Université d'Ottawa · University of Ottawa

Faculté de génie
Génie chimique

Faculty of Engineering
Chemical Engineering

CHG 2314
Midterm Exam

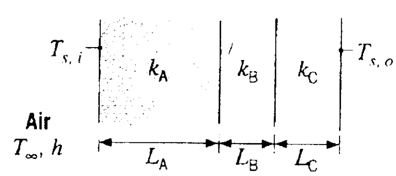
March 4, 2005

Duration: 85 min
One textbook and one double-sided reference or crib page are allowed. The crib page must be submitted with the exam paper.
Do any 2 problems; the exam will be marked out of 25
State clearly all assumptions

Good Luck!!!

(12.5) Problem 1

The composite wall of an oven consists of three materials, two which are of known thermal conductivity, k_A = 20 W/m K and k_C = 50 W/m K, and known thickness, L_A = 0.30 m and L_C = 0.15 m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B = 0.15 m, but unknown thermal conductivity k_B. Under steady state operating conditions, measurements reveal an outer surface temperature of $T_{s,o}$ = 20°C, and inner surface temperature $T_{s,i}$ = 600°C and an oven air temperature of T_∞ = 800°C. If the inside convection heat transfer coefficient h is known to be 25 W/m^2 K, what is the value of k_B ?



If the thickness of material B were increased to L_B = 0.20 m, while $T_{s,o}$, T_∞, and h remained unchanged, what would be the new $T_{s,i}$?

(12.5) Problem 2

A circular copper rod (k = 400 W/m K) of diameter D = 1 mm and length L = 25 mm is used to enhance heat transfer from a surface that is maintained at $T_{s,1}$ = 100°C. One end of the rod is attached to this surface (at x = 0), while the other end (at x = 25 mm) is joined to the second surface, which is maintained at $T_{s,2}$ = 0°C. Air flowing between the surfaces and over the rods is also at temperature of T_∞ = 0°C, and the convection coefficient of h = 100 W/m^2 K is maintained. What is the rate of heat transfer by convection from the rod?

NB. The heat is transferred by convection from the side of the rod and by conduction from the tip of the rod to the cold wall.

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(12.5) Problem 3

The 150-mm-thick wall of a gas-fired furnace is constructed of fire-clay brick (k = 1.5 W/m K, ρ = 2600 kg/m^3, c_p = 1000 J/kg K) and is well insulated at its outer surface. The wall is at uniform initial temperature of 20°C, when the burners are fired and the inner surface is exposed to products of combustion for which T_∞ = 950°C and h = 100 W/m^2 K.

a) How long does it take for the outer surface of the wall to reach a temperature of 750°C?

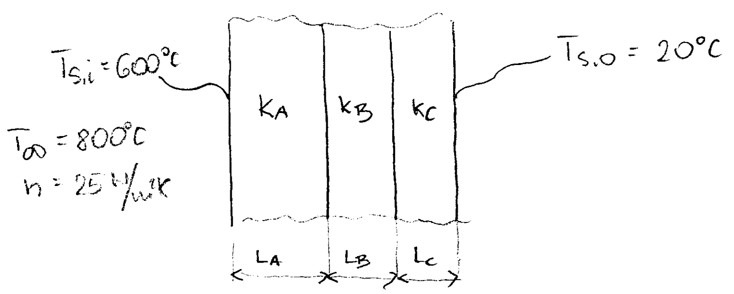
b) What is the corresponding average temperature of the wall at that time?

