# Problem Set #1 Solutions

2.41/1 Given: $\forall_{tot} = 1 m^3$

$m_{stone} = 400\ kg$

$m_{sand} = 200 kg$

$\forall_{liq} = 0.2 m^3$

$T_{liq} = 25°C$

$\rho_{air} = 1.1 kg/m^3$

find: $\rho$ + $v$

Sol'n: $\boxed{v = \forall_{tot} / m_{tot}}$ and $\boxed{\rho = \frac{1}{v}}$

$\forall_{tot} = 1 m^3$

$m_{tot} = m_{stone} + m_{sand} + m_{liq} + m_{air}$

we have $m_{stone}$ + $m_{sand}$.

$m_{liq} = \forall_{liq} / v_{liq} = \forall_{liq} \rho_{liq}$ $\rho_{liq} = 997\ kg/m^3$ from table A.4

$\therefore m_{liq} = (0.2)(997) = 199.4 kg$

$m_{air} = \forall_{air} / v_{air} = \forall_{air} \rho_{air}$

Need to find $\forall_{air}$

$\forall_{air} = \forall_{tot} - \forall_{stone} - \forall_{sand} - \forall_{liq}$

$\forall_{stone} = mv = \frac{m}{\rho} = \frac{400}{2750} = 0.1455 m^3$

(from table A.3)

$\forall_{sand} = mv = \frac{m}{\rho} = \frac{200}{1500} = 0.1333 m^3$

(from A.3)

$\therefore\ V_{air} = 1 - 0.1455 - 0.1333 - 0.2 = 0.5212\,m^3$

$\therefore\ M_{air} = \dfrac{V_{air}}{v_{air}} = V_{air}\,\rho_{air} = (0.5212)(1.1) = 0.573\,kg$

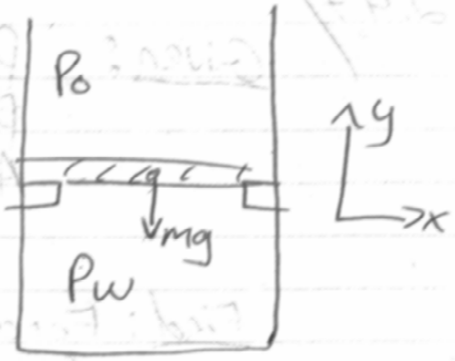
$\therefore\ M_{tot} = 400 + 200 + 199.4 + 0.573 \approx 800\,kg$

$$\boxed{\begin{array}{l} \therefore\quad v = \dfrac{V_{tot}}{M_{tot}} = \dfrac{1}{800} = 0.00125\,m^3/kg \\[2ex] \text{and}\quad \rho = \dfrac{1}{v} = 800\,kg/m^3. \end{array}}$$

2.45/ Given: $A = 0.01 m^2$

$M = 100 kg$

$P_o = 100 kPa$




Find: $P_w$.

SOLN: $\Sigma F_y = 0 = \Sigma F_\uparrow - \Sigma F_\downarrow$

$$0 = P_o A + mg - P_w A.$$

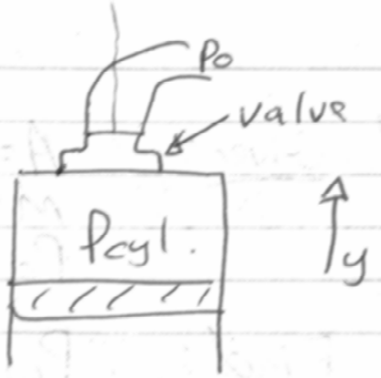
$$P_w = \frac{P_o A + mg}{A} = P_o + \frac{mg}{A}$$

$$= 100 kPa + \frac{(100 kg)(9.81 m/s^2)}{(0.01 m^2)(1000 Pa/kPa)}$$

$$\boxed{P_w \approx 198 kPa.}$$

2.47/

<u>Given:</u> $P_o = 99\,kPa$
$P_{cyl} = 735\,kPa$
$A_{valve} = 11\,cm^2$




<u>Find:</u> Force required to open valve
(ie. force required to overcome force from $P_{cyl}$)

