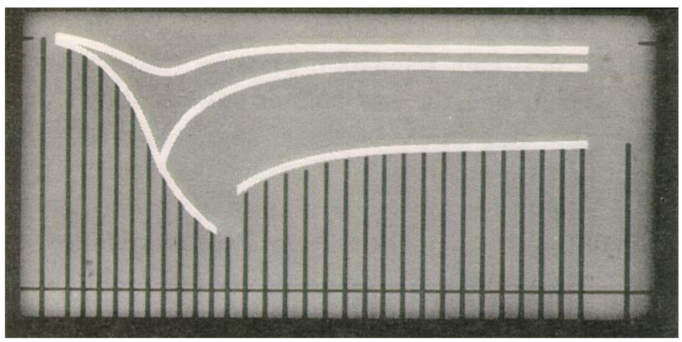
MCG 3341 FLUID MECHANICS II

LABORATORY MANUAL



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FLOW METER EXPERIMENTS

1. Objective

The objective of this experiment is to demonstrate the use of different instruments for measuring water flow rate in pipes.

2. Apparatus and Instrumentation

Water flow is produced by a pump in a closed-circuit flow loop that contains a surge tank with a free surface open to the atmosphere. The flow rate is controlled by various valves and a by-pass loop. All measuring devices have been connected in series and, therefore, measure the same mass (and volume) flow rate. The flow measuring devices include the following.

a) An **axisymmetric contraction** with a 16:1 area ratio. The contraction is fed through a cylindrical tube inserted in the surge tank. Its inlet diameter is D_1 = 6.00 in and its outlet diameter is D_2 = 1.50 in. Assuming negligible losses in the contraction, Bernoulli's equation provides

$$\dot{m} = \frac{A_2}{\sqrt{1-\left(A_2 / A_1\right)^2}}\sqrt{2\rho\Delta p}$$

where A_1 and A_2 are the corresponding cross-sectional areas. Notice that the elevation difference between the two pressure taps is balanced by the difference in the heights of the columns of water in the manometer tubes and that $\Delta p = 0$ when there is no flow. The pressure difference is measured with a liquid manometer containing a fluid (blue colour) with specific gravity SG_1 = 1.75. When taking manometers readings, be sure to take into account the pressure of the water above the manometer fluid, i.e. $\Delta p = \rho_{H_2O}\, g\, \Delta h\, (SG_1 - 1)$.

b) A **variable area flow meter** (**Rotameter**), connected to a PVC pipe with a nominal diameter of 1.5 in (ID = 1.572 ± 0.007 in). It consists of a vertical conical tube containing an element ("float") that rises to a level that depends on the flow rate. Because the area of the tube increases with increasing height, the speed V of the flow surrounding the float decreases as the float moves upwards. The float will stop at a position for which the drag force $F_D = C_D \tfrac{1}{2} \rho V^2 A$ is equal to its weight. The present device has been calibrated to indicate flow rate in US GPM (US gallons per minute), when used with a liquid with a specific gravity of 1.0.

c) An **electromagnetic flow meter** (Omega, Model FMG-423), connected to a PVC pipe with a nominal diameter of 1.5 in (ID = 1.572 ± 0.007 in). Its operation is based on Faraday's law of electromagnetic induction, which states that, when a conductor with length l moves with speed V in a direction normal to the direction of a magnetic field with magnetic flux density B, an electric potential E is generated across it as $E = BlV$. The flow meter consists of a teflon-insulated pipe section of the same diameter D as the pipe, an electromagnet producing a pulsed magnetic field and two surface electrodes located on the wall across a diameter normal to the magnetic field direction. The voltage difference between these electrodes is related to the volume flow rate Q as $E = [(4kB)/(\pi D)]Q$, where k is a numerical coefficient. The instrument has been calibrated to provide the volume flow rate in GPM. Electromagnetic flow meters have an accuracy that exceeds 0.5% and are not overly sensitive to the velocity profile. On the other hand, they are bulky, heavy and relatively expensive.

d) A **turbine flow meter** (Omega, Model FTB793), connected to a PVC pipe with a nominal diameter of 1 in (ID = 1.038 ± 0.007 in). As water flows through the meter, it causes a rotor to spin. The rotor contains an electric coil, which, as it spins, generates an electric voltage. An electronic circuit provides an output in the form of a 6-digit number, displayed on an LCD readout device. By pushing the "DISPLAY" button on the front of the device, it is possible to select display of the flow rate (precalibrated to indicate GPM) or the total flow through the meter.

