4.6 Laminar Boundary Layer Separation – Stratford Criterion

Upstream of the separation point, the flow is moving forward and the wall shear stress $\tau_w$ is positive. Downstream of the separation point, the near-wall flow is reverse and $\tau_w < 0$. At the separation point, $\tau_w = 0$.

Separation may only occur if $\frac{dp}{dx} > 0$ (or, equivalently, $\frac{du}{dx} < 0$).

Criteria for separation:

- Falkner-Skan solutions: $m = -0.091$
- Pohlhausen’s solution: $\lambda_P = -7$ to $-8$ (the solution gives $\lambda = -12$)
- Thwaites’ method: $\lambda = -0.0842$
- Stratford criterion: see below

- Beyond the separation point, the boundary layer approximation is no longer valid and none of these methods can be used for calculating $\tau_w$. The main type of drag in the separated region is form (pressure) drag.
Stratford criterion:

Separation occurs when \[ C_p \left( \frac{dC_p}{dx} \frac{C_p}{x} \right)^2 = 0.0104 \]

where \( \bar{x} \) is an effective origin and the pressure recovery coefficient \( C_p \) is defined in terms of the minimum pressure \( p_m \) along the wall and the corresponding free stream velocity \( u_m \) as

\[ C_p = \frac{p - p_m}{\frac{1}{2} \rho u_m^2} = 1 - \left( \frac{u_e}{u_m} \right)^2 \]

The definition of \( \bar{x} \) depends on the variation of \( dp/dx > 0 \) along the b.l. Three different cases may be considered, as follows.

(a) \( dp/dx > 0 \) from the start of the boundary layer (\( x = 0 \)):

\[ \bar{x} = x ; \; u_m, p_m \text{ are the values at } x = 0. \]

At \( \bar{x} = 0 \), \( C_p = 0 \). Compute \( C_p \left( \frac{dC_p}{dx} \frac{C_p}{x} \right)^2 \) as it increases with increasing \( \bar{x} \).

When this becomes 0.0104, separation occurs.

(b) \[ \begin{cases} \frac{dp}{dx} = 0, & 0 \leq x \leq x_m \\ \frac{dp}{dx} > 0, & x_m < x \end{cases} \]

then use \( u_m, p_m \) at \( x_m \) but \( \bar{x} = x \) (from start).
(c) The pressure gradient is initially favourable and then adverse, as on the top surface of an airfoil, i.e. \( \frac{dp}{dx} < 0 \), \( 0 \leq x \leq x_m \)
\( \frac{dp}{dx} > 0 \), \( x_m < x \)

To define \( \bar{x} \), consider an equivalent problem with \( \frac{dp}{dx} = 0 \) upstream of \( x_m \) and such that it has the same \( \theta \) (momentum thickness) at \( x_m \) as the actual problem. Then, from Thwaites’ method:

\[
\bar{x}_m = \int_0^{x_m} \left( \frac{u_e}{u_m} \right)^5 \, dx = \ldots \quad \text{(can be computed for a given } u_e(x)\text{)}
\]

and \( \bar{x} = x - (x_m - \bar{x}_m) \)

Notice that \( \overline{C_p} = 0 \) at \( x = x_m \). Compute \( \overline{C_p} \left( \frac{\overline{x} C_p}{dx} \right)^2 \) till it becomes 0.014.