<u>Soln:</u> $\sum F_y = F_{net} = P_{cyl} A - P_o A$

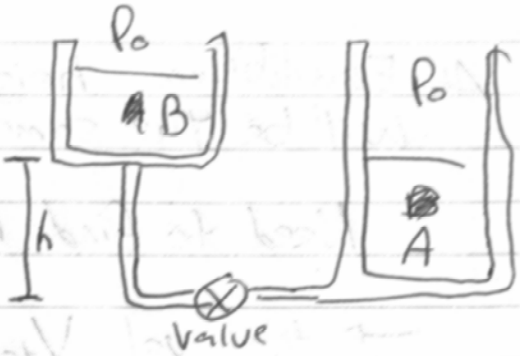
$$= (735-99)\,kPa \times 11\,cm^2$$

$$= 6996\ kPa\ cm^2$$

$$= 6996 \times \frac{kN}{m^2} \times 10^{-4} m^2$$

$$\boxed{F_{net} = 700\,N}$$

2.76/ **Given:** $\rho = 1000\ kg/m^3$
$m_A = 100\ kg$
$m_B = 500\ kg$
$A_A = 0.1\ m^2$
$A_B = 0.25\ m^2$
$h = 1\ m$




**Find:** Pressure on both sides of Value
$P_A$ & $P_B$

**Assumption:** Volume in connecting pipe is small compared to volume of fluid in cylinders

**Soln:** Vol. in cyl. A: $V_A = v\, m_A = \frac{m_A}{\rho} = 0.1\ m^3$

also $V_A = A_A h_A$

$\therefore h_A = \frac{0.1}{0.1} = 1\ m$ (height of liquid in cyl A)

Vol in cyl. B: $V_B = \frac{m_B}{\rho} = 0.5\ m^3 = A_B h_B$

$\therefore h_B = 2\ m$

$\therefore$ when value is closed

$$P_A = P_o + \rho g h_A = 101.3\ kPa + \frac{(1000)(9.81)(1m)}{(1000\ Pa/kPa)}$$

$$\boxed{P_A = 111.135\ kPa}$$

$$P_B = P_o + \rho g(h_A + h) = 101.3\ kPa + \frac{(1000)(9.81)(3m)}{1000\ Pa/kPa}$$

$$\boxed{P_B = 130.755\ kPa}$$

At Equilibrium heights of liquid in both cylinders will be the same.

Need to find height, $h_2$

$\rightarrow$ total vol $V_{tot} = V_A + V_B \approx h_2 A_A + (h_2 - h_2) A_B$

$$0.1 + 0.5 = h_2(0.1) + (h_2 - h)(0.25)$$

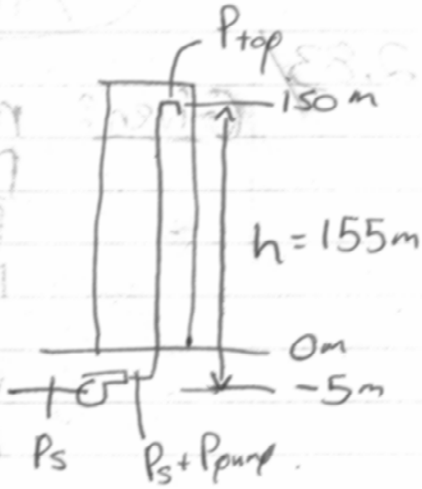
$$h_2 = 2.43\,m$$

$\therefore$ Pressure at Valve $P = P_A = P_B$

$$P = P_0 + \rho g h_2 = 101.3\,kPa + \frac{(1000)(9.81)(2.43)}{1000\,Pa/kPa}$$

$$\boxed{P = 125.2\,kPa}$$

2.82/ Given: $P_{supply} = P_S = 600\,kPa$.
$P_{top} = 200\,kPa$
$h = 155\,m$




Find: $P_{pump}$

SOL'N.

$P_{after\ pump} = P_S + P_{pump} = P_{top} + \Delta P$

$\Delta P = \rho g h = (997\,kg/m^3)(9.807\,m/s^2)(155\,m)$
$= 1516\,kPa$.

$\therefore P_{pump} = P_{top} + \Delta P - P_S$
$= 200\,kPa + 1516\,kPa - 600\,kPa$

$\boxed{P_{ump} = 1116\,kPa.}$

2.83/ GIVEN: $m = 5\,kg$

$D = 100\,mm \Rightarrow$ Piston Area $A = \frac{\pi D^2}{4} = 0.00785\,m^2$

$P_0 = 100\,kPa$

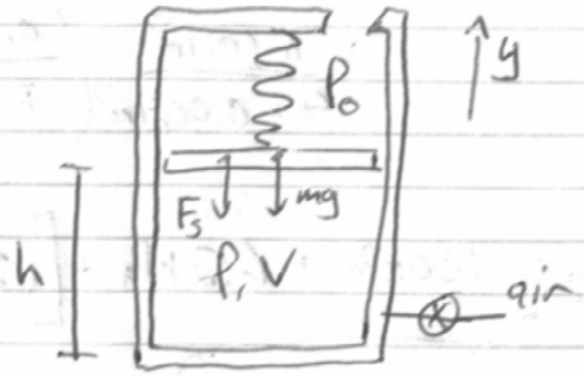
$F_{spring} = 0$ when $h = 0$

$P_1 = 400\,kPa$

$V_1 = 0.4\,L = 0.0004\,m^3$

$h_2 = h_1 + 2\,cm$

Find: $P_2$




SOL'N:

