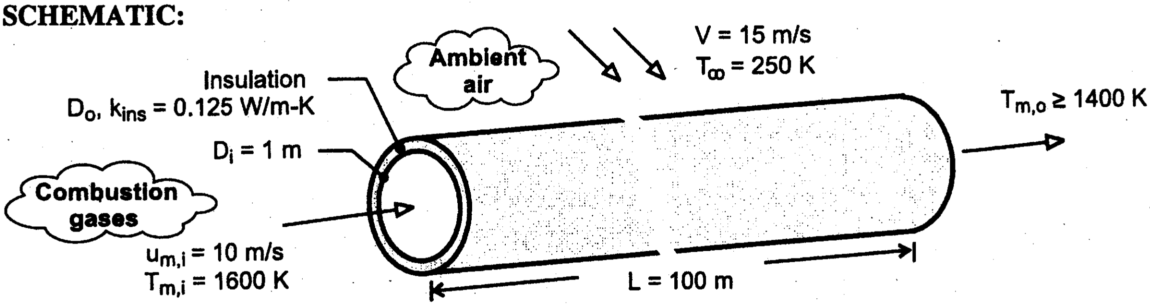
PROBLEM 8.41

KNOWN: Duct diameter and length. Thermal conductivity of insulation. Gas inlet temperature and velocity and minimum allowable outlet temperature. Temperature and velocity of air in cross flow.

FIND: Minimum allowable insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible potential and kinetic energy and flow work changes for gas flow through duct, (2) Fully developed flow throughout duct, (3) Negligible duct wall conduction resistance, (4) Negligible effect of insulation thickness on outer convection coefficient and thermal resistance, (5) Properties of gas may be approximated as those of air.

PROPERTIES: *Table A-4*, air (p = 1 atm). $T_{m,i} = 1600K$: ($\rho_i = 0.218$ kg/m^3). $\bar{T}_m = (T_{m,i} + T_{m,o})/2 = 1500K$: ($\rho = 0.232$ kg/m^3, $c_p = 1230$ J/kg·K, $\mu = 557 \times 10^{-7}$ N·s/m^2, k = 0.100 W/m·K, Pr = 0.685). $T_f \approx 300K$ (assumed): $\nu = 15.89 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707.

ANALYSIS: From Eqs. (8.46a) and (3.19),

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \frac{-1150\,K}{-1350\,K} = 0.852 = \exp\left(-\frac{\bar{U}A_s}{\dot{m}c_p}\right) = \exp\left(-\frac{1}{R_{tot}\dot{m}c_p}\right)$$

Hence, with $\dot{m} = (\rho u_m A_c)_i = 0.218\,kg/m^3 \times 10\,m/s \times \pi(1m)^2/4 = 1.712\,kg/s$,

$$R_{tot} = -\left[\dot{m}c_p \ln(0.852)\right]^{-1} = -\left[1.712\,kg/s \times 1230\,J/kg\cdot K \times (-0.160)\right]^{-1} = 2.96 \times 10^{-3}\,K/W$$

The total thermal resistance is

$$R_{tot} = R_{conv,i} + R_{cond,ins} + R_{conv,o} = (h_i\pi D_i L)^{-1} + \frac{\ln(D_o/D_i)}{2\pi k_{ins} L} + (h_o\pi D_o L)^{-1} \qquad (1)$$

With $Re_{D,i} = 4\dot{m}/\pi D_i\mu = (4 \times 1.712\,kg/s)/\left(\pi \times 1m \times 557 \times 10^{-7}\,N\cdot s/m^2\right) = 39{,}130$, the Dittus-Boelter correlation yields

Students may use alternative correlations e.g. Colburn eq., Sieder-Tate, or Gnielinski's correlations

$$h_i = \left(\frac{k}{D}\right)0.023 Re_D^{4/5} Pr^{1/3} = \left(\frac{0.100\,W/m\cdot K}{1m}\right)0.023(39{,}130)^{4/5}(0.685)^{1/3} = 9.57\,W/m^2\cdot K$$

In your solution, please calculate $\bar{h}_i$ using these alternative correlations

The internal resistance is then

$$R_{conv,i} = (h_i\pi D_i L)^{-1} = \left(9.57\,W/m^2\cdot K \times \pi \times 1m \times 100m\right)^{-1} = 3.33 \times 10^{-4}\,K/W$$

With $Re_D \approx VD_i/\nu = 15$ m/s $\times$ 1m/15.89 $\times 10^{-6}$ m^2/s $= 9.44 \times 10^5$, the Churchill-Bernstein correlation yields

Continued .....