Problem 1

Known: Thicknesses of three materials which form a composite wall and thermal conductivities of two materials, inner and outer surface temperatures of the composite, also temperature and convection coefficient associated with adjoining gas

Unknown: (i) Value of unknown thermal conductivity k_B.
(ii) Inside surface temperature $T_{s,i}$ when the thickness of B is increased to $L_B = 0.20$

Schematic:



$L_A = 0.3\ m$

$L_B = 0.15\ m$ (i) and $L_B = 0.2\ m$ (ii)

$L_C = 0.15\ m$

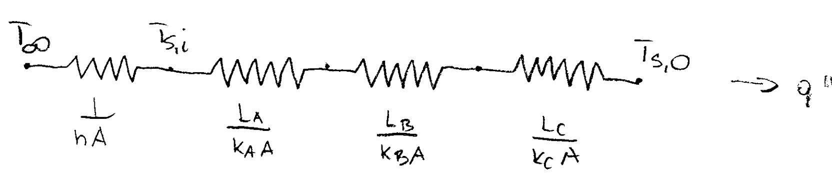
$k_A = 20\ W/mK$

$k_C = 50\ W/mK$

Assumptions:

1. Steady state conditions
2. One-dimensional conduction
3. Constant properties
4. Negligible contact resistance
5. Negligible radiation effects

Analysis: Thermal circuit:



Since we are in Cartesian coordinates. A does not depend on x, i.e. const so q'' = const and can be expressed in different ways

$$q'' = \frac{T_\infty - T_{s,i}}{\frac{1}{h}} = \frac{T_{s,i} - T_\infty}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}}$$ ⇐ k_B is the only unknown

Substituting numbers:

$$q'' = \frac{800 - 600}{\frac{1}{25}} = 5000\ \frac{W}{m^2} = \frac{600 - 20}{\frac{0.3}{20} + \frac{0.15}{k_B} + \frac{0.15}{50}} = \frac{580}{0.018 + \frac{0.15}{k_B}} \Rightarrow k_B = 1.53\ W/mK$$

ii) If $L_B = 0.2$ while T_∞, $T_{s,o}$ and h remain unchanged $T_{s,i}$ will change

we use again the fact that $q'' = $ const

$$q'' = \frac{T_\infty - T_{s,i}}{\frac{1}{h}} = \frac{T_\infty - T_{s,o}}{\frac{1}{h} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{800 - 20}{\frac{1}{25} + \frac{0.3}{20} + \frac{0.2}{1.53} + \frac{0.15}{50}} = 4133 \ W/m^2$$

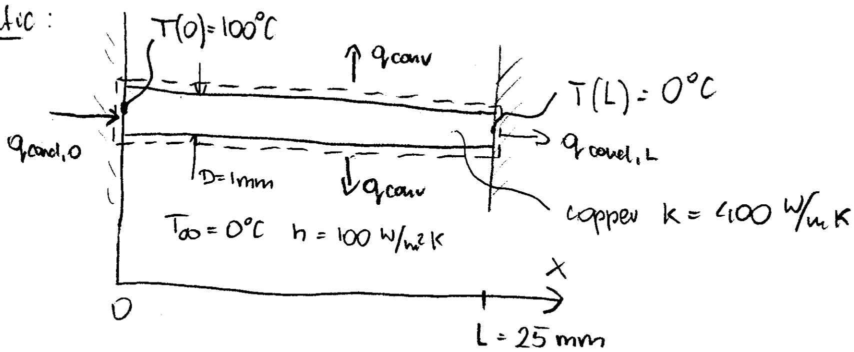
$$\Rightarrow T_{s,i} = T_\infty - \frac{q''}{h} = 800 - \frac{4133}{25} = \underline{634.7\,^\circ C} \quad \text{(ii) ans}$$

Problem 2

Known: Dimensions and end temperatures of the circular rod (pin fin); temperature and heat transfer coefficient of air.