$$\Sigma F_y = 0 = PA - P_0 A - mg - F_{spring}$$

$$F_{spring} = kh$$

$$\therefore \quad P = P_0 + \frac{mg}{A} + \frac{kh}{A}$$

P varies linearly with h but $h = \frac{V}{A}$ and $A =$ constant

$\therefore$ P varies linearly with volume $V$

$$\therefore \quad P = a + bV$$

where $a = P_0 + \frac{mg}{A}$

and $b = \frac{k}{A^2}$

When $h=0,\ V=0$

$$\therefore P_{h=0} = a = P_0 + \frac{mg}{A} = 100\,kPa + \frac{(5)(9.81)}{(0.00785)(1000)}$$

$$a = 106.2\ kPa$$

$$\text{also}\quad V_2 = h_2 A = (h_1 + 0.02)A$$

$$V_2 = \left(\frac{V_1}{A} + 0.02\right)A$$

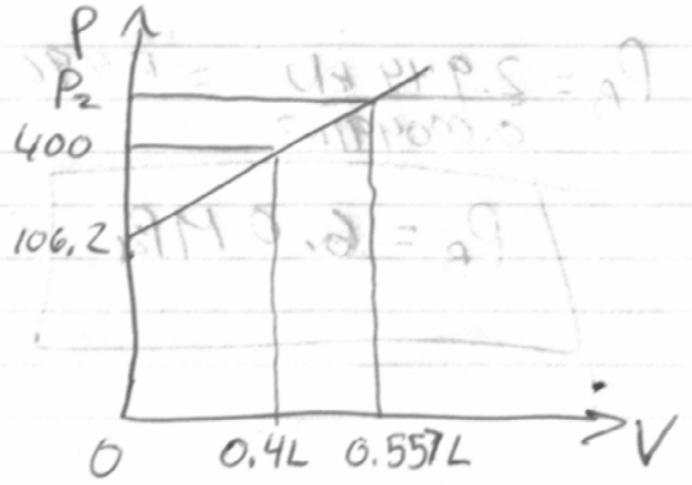
$$= \left(\frac{0.0004}{0.00785} + 0.02\right)(0.00785)$$

$$V_2 = 0.000557$$

$$\therefore P_2 = P_1 + \frac{dP}{dV}\Delta V$$

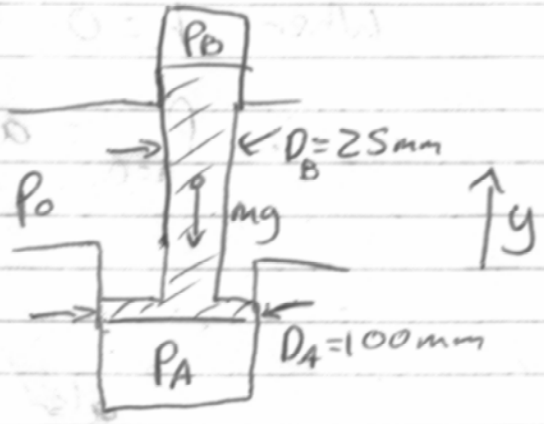
$$= 400\,kPa + \frac{(400-106.2)}{(0.4-0)}(0.557-0.4)$$

$$\boxed{P_2 = 515.3\ kPa}$$



2.85/ Given: $D_A = 100\,mm$
$P_0 = 100\,kPa$
$P_A = 500\,kPa$
$m = 25\,kg$
$D_B = 25\,mm$

Find: $P_B$




SOL'N:

Force balance on piston

(1) $\Sigma F_y = 0 = P_A A_A - P_B A_B - mg - P_0(A_A - A_B)$

Piston Areas $A_A = \frac{\pi D_A^2}{4} = \frac{\pi}{4}(0.1)^2 = 0.00785\,m^2$

$A_B = \frac{\pi D_B^2}{4} = \frac{\pi}{4}(0.025)^2 = 0.000491\,m^2$

from eqn (1) we have:

$$P_B A_B = P_A A_A - mg - P_0(A_A - A_B)$$

$$= (500)(0.00785) - \frac{(25)(9.81)}{1000\ Pa/kPa} - (100)(0.00785 - 0.000491)$$

$$= 2.944\,kN$$

$$\therefore P_B = \frac{2.944\,kN}{0.000491\,m^2} = 5996\,kPa$$

$$\boxed{P_B = 6.0\,MPa}$$