Students may use alternatively Hilpert correlation along with Table 7.2 or Zhukauskas correlation along with Table 7.4

In your solution please calculate $\bar{h}_o$ using the alternative correlations

$$h_o \approx \left(\frac{k}{D}\right)\left\{0.3+\frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}\right\} = 30.9\,W/m^2\cdot K$$

$$R_{conv,o} \approx (h_o \pi D_i L)^{-1} = \left(30.9\,W/m^2\cdot K\times\pi\times 1m\times 100m\right)^{-1} = 1.03\times10^{-4}\,K/W$$

Hence, from Eq. (1)

$$\frac{\ln(D_o/D_i)}{2\pi\,k_{ins}\,L} = \left(2.96\times10^{-3} - 3.33\times10^{-4} - 1.03\times10^{-4}\right)K/W = 2.52\times10^{-3}\,K/W$$

$$D_o = D_i \exp\left(2\pi k_{ins} L\times 2.52\times10^{-3}\,K/W\right) = 1m\times\exp\left(1.58\times10^{-2}\,K/W\times 0.125\,W/m\cdot K\times 100m\right) = 1.22m$$

Hence, the minimum insulation thickness is

$$t_{min} = (D_o - D_i)/2 = 0.11m$$ <

COMMENTS: With D_o = 1.22m, use of D_i = 1m to evaluate the outer convection coefficient and thermal resistance is a reasonable approximation. However, improved accuracy may be obtained by using the calculated value of D_o to determine conditions at the outer surface and iterating on the solution.

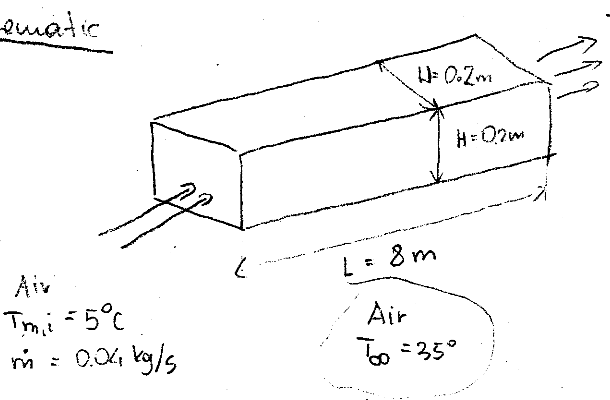
Problem 2 - Solution

Known: Dimensions of air conditioning duct, inlet air temperature and external conditions

Find: Outlet air temperature and effect of proposed simplifying assumptions

Assumptions:
1) Steady state conditions
2) Negligible kinetic energy, potential energy and flow work changes for air inside the duct
3) Negligible radiation effects
4) Ambient air is quiescent
5) Negligible entrance effects
6) -"- resistance to conduction through the walls of the duct (thin walls)

Schematic



s → side of duct
t → top of duct
b → bottom of duct

Properties

i) For inside heat transfer coefficient $\bar{h}_i$ the properties are to be evaluated at 5 °C as requested in problem statement

For air at 5 °C = 278 K:
$k = 24.54 \cdot 10^{-3}$ W/m K
$\mu = 173.6 \cdot 10^{-7}$ kg/m.s
$\rho = 1.2641$ kg/m³
Pr = 0.713
c_p = 1007 J/kg K

ii) For outside heat transfer coefficient $\bar{h}_o$, the properties should be evaluated at film temperature $T_f = \frac{\bar{T}_s + T_\infty}{2} = 295.5 \approx 300$ K

$\bar{T}_s = 10$ °C is given in the problem statement

$k = 26.3 \cdot 10^{-3}$ W/m K
$\nu = 15.89 \cdot 10^{-6}$ m²/s
Pr = 0.707
$\beta = \frac{1}{T_f} = 0.00338$

Analysis:

The temperature of air along the duct changes according to

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m} c_p} \bar{U}\right) \quad \cdots \quad (1)$$

where $\bar{U}$, $\Rightarrow$ $\frac{1}{\bar{U}} = \frac{1}{\bar{h}_i} + \frac{1}{\bar{h}_o}$ ...(2) $\Rightarrow$ because negligible resistance to conduction through walls of duct and since the wall is thin $A_i \approx A_o$