Find: Rate of heat transfer by convection from the rod, q_{conv}

Schematic:



Assumptions:
1) Steady state conditions
2) 1-D conduction along the rod
3) Constant properties
4) No internal heat generation
5) Negligible radiation effects

Analysis: This is the case of pin fin with prescribed tip temperature. In this case temperature distribution along the fin is given by Eq (3.77)

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh[m(L-x)]}{\sinh mL} \qquad \text{where} \quad \theta_L = T_L - T_\infty = 0$$

$$\theta_b = T(0) - T_\infty = 100°C$$

$$\Downarrow$$

$$\frac{\theta}{\theta_b} = \frac{\sinh[m(L-x)]}{\sinh(mL)} = \frac{T - T_\infty}{T_b - T_\infty} \Rightarrow \boxed{T(x) = \frac{T_b - T_\infty}{\sinh(mL)}\sinh[m(L-x)]}$$

- Energy balance on the fin at steady state gives:

$$q_{cond,0} - q_{conv} - q_{cond,L} = 0 \Rightarrow q_{conv} = q_{cond,0} - q_{cond,L}$$

- Fourier's Law of conduction: $q_{cond} = -k A_c \frac{dT}{dx}$

  - differentiation of temp. distribution $\frac{dT(x)}{dx} = \frac{T_b - T_\infty}{\sinh(mL)}\cosh[m(L-x)] \cdot (-m)$

$$-m\frac{(T_b - T_\infty)}{\sinh(mL)}\cosh[m(L-x)]$$

Therefore: $q_{cond,0} = q_{cond,x=0} = kA_c m \frac{(T_b - T_\infty)}{\sinh mL} \cosh(mL)$

$$q_{cond,L} = q_{cond,x=L} = kA_c m \frac{(T_b - T_\infty)}{\sinh mL} \cosh 0 = \frac{kA_c m(T_b - T_\infty)}{\sinh(mL)}$$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4h}{kD}} \quad \text{since } A_c = \frac{\pi}{4}D^2, \; P = \pi D$$

$$m = \sqrt{\frac{4 \cdot 100}{400 \cdot 0.001}} = 31.62\,m^{-1} \qquad mL = 31.62 \cdot 0.025 = 0.791$$

$$kA_c m = 400 \cdot \frac{\pi}{4} \cdot 0.001^2 \cdot 31.62 = 0.00993 \left[\frac{W}{K}\right]$$

$$\therefore \; q_{cond,0} = 0.00993 \cdot 100 \frac{\cosh 0.791}{\sinh 0.791} = 1.51\,W$$

$$\therefore \; q_{cond,L} = \frac{0.00993 \cdot 100}{\sinh 0.791} = 1.13\,W$$

$$\therefore \; q_{conv} = 1.51 - 1.13 = \boxed{0.38\,W} \quad \text{ans.}$$

Alternatively, at steady state $q_{conv} = hA(T - T_\infty)$. However since T is function of x

$$q_{conv} = hP\frac{1}{L}\int_0^L (T - T_\infty)\,dx = hP\frac{1}{L}\int_0^L \frac{T_b - T_\infty}{\sinh(mL)} \sinh[m(L-x)]\,dx$$

$$= hP\frac{1}{L}\frac{T_b - T_\infty}{\sinh(mL)} \int_0^L \sinh[m(L-x)]\,dx \; \ldots\ldots \Rightarrow \text{would lead to } 0.38\,W$$

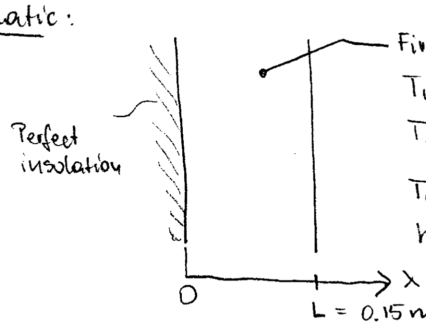
where $P = \pi D$ i.e., the perimeter.