e) A **Venturi flow meter** (FLO FAB, Model VT200H), connected to PVC pipe with a nominal diameter of 2 in (ID = 2.023 ± 0.008 in). This is a compact, industrial design of Venturi, rated as having a permanent pressure loss of about 10% of the pressure difference Δp. The manufacturer provides a formula for the flow rate Q (in US GPM) in terms of Δp (in inches of water) as

$$Q = 4.62\sqrt{\Delta p}$$

Its recommended range of operation is between 12 and 467 GPM. The pressure difference is measured with a liquid manometer containing a fluid (orange colour) with specific gravity SG_2 = 2.95. When taking manometers readings, be sure to take into account the pressure of the water above the manometer fluid, i.e. $\Delta p = \rho_{H_2O}\, g\, \Delta h\ (SG_2 - 1)$.

f) A **Circuit Setter Balance Valve** (Bell & Gossett, Model CB-2) and a **Readout Kit** (Bell & Gossett, Model RO-5). The valve is connected to a PVC pipe with a nominal diameter of 2 in (ID = 2.023 ± 0.008 in). This device functions both as a piping system balancing valve and as a variable orifice flow meter. The correspondence between the valve setting, the flow rate and the indicated pressure difference may be found with the use of a rotary "calculator" (Model V91483).

g) An **ultrasonic flow meter** (Omega, Model FD613). This instrument utilizes a non-invasive transducer, which is hand-held or strapped to the outside of a pipe with an inner diameter equal to or larger than 1 in. This instrument utilizes two piezoelectric crystals, one of which transmits an ultrasonic wave through the pipe while the other one receives the ultrasound reflected by solid particles suspended in the flowing liquid or entrained gas bubbles. The frequency of the reflected sound is shifted from the frequency of the transmitted sound by an amount ("Doppler" shift), that is proportional to the velocity of the reflector. The device provides an output that is equal to the average velocity of the fluid in the pipe. The range of this flow meter is between 0.10 and 9.00 m/s and its accuracy is specified as ±2% of full scale. To use the instrument, apply a 1/8 in layer of silicone grease to the transducer face and hold the transducer parallel to the pipe with the cable pointing downstream of the flow direction. Then press the ON/OFF key and wait for at least 30 s before taking a reading. The velocity may be selected in m/s or ft/s. The transducer reading will represent the correct flow rate only if the flow in the pipe is fully developed. This means that the reading must be taken at locations sufficiently downstream (at least 10 pipe diameters) and/or upstream (at least 5 diameters) of elbows, valves etc.

3. Instructions for the Experiments

Details for the use of the apparatus and specifications for the experiments to be conducted will be provided by the instructor. Among the required tasks are the following.

a) Study carefully the layout and components of the apparatus and familiarize yourselves with all controls and measuring instrumentation. Identify the measured properties and their units.

b) Turn on the pump motor and adjust the flow rate to a relatively high value (e.g. such that the rotameter indicates its maximum reading). Make sure that the angle on the Circuit Setter Balance Valve is at a low value, but that the reading on the pressure gauge (Readout Kit) is measurable.

c) Record the readings of the various measuring devices. The ultrasonic flow meter should be used on the three different diameter pipes, and each person in each group should take one reading separately. Be sure to record the location of the readings (specifically the pipe diameter and any nearby disturbances). Calculate the volume flow rate and compare the results for each device.

d) In addition to the previous angle, adjust the Circuit Setter Balance Valve to two larger angles. For each angle, record the pressure drop, and the reading of the manometer for the axisymmetric contraction. Calculate the flow rate using the calculator provided by the Instructor ***during the lab***. Compare the volume flow rates for the three angles with those calculated for the axisymmetric contraction, and comment on the effect of the angle setting.

e) Reduce the flow rate, and reset the Circuit Setter Balance Valve to a low angle, making sure that the reading on the associated pressure gauge is measurable. Repeat steps (c) and (d) for this flow rate. Comment on the accuracy of the devices at different points in their range.

f) Provide an informal report of your results, following the guidelines posted on the course webpage. Each student should submit a separate report.

4. Guidelines for Lab Reports

The report must be typed, but sample calculations can be made by hand.