- Determination of $\bar{h}_i$

  - Case: forced flow through a noncircular duct $\Rightarrow$ $L_c = D_h = \frac{4A_c}{P} = \frac{4 H \cdot W}{2H + 2W} = \frac{4H^2}{4H} = H = 0.2\ m$

N.B $H = W = 0.2\ m$

  - $Re_{D_h} = \frac{U_m D_h \rho}{\mu}$ where $U_m = \frac{\dot{m}}{\rho H^2}$

$$= \frac{\frac{\dot{m}}{\rho H^2} H \rho}{\mu} = \frac{\dot{m}}{H \mu} = \frac{0.04}{0.2 \cdot 173.6 \cdot 10^{-7}} = 11520 \Rightarrow \text{turbulent flow}$$

  - Since the flow is turbulent, one can use correlations for circular pipes. Dittus-Boelter equation may be used since T_s is not significantly different from $\bar{T}_m$. with $T_s > T_m$ $n = 0.4$. Neglecting entrance effects, $\bar{Nu}_{D_h} = Nu_{D_h}$

$$\bar{Nu}_D = 0.023 Re^{4/5} Pr^{0.4} = 0.023 \cdot (11520)^{4/5} \cdot (0.713)^{0.4} = 35.66 = \frac{\bar{h}_i D_h}{k}$$

$$\therefore \bar{h}_i = \frac{35.66 \cdot 24.54 \cdot 10^{-3}}{0.2} = 4.38\ W/m^2K$$

- Determination of $\bar{h}_o$

$\bar{h}_o$ is not the same for bottom, top and side surfaces

$$\bar{h}_o = \left(\bar{h}_s \cdot 2 H \cdot L + \bar{h}_b W \cdot L + \bar{h}_t WL\right) / \left(2 W \cdot L + 2 W H\right) \overset{\text{since } H = L}{=} \frac{1}{2}\bar{h}_s + \frac{1}{4}\bar{h}_b + \frac{1}{4}\bar{h}_t \quad ...(3)$$

  - For side surface, since the duct is horizontal, we have free convection over vertical plate with $L_c = H = 0.2\ m$

$$Ra_L = \frac{g \beta (T_\infty - T_s) H^3}{\nu^2} \cdot Pr = \frac{9.8 \cdot 0.00338 (35 - 10) \cdot 0.2^3}{(15.89 \cdot 10^{-6})^2} \cdot 0.707 = 1.855 \cdot 10^7$$

For $Ra_L < 10^9$ Eq. (9.27) may be used

$$\bar{Nu}_L = 0.68 + \frac{0.670\, Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} = 0.69 + \frac{0.670 (1.855 \cdot 10^7)^{1/4}}{\left[1 + (0.492/0.707)^{9/16}\right]^{4/9}} = 34.42$$

$$\therefore \bar{h}_s = \frac{34.49 \cdot 26.3 \cdot 10^{-3}}{0.2} = 4.53\ W/m^2K$$

- For top surface. Since the duct is horizontal, we have natural convection from upper surface of cold plate for which Eq (9.32) is applicable

$$\overline{Nu}_L = 0.27 \cdot Ra_L^{1/4} \qquad \text{where } L_c = \frac{A_s}{P} = \frac{W \cdot L}{2L + 2W} = \frac{0.2 \cdot 8}{2 \cdot 8 + 2 \cdot 0.2} = 0.0976 \text{ m}$$

with different L_c we need to recalculate Ra_L.

$$Ra_L = \frac{9.8 \cdot 0.00338 (35 - 10)(0.0976)^3}{(15.89 \cdot 10^{-6})^2} \cdot 0.707 = 2.153 \cdot 10^6$$

$$\overline{Nu}_L = 0.27 \cdot (2.153 \cdot 10^6)^{1/4} = 10.34 = \frac{\overline{h}_t \cdot L_c}{k} \Rightarrow \overline{h}_t = \frac{10.34 \cdot 26.3 \cdot 10^{-3}}{0.0976} = 2.79 \frac{W}{m^2 K}$$

- For bottom surface. Since the duct is horizontal, we have free convection from lower surface of cooled plate. L_c and thus Ra_L are the same as above and Eq (9.30) is applicable

$$\overline{Nu}_L = 0.54 \cdot Ra_L^{1/4} = 20.68 = \frac{\overline{h}_b \cdot L_c}{k} \Rightarrow \overline{h}_b = \frac{20.68 \cdot 26.3 \cdot 10^{-3}}{0.0976} = 5.57 \frac{W}{m^2 K}$$