Problem 3

Known: Thickness, initial temperature and properties of furnace wall. Convection conditions at inner surface

Find: i) Time required for outer surface to reach a prescribed temperature
ii) Corresponding average temperature of the wall

Schematic:



Fire-clay brick:
$T_i = 20°C$
$T_f(0,t) = 750°C$

$T_\infty = 950°C$
$h = 100\ W/m^2K$

$\left.\begin{array}{l} \rho = 2600\ kg/m^3 \\ c_p = 1000\ J/kgK \\ k = 1.5\ W/mK \end{array}\right\} \alpha = \frac{k}{\rho c_p} = \frac{1.5}{2600 \cdot 1000}$

$= 5.77 \cdot 10^{-7}\ \frac{m^2}{s}$

Assumptions:
(1) 1-D conduction in a plane wall
(2) Constant properties
(3) Adiabatic outer surface (at $x = 0$)
(4) $Fo > 0.2$ so that one-term approximation is applicable $\Rightarrow$ to be verified
(5) Negligible radiation from convection gases

Analysis Because of adiabatic surface, the wall can be seen as a half of symmetrically heated plane wall $\therefore L_c = L = 0.15\ m$. The adiabatic surface at $x=0$ is equivalent to a midplane in symmetrically heated plane wall.

Using Assumption 4; $\theta^* = C_1 \exp(-\xi_1^2 Fo)\cos(\xi_1 x^*)$ .... (5.40a)

at $x = 0 \Rightarrow \theta^* = \theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 Fo)$. since $\cos 0 = 1$

$\theta_0^* = \frac{750 - 950}{20 - 950} = 0.215 = C_1 \exp(-\xi_1^2 Fo)$

To determine C_1 and ξ_1 we need Bi, $Bi = \frac{hL}{k} = \frac{100 \cdot 0.15}{1.5} = 10 \Rightarrow$ LTCM is not applicable

For a plane wall with $Bi = 10$: $\xi_1 = 1.4289$ $C_1 = 1.2620$

Rearranging above equation: $\ln\frac{0.215}{C_1} = -\xi_1^2 Fo \Rightarrow Fo = -\ln\frac{0.215}{C_1} / \xi_1^2$

$\therefore Fo = -\ln\frac{0.215}{1.2620} / (1.4289)^2 = 0.867 > 0.2 \Rightarrow$ indeed one term approximation is valid (Assumption 4 is satisfied)

$Fo = \frac{\alpha t}{L^2} \Rightarrow t = Fo\, L^2 / \alpha = \frac{0.876 \cdot (0.15)^2}{5.77 \cdot 10^{-7}} = \underline{\underline{34164\ s}}$

Problem 3 continued...

$$\phi = \frac{Q}{Q_0} = 1 - \frac{\sin \zeta_1}{\zeta_1} \theta_0^* = 1 - \frac{\overline{T} - T_\infty}{T_i - T_\infty}$$

$$\Rightarrow \frac{\sin \zeta_1}{\zeta_1} \theta_0^* = \frac{\overline{T} - T_\infty}{T_i - T_\infty} \Rightarrow \overline{T} = (T_i - T_\infty) \frac{\sin \zeta_1}{\zeta_1} \theta_0^* + T_\infty$$

$$\therefore \overline{T} = (20 - 950) \frac{\sin(1.4286)}{1.4286} \cdot 0.215 + 950 = \underline{\underline{811 °C}}$$ ans part (ii)

Alternative Graphical solution:

For $Bi = 10 \Rightarrow Bi^{-1} = 0.1$ and $\theta_0^* = 0.215$ according to Fig D1 (For plane wall)

$$Fo \approx 0.9 \Rightarrow t = \frac{Fo\, L^2}{\alpha} = \frac{0.9 \cdot 0.15^2}{5.77 \cdot 10^{-7}} = \underline{\underline{35 \cdot 10^3 s}}$$

Note this solution utilizes determined earlier θ_0^*, Bi and α

The average temp after $t = 35 \cdot 10^3$ s $\Rightarrow Fo = 0.9$

Using Fig D3 for $Fo = 0.9$ and $Bi = 10$ i.e., $Fo \cdot Bi^2 = 90$ and $Bi = 10 \Rightarrow \frac{Q}{Q_0} = \phi \approx$

$$\approx 0.825 = \frac{\overline{T} - T_i}{T_\infty - T_i} \Rightarrow \overline{T} = (950 - 20) \cdot 0.825 + 20 = \underline{\underline{787 °C}}$$