The report must contain the following parts in order to receive full marks:

- Title Page, including course name and code, group number, and the names of all group members
- Introduction -keep it short, no longer than one page, summarizing the experiment and procedures in your own words
- Sample Calculations for all major results -not just the equations used, but a sample of the calculations performed
- Results should include the following :
    - Tables with all raw data
    - Tables with all calculated results
    - A single (and separate) summary table of the flow rates obtained with the various instruments
- Discussion should be kept short (typically 1-2 pages), and include the following:
    - Comments on the use of each device and its apparent accuracy, compared with the axisymmetric contraction
    - Comments on the range of the devices with respect to the accuracy obtained at low and high flow rates
    - Comments on the effect of the balance valve angle on the flow rate
    - The choice of a flow measuring device that is the most useful, and give reasons for this choice
    - The sources of error encountered
- Conclusion - typically three or four sentences, summarizing the results
- References - a separate section where you include any sources of information that are used in the lab, be sure to include the lab manual, do not neglect to include any web pages if applicable and be sure to cite your source when you use information from that source in your discussion, introduction, etc.

DRAG AND LIFT EXPERIMENTS

1. Objective

The objective of this experiment is to measure the aerodynamic drag and lift on two-dimensional objects.

2. Apparatus and Instrumentation

Wind Tunnel: The tests are conducted in a suction wind-tunnel, custom designed and fabricated by staff of the Department of Mechanical Engineering of the University of Ottawa. The flow is generated by an axial fan (Woods), with a diameter of 483 mm and fixed blades at an angle of 29 deg, driven by a three-phase, 1.1 kW (1.5HP), variable speed electric motor. The flow rate is adjusted by a digital motor speed controller. The test section is 1,220 mm long and has a 305 mm × 305 mm square cross section. It is equipped with two circular windows and several circular ports for the insertion of the tested objects and measuring probes, respectively. For a smooth, low-turbulence entrance to the test section, the room air first passes through a fibreglass screen and then through a specially designed, square, 6.25:1 contraction. The test section is connected to the fan inlet through a square-to-circular diffuser section.

Pressure and Velocity Measurement: The static pressure in the wind tunnel is measured with the use of pressure taps on the top wall of the tunnel. The total pressure in the flow is measured with a Pitot tube, having an outer diameter of 1.26 mm and an inner-to-outer diameter ratio of 0.59. The Pitot tube is inserted through a port at the top wall and traversed across the test section with the use of a Unislide leadscrew assembly (Velmex Inc.), driven by an electric motor and equipped with a vernier having a resolution of 0.01 mm. All pressures are measured with a digital manometer (Aerolab, Model M-100), having a nominal full-scale of 4,982 Pa (20 in H_2O), a resolution of 0.25 Pa (0.001 in H_2O) and an accuracy of 0.25% of full-scale (12.5 Pa). A Pitot tube rake is also available for wake measurements; it consists of 18 tubes with an outer diameter of 1.26 mm, an inner-to-outer diameter ratio of 0.59 and a centre-to-centre spacing of 2.55 ± 0.2 mm.

Tested Objects: All tested objects have a uniform cross section and extend across the entire width of the wind-tunnel test section. Each object is mounted on a separate window cover and some objects can be rotated about a longitudinal axis by a measurable angle. The following objects are available.

- A *circular cylinder*, with a diameter of 25.4 mm. This object is equipped with three wall pressure taps for the measurement of the surface pressure and the testing of the two-dimensionality assumption. The taps are located at distances of 0.167b, 0.300b and 0.500b from the one end (b = 305 mm is the span of the cylinder)

- A *rectangular plate* with a thickness of 2.0 mm and a width of 50.8 mm.

- A *strut* (symmetric airfoil) with a thickness of 36 mm and a chord of 85 mm.

- An *airfoil*, with a Clark Y-14 profile, which is a design developed for airplane propellers. It has a uniform cross section with a chord of $c = 88.8 \pm 0.5$ mm, a maximum camber of 4.6%c and a maximum thickness of 14%c. Nominal specifications for this airfoil are: maximum lift coefficient, 1.72; lift coefficient at small α, $dc_L/d\alpha = 0.096$ deg^{-1} (less than the ideal value of 0.110 for thin airfoils); angle of zero lift, -6.2 deg; minimum drag coefficient, 0.0077. The airfoil is equipped with 18 pressure taps on its middle section, positioned as follows (x is the distance from the leading edge, measured along the chord; the uncertainty in tap position is ± 0.1 mm).

