# WEEK 10 SOL'NS

9.39/ a) C.V. turbine Full load reversible + adiabatic
- Entropy eqn 9.8 reduces to constant S, so from table B.1.3 + B.1.2

$$S_3 = S_1 = 7.1271 = 0.6493 + X_{3a}(7.5009)$$

$$X_{3a} = 0.8636$$

$$h_{3a} = 191.83 + 0.8636(2392.8)$$
$$h_{3a} = 2258.3 \text{ kJ/kg}$$

from energy eqn 6.13 for turbine

$$\boxed{w_{1\to 3a} = h_1 - h_{3a} = 3247.6 - 2258.3 = 989.3 \text{ kJ/kg}}$$

b) - use energy eqn for part load operation and notice that we have constant h in throttle process

$$w_T = 0.80(989.3) = 791.4 = 3247.6 - h_{3b}$$

$$h_{3b} = 2456.2 = 191.83 + X_{3b}(2392.8)$$
$$X_{3b} = 0.9463$$

$$S_{3b} = 0.6492 + 0.9463(7.501) = 7.7474 \text{ kJ/kg}$$

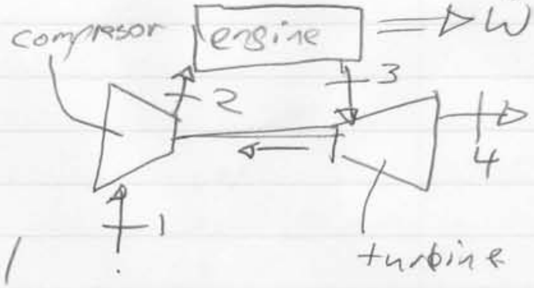
$$\left.\begin{aligned} S_{2b} &= S_{3b} = 7.7474 \\ h_{2b} &= h_1 = 3247.6 \end{aligned}\right\} \quad \boxed{\begin{aligned} P_{2b} &= 510 \text{ kPa} \\ T_{2b} &= 388.4\,^\circ C \end{aligned}}$$

9.43/ C.V turbine, Steady single inlet & exit flows.

Process: adiabatic $q=0$

reversible $S_{gen}=0$,

or $\Delta S = \frac{Q}{T}$.




in rate form $\Rightarrow \frac{ds}{dt} = \frac{dQ}{dt} \cdot \frac{1}{T}$

from the energy eqn (1st law) $w_T = h_3 - h_4$

from second law. $s_4 = s_3$

- for an ideal gas and isentropic process.

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

getting values from table A.5 for K.

$$\boxed{T_4 = 923.2 \left(\frac{100}{170}\right)^{0.286} = 793.2\,K}$$

from 1st Law

$$w_T = h_3 - h_4 = C_{p0}(T_3 - T_4) = 1.004(923.2 - 793.2)$$
$$= 130.5\ kJ/kg$$

$$\boxed{\dot{W}_T = \dot{m}\, w_T = 13.05\ kW}$$

~~C.V. Compressor, Steady 1 in~~

C.V compressor, steady 1 inlet and 1 exit
continuity → same flow rate as turbine

1st law: $-w_C = h_2 - h_1$
second law: $s_2 = s_1$

if the turbine is used only to power the compressor, then,

$$-w_C = w_T = 130.5 = C_{p0}(T_2 - T_1)$$
$$= 1.004(T_2 - 303.2)$$

$$\boxed{T_2 = 433.2\ k.}$$

also, for ideal gas and isentropic eqn.

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = 100\left(\frac{433.2}{303.2}\right)^{3.5}$$

$$\boxed{P_2 = 348.7\ kPa}$$

9.51/ Soln) continuity: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

1st law: $\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{m}_3 h_3$

2nd law: $\dot{m}_1 s_1 + \dot{S}_{gen} + \int \frac{d\dot{Q}}{T} = \dot{m}_2 s_2 + \dot{m}_3 s_3$

process: $\dot{Q} = 0$ irreversible (throttle)


from table B.1.1 $h_1 = 632.18$ kJ/kg $s_1 = 1.8417$ kJ/kg K

B.1.2 $h_3 = 2706.63$ kJ/kg $S_3 = 7.1271$ kJ/kg K

$h_2 = 504.68$ kJ/kg $S_2 = 1.53$ kJ/kg K

from 1st law: $\dot{m}_3 = \dot{m}_1 \frac{(h_1 - h_2)}{h_3 - h_2} = 1.5\,(0.0579)$

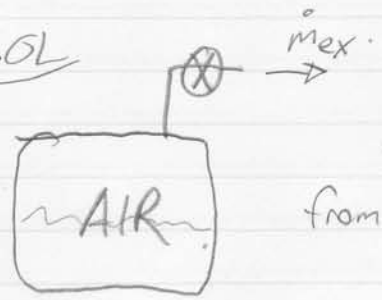
$$\dot{m}_3 = 0.08685 \text{ kg/s}$$

from continuity $\dot{m}_2 = \dot{m}_1 - \dot{m}_2 = 1.41315$ kg/s

$\therefore$ from second law: $\dot{S}_{gen} = \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1$
$= 1.41315(1.53) + (0.8685)(7.127) - 1.5(1.8417)$

$$\boxed{\dot{S}_{gen} = 0.017 \text{ kW/K}}$$

9.67/ SOL




C.V. total tank

from continuity $m_2 - m_1 = -m_{ex}$

from 1st Law: $m_2 u_2 - m_1 u_1 = -m_{ex} h_{ex} + Q_{12} - W_{12}$

2nd Law: $m_2 s_2 - m_1 s_1 = -m_{ex} s_{ex} + \int \frac{dQ}{T} + S_{gen,12}$

process: Adiabatic, $Q = 0$, rigid tank $W = 0$

- There are too many unknowns, we do not know state 2.

- Consider C.V as $m_2$, the remaining mass in the tank.

1st Law: $m_2 (u_2 - u_1) = Q_{12} - W_{12}$

2nd Law: $m_2 (s_2 - s_1) = \int \frac{dQ}{T} + S_{12,gen}$

process: adiabatic, $Q_{12} = 0$, reversible $S_{gen,12} = 0$

Assume Ideal gas
since process is isentropic, $s_2 = s_1$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 400 \left(\frac{200}{300}\right)^{0.2857} = \underline{356.25\,K}$$

$$m_2 = \frac{P_2 V}{R T_2} = \frac{200 \times 2}{0.287\,(356.25)} = \underline{3.912\ kg}$$

(Note: work is not zero for the control mass since the work goes into pushing the mass out)

9.109/ soln/ C.V. compressor:

given: $P_1$, $T_1$, $T_e$(real), $\eta_{s,comp}$

Assume constant $C_