Substituting calculated $\overline{h}_s$; $\overline{h}_t$ and $\overline{h}_b$ into Eq. (3)

$$\overline{h}_o = \frac{1}{2} 4.53 + \frac{1}{4} 2.79 + \frac{1}{4} 5.57 = \boxed{4.36 \ W/m^2 K}$$

$\Rightarrow$ Now substituting $\overline{h}_i$ and $\overline{h}_o$ into Eq (2)

$$\frac{1}{\overline{U}} = \frac{1}{4.38} + \frac{1}{4.36} \Rightarrow \boxed{\overline{U} = 2.18 \ W/m^2 k}$$

$\Rightarrow$ Rearranging Eq (1) and substituting numerical values

$$T_{m,o} = T_\infty - (T_\infty - T_{m,i}) \exp\left(-\frac{PL}{\dot{m} c_p} \overline{U}\right) = 35 - (35 - 5) \exp\left(\frac{4 \cdot 0.2 \cdot 8}{0.04 \cdot 1007} \cdot 2.18\right)$$

$$= 35 - 30 \exp(-0.0346) = \underline{\underline{6.0 \ ^\circ C}}$$

Final answer

Comments:

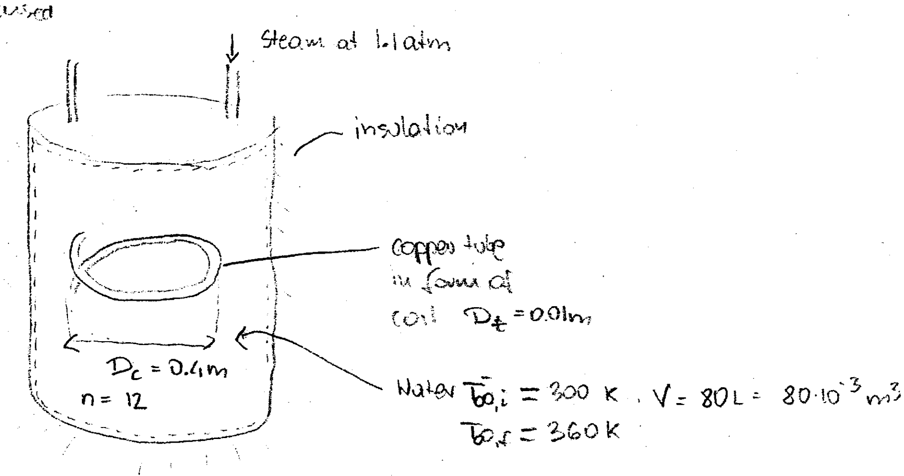
1) Since $T_{m,o}$ is close $T_{m,i}$ evaluating properties at $T_{m,i}$ was not a bad idea
2) In this problem $L/D_h = 8/0.2 = 16 \Rightarrow$ entrance effects might not be negligible. The entrance effects would lead to some increase in $\overline{h}_i$, and thus higher $T_{m,o}$
3) Since $\overline{h}_o \approx \overline{h}_i$, $\overline{T}_s$ will be approximately $(\overline{T}_m + T_\infty)/2 \approx 20°C$ rather than $10°C$. Higher $\overline{T}_s$ would lead to decrease (very slight) of $\overline{h}_o$ thus lower $T_{m,o}$

Problem 3 - Solution

Known: Water in an isolated tank being heated by a coil inside the tank in which saturated steam condenses

Find: Time required to heat water from 300 to 360 K and the amount of steam condensed

Schematic



Assumptions

1) Thermal resistances of the tube wall and on the condensing side (inside the tube) are negligible
2) Tank is perfectly insulated so that all heat released by the condensing steam is used to heat water inside the tank
3) The water inside the tank can be treated as quiescent medium which changes its temperature from 300 to 360
4) Heat transfer between the coil and water can be modelled as natural convection over horizontal cylinder
5) Constant properties

Properties

For natural convection over horizontal cylinder, the properties should be evaluated at film temperature $T_f = (T_s + T_\infty)/2$

Because of assumption 1, $T_s = T_{sat,steam}$. From Table A6 for sat steam at 1.1 atm, $T_{sat} \simeq 374.5K$. Also heat of vaporization $h_{fg} = 2498 \cdot 10^3$ J/kg