Top Surface	Tap #	0	1	2	3	4	5	6	7	8	9
	x [mm]	0	5.1	9.2	18.1	27.1	36	44.8	53.7	62.5	71.4
Bottom Surface	Tap #	10	11	12	13	14	15	16	17		
	x [mm]	61.5	52.6	43.7	35.7	26.8	17.9	9.1	5.6		

3. Instructions for the Experiments

Details for the use of the apparatus and specifications for the experiments to be conducted will be provided by the Instructor. Among the required tasks are the following.

a) Study carefully the layout and components of the apparatus and familiarize yourselves with all controls and measuring instrumentation. Identify the measured properties and their units.

b) Insert the circular cylinder. Using the static tap in the middle of the cylinder, measure the surface pressure by rotating the cylinder in 10 deg increments up to the separation points on the top and bottom surfaces, and, in the separated zone, in 30 deg increments to ±180°. Plot the surface pressure coefficient vs. angle around the cylinder. Integrate the horizontal component of the surface pressure coefficient around the cylinder (e.g. by using Simpson's Rule) and determine the drag coefficient. More details for drag computation can be found on the course webpage.

c) Measure the base surface pressure at the two off-centre pressure taps on the cylinder at 180°. Comment on the two-dimensionality of the flow.

d) Using the Pitot tube, take total pressure measurements in the wake of the cylinder. Start from a vertical position that is clearly outside the wake and traverse the tube vertically in steps of 10 mm until the wake is reached, then in steps of 5 mm throughout the wake, and, finally, in steps of 10 mm once outside the wake again. Define the wake as the region in which the change in pressure between two successive points is measurable. Measure the local static pressure using a tap on the upper wall of the wind tunnel, at the streamwise location of the Pitot tube. Take total pressure measurements upstream of the cylinder to determine the free stream pressure. Use the momentum equation to determine the drag coefficient of the cylinder. More details for drag computation can be found on the course webpage.

e) Insert the strut into the wind tunnel. Repeat step (d) using the strut.

f) Compare the cylinder drag coefficients calculated using the direct method and the momentum equation method. Compare the measured drag coefficients of the cylinder and the strut. Compare the measured cylinder and strut drag coefficients with data available in the literature.

g) Insert the airfoil into the wind tunnel. Repeat the following steps for angles of attack of –5, 0, 5, 10, and 15 deg, or as specified by the Instructor. Measure the surface pressure around the airfoil at all 18 taps. Using the Pitot tube, measure the total pressure upstream of the airfoil. Using a tap on the upper wall of the wind tunnel, upstream of the airfoil, measure the free stream static pressure. Calculate the free stream flow velocity. Plot the surface pressure coefficient vs. the dimensionless distance along the airfoil chord for the range $0 \leq x/c \leq 1$ (extrapolate values up to the trailing edge). Integrate the pressure (e.g. by using Simpson's Rule) to determine the airfoil's lift coefficient, pitching moment coefficient about the leading edge and centre of pressure. Plot the lift and moment coefficients vs. angle of attack, and compare the values to the nominal specifications for this airfoil.

h) Provide an informal report of your results, following the guidelines posted on the course webpage. Each student should submit a separate report.

4. Guidelines for Lab Reports

The report must be typed, but sample calculations can be made by hand.

To perform the integrations, it is suggested that you use a simple numerical integration technique such as the midpoint rule or Simpson's rule. Do not forget that your data are not taken at evenly spaced intervals. Also, when integrating the pressure to determine lift and moment on the wing, do not forget to extrapolate your results to the trailing edge.

Note that marks will be awarded for the appearance of your graphs. Do not put colour graphs into the report if you print it out in black and white or gray scale (this makes it difficult to distinguish between different data series). Backgrounds of the graphs should be white, not gray (white background makes the chart easier to read and also saves your printer toner). Use symbols, not lines, to represent experimental data (this prevents misinterpretation of the results).

However, you may include lines to connect the symbols if you feel that this makes the chart easier to read.

The following components must be present to obtain full marks:

- Title Page, including course name and code, group number, and the names of all group members
- Introduction - keep it short, no longer than one page, summarizing the experiment and procedures in your own words
- Sample Calculations for all major results -not just the equations used, but a sample of the calculations performed. Note that for the integrations you need only to provide an outline of the integration method (i.e. provide only enough information that the method could be repeated by the instructor if necessary)
- Results should include the following:
    - Tables
        - all raw data
        - all calculated results (give the freestream speed for every trial), including drag and lift coefficients. Provide only the final calculated result and not intermediate steps. For example, include C_P, V_2, C_L, C_{MLE} and C_D but do not include $C_P \cos q$, V_2^2 etc.
        - a list of the values of the maximum lift coefficient, angle of zero lift, the slope of the lift curve, and the approximate stall angle for the airfoil
        - a separate summary of all the results for all the experiments
    - Graphs for the drag experiments
        - C_P vs. q for the cylinder, and indicate stagnation and separation points on this graph,
        - C_p vs. z/b (the length axis) for the cylinder at q =180 degrees (3 data points)
        - V_2 vs. y for the wakes of both cylinder and strut
    - Graphs for the lift experiments (two to four graphs per page, if possible)
        - C_P vs. x/c for all angles of attack (identify the upper and lower surfaces, and remember to extrapolate the graphs to $0 < x/c < 1$)
        - C_L vs. a
        - C_{MLE} vs. a
- Discussion should be kept short (typically 2-3 pages), and include the following:
    - Comparison of the *shape* (no numbers necessary) of the cylinder pressure distribution with those of other bluff bodies from the literature (sphere and cylinder wakes are similar; see the course textbook),
    - Comparison of the cylinder drag coefficients calculated using both methods, with data obtained from the literature, including error estimates
    - Approximate location of separation and stagnation points around the cylinder
    - Comments on the two-dimensionality of the flow around the cylinder
    - Comparison of the calculated strut drag coefficient with those calculated for the cylinder and the data obtained from the literature

- Comparison of the shape of the airfoil pressure distribution with typical data obtained in the literature (no numbers necessary, if comparing to different airfoils)
- Comparison of the airfoil lift coefficient curve with the expected shape
- Comparison of the maximum lift coefficient, angle of zero lift, and the slope of the lift curve with the nominal specifications given in the lab manual
- Short discussion of sources of error

- Conclusion - typically three or four sentences, summarizing the results
- References - be specific in the discussion about the figure numbers used from the textbook or other sources and be sure to include those sources in a separate reference section

5. Calculation Procedures

5.1 Pressure Coefficient and Velocity Calculations

The Pitot tube measures the stagnation pressure, P_o, whereas the pressure taps along the top surface of the wind tunnel measure the static pressure, P_s. We may assume (with some error) that the taps along the top surface of the wind tunnel are infinitely far away from the objects being tested, so that $P_s = P_\infty$. The absolute static pressure at any tap on the object surface is denoted as P. In all cases, the digital multi-manometer shows a gauge pressure reading in inches of water (i.e., $P - P_{atm}$).

The free stream velocity and the pressure coefficient can be calculated based on the measured stagnation and static pressures, as follows:

$$P_o = P_s + \frac{1}{2}\rho V^2$$

$$V = \sqrt{\frac{2\left(P_o - P_s\right)}{\rho}} \quad \text{and} \quad C_p = \frac{P - P_s}{\frac{1}{2}\rho V_\infty^{\,2}} = \frac{P - P_s}{P_o - P_s}$$

Note that $(P - P_s)$ can be calculated as $(P - P_{atm}) - (P_s - P_{atm})$.

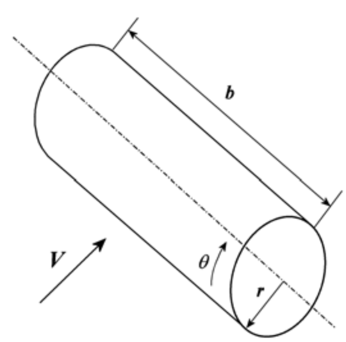
5.2 Drag on a Cylinder Using Surface Pressures

The drag force may be calculated as the integral of the horizontal components of the distributed pressure forces, as follows:

$$F_D = \int_0^{2\pi} P_g \cos\theta\, brd\theta$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^{\,2}(2rb)} = \frac{\int_0^{2\pi}\left(P - P_\infty\right)\cos\theta\, brd\theta}{\frac{1}{2}\rho V_\infty^{\,2}(2rb)}$$

$$C_D = \frac{1}{2}\int_0^{2\pi} \frac{P - P_s}{P_o - P_s}\cos\theta d\theta = \frac{1}{2}\int_0^{2\pi} C_p \cos\theta d\theta$$