T_∞ changes from 300 to 360 $\Rightarrow \overline{T_\infty} = 330$ K

$\therefore T_f = (330 + 374.5)/2 = 352.2$ K

For water at 352.2 K: $k = 0.669$ W/m K, $Pr = 2.22$, $\frac{1}{\rho} = 1.028 \frac{m^3}{kg}$

$\mu = 355 \cdot 10^{-6} \frac{kg}{m \cdot s}$, $\beta = 636.6 \cdot 10^{-6} \frac{1}{K}$ $\Leftrightarrow \rho = 972.8 \frac{kg}{m^3}$

Analysis

- The system in this problem is water in the tank. This is a closed system

Energy balance on water:

$$\dot{E}_{in} + \cancel{\dot{E}_g} - \cancel{\dot{E}_{out}} = \dot{E}_{st} \Rightarrow \bar{h}_D A_{coil} (T_s - T_\infty) = \rho V c_p \frac{dT_\infty}{dt} \quad (1)$$

0 0

where T_∞ is variable dependent on t

$A_{coil} = PL$ where $P = \pi D_t$ $L = \pi D_c n$

$\therefore A_{coil} = (\pi \ 0.01)(\pi \cdot 0.4) \cdot 12 = 0.474 \, m^2$

- Evaluation of $\bar{h}_D$ ⇒ For free convection on horizontal tube $L_c = D = 0.01m$

$$Ra_D = Gr_D Pr = \frac{g\beta(T_s - T_\infty) D^3}{\nu^2} Pr$$

T_∞ changes so Ra_D changes but we will use $\overline{T_\infty}$

$$Ra_D = \frac{9.8 \cdot 636.6 \cdot 10^{-6} (374.5 - 330) \cdot 0.01^3}{(355 \cdot 10^{-6}/972.8)^2} \cdot 2.22 = 4.62 \cdot 10^6$$

$\overline{Nu}_D$ may be determined using Eq (9.33) along with table 9.1 or Eq. (9.34)

⇒ Eq (9.33) $\overline{Nu}_D = C \, Ra_D^n$ where for $Ra_D = 4.62 \cdot 10^6$: $C = 0.480$, $n = 0.250$

$$\overline{Nu}_D = 0.480 (4.62 \cdot 10^6)^{0.25} = \underline{22.26}$$

⇒ Eq (9.34)

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2 = \left\{ 0.60 + \frac{0.387 (4.62 \cdot 10^6)^{1/6}}{[1 + (0.559/2.22)^{9/16}]^{8/27}} \right\}^2$$

$\therefore \overline{Nu}_D = \underline{25.65}$ ◀ ⇐ we will use this value

$$\therefore h_D = \frac{\overline{Nu}_D \cdot k}{D} = \frac{25.65 \cdot 0.669}{0.01} = \underline{1716 \ W/m^2 K}$$ ◀

- In order to solve Eq (1) we need initial condition $T_\infty(t=0) = T_{\infty,i} = 300$

$$\frac{dT_\infty}{T_s - T_\infty} = \frac{\bar{h}_D A_{coil}}{\rho V c_p} dt \Rightarrow \ln(T_s - T_\infty) = -\frac{\bar{h}_D A_{coil}}{\rho V c_p} t + C$$

Substituting initial condition $C = \ln(T_s - T_{\infty,i}) \Rightarrow \ln \frac{(T_s - T_\infty)}{(T_s - T_{\infty,i})} = -\frac{\bar{h}_D A_{coil}}{\rho V c_p} t$

$$\therefore \quad t = -\ln\left(\frac{T_s - T_\infty}{T_s - T_{\infty,i}}\right) \Big/ \frac{\overline{h}_D A_{coil}}{\rho V c_p}$$

ρ and c_p should be at $\overline{T}_\infty = 330\ K$ => From Table AC

$$\frac{1}{\rho} = 1.016\ 10^{-3}\ \frac{m^3}{kg} \quad = 984.2\ \frac{kg}{m^3} \quad , \quad c_p = 4184\ \frac{J}{kg\ K}$$

$$t = -\ln\left(\frac{374.5 - 360}{374.5 - 300}\right) \Big/ \left(\frac{1716 \cdot 0.474}{984.3\ 86\ 10^{-3} \cdot 4184}\right) = \underline{\underline{663\ s}}$$ ans

Note: 1) In principle $\overline{h}_D$ is not constant, it changes with time so that the value determined above based on $\overline{h}_D$ at $\overline{T}_\infty$ might be in error.

2) This is transient heat problem and since heat losses from the tank are negligible, one could use LTCM approach, for water, which would yield identical equation for t