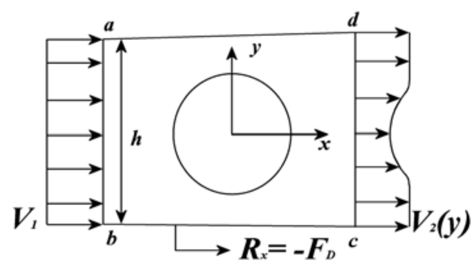
To calculate C_D using the data obtained in the lab, plot $C_p \cos\theta$ vs. θ and integrate the area under the curve. Use any numerical or graphical integration technique.

5.3 Drag on a Cylinder and Strut Using the Momentum Equation

Consider the wake of a two-dimensional object with a span b in an *unbounded* fluid stream. To apply the momentum principle, we use a control volume (C.V.) that surrounds the object. Let *abcd* be the control surface (C.S.) intersection with the plane normal to the object axis. The left and right sides of this surface are normal to the flow direction, however, the top and bottom sides are not parallel to the flow direction but diverge slightly. The reason for this divergence is that, in the wake of the object, the velocity $V_2(y)$ is lower than the upstream velocity V_1, which is assumed to be uniform. If the top and bottom sides of the C.S. were parallel, then some flow entering the C.V. would leave the C.V. from the top and bottom, transporting away its own momentum and it would be necessary to consider momentum flux from all four sides. To simplify the analysis, we select a C.V. such that the mass entering from the left side is equal to the mass exiting from the right side and assume that the top and bottom sides are streamlines, so that no fluid crosses them. Let us assume that we measure the wake velocity between points c and d. Then, we can find the inlet height $h = ab$ by applying the conservation of mass equation in the C.S. *abcd*, as

$$\rho_1 V_1 bh = \rho_2 \int_c^d V_2 bdy$$

$$h = \frac{1}{V_1}\int_c^d V_2 dy$$



The momentum equation can then be used to calculate the drag coefficient, as follows:

$$R_x = -F_D = \int_{C.S.} \rho u \vec{V} \cdot d\vec{A} = \rho V_1^2 bh - \int_c^d \rho V_2^2 bdy$$

$$F_D = \rho b \left(V_1 \int_c^d V_2 dy - \int_c^d V_2^2 dy \right)$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_\infty^{\ 2}(2rb)} = \frac{\rho b}{\frac{1}{2}\rho V_\infty^{\ 2}(2rb)} \left(V_1 \int_c^d V_2 dy - \int_c^d V_2^2 dy \right)$$

$$C_D = \frac{\rho}{2r(P_o - P_s)} \left(V_1 \int_c^d V_2 dy - \int_c^d V_2^2 dy \right)$$

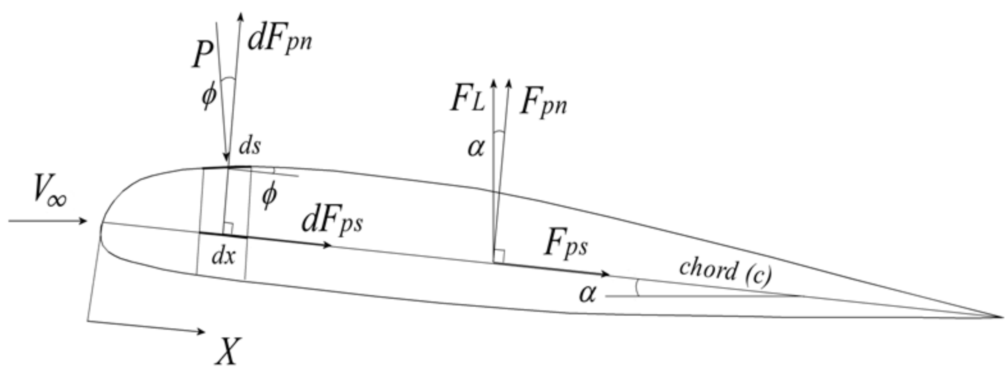
To calculate the drag coefficient from the data obtained in the lab, plot V_2 and V_2^2 vs. y and integrate the areas under these curves. Do this using a numerical or graphical integration technique. For the cylinder, r is the radius, while, for the strut, r is half the maximum thickness.

Note: Some differences from the idealized expressions should be expected in the present experiments. Because of the walls, the flow velocity outside the wake must be higher than the upstream uniform velocity, so that the mass flow rate remains constant. Furthermore, because of the proximity of the object, the measured upstream velocity may not be uniform across the cross-section.

5.4 Lift on an Airfoil Using Surface Pressures

The lift force on an airfoil of span *b* and chord *c* is calculated as the component of the total pressure force normal to the relative velocity V_∞ of the far stream. Pressure acts on the upper and lower surfaces of the airfoil, denoted by subscripts *u* and *l*, respectively. To compute the lift accurately, one must consider that neither the airfoil surfaces nor its chord are, in general, parallel to V_∞.

If an infinitesimal portion of the airfoil surface, *ds*, is oriented at an angle ϕ to the chord, *dx*, then $dx = ds\cos\phi$.



The total pressure force normal to the chord is the difference between the corresponding pressure forces acting on the lower and upper surfaces,

$$dF_{pn} = P_l \cos\phi_l b ds_l - P_u \cos\phi_u b ds_u$$

$$dF_{pn} = P_l b dx - P_u b dx$$

$$dF_{pn} = (P_l - P_u) b dx$$

The total pressure force parallel to the chord is:

$$dF_{ps} = P_l \sin\phi_l b ds_l - P_u \sin\phi_u b ds_u$$

$$dF_{ps} = P_l \tan\phi_l b dx - P_u \tan\phi_u b dx$$

$$dF_{ps} = (P_l \tan\phi_l - P_u \tan\phi_u) b dx$$

The total lift force is: $F_L = \int dF_{pn} \cos\alpha - \int dF_{ps} \sin\alpha$.

But note that | tanϕ| << 1 for a thin airfoil with a small camber, so | dF_{ps} | << | dF_{pn} |. Furthermore, | sin α| << | cos α | for small angles of attack. Therefore, the second integral is negligible compared to the first one, and the total lift force then becomes: $F_L \approx \int dF_{pn} \cos\alpha$.

Using the data acquired during the lab, where P_g is the gauge pressure measured by the digital multi-manometer,

$$F_L = \int \left(P_{gl} - P_{gu}\right)\cos\alpha b dx$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V_\infty{}^2 (bc)} = \frac{\int_0^c \left[(P - P_\infty)_l - (P - P_\infty)_u\right] b \cos\alpha dx}{\frac{1}{2}\rho V_\infty{}^2 (bc)}$$

$$C_L = \cos\alpha \int_0^1 \frac{\left[(P - P_s)_l - (P - P_s)_u\right]}{P_o - P_s} d\left(\tfrac{x}{c}\right) = \cos\alpha \left[\int_0^1 C_{Pl}\, d\left(\tfrac{x}{c}\right) - \int_0^1 C_{Pu}\, d\left(\tfrac{x}{c}\right)\right]$$

For each angle of attack, plot C_P vs. x/c and integrate the area between the curves (extrapolate results to the trailing edge, so that your integral is in the range $0 \le x/c \le 1$). Do this using any numerical technique.

The **Pitching Moment Coefficient** about the leading edge can be found as follows:

$$M_{LE} = -\int_A x P_g b dx$$

$$C_{M_{LE}} = \frac{M_{LE}}{\frac{1}{2}\rho V_\infty{}^2 \left(bc^2\right)} = -\frac{\int_0^c x(P - P_\infty) b dx}{\frac{1}{2}\rho V_\infty{}^2 \left(bc^2\right)}$$

$$C_{M_{LE}} = -\int_0^1 \left(C_p \tfrac{x}{c}\right) d\left(\tfrac{x}{c}\right)$$

Once again, plot $C_P x/c$ vs. x/c, and integrate the area between the two curves using a numerical or graphical technique.

The **Centre of Pressure** is the point where the moment about the leading edge is balanced by the lift force:

$$M_{LE} + x_{cp} F_L = 0$$

$$x_{cp} = -\frac{M_{LE}}{\frac{1}{2}\rho V_\infty^2 bc^2} \frac{\frac{1}{2}\rho V_\infty^2 bc^2}{F_L}$$

$$x_{cp} = -\frac{C_{M_{LE}} c}{C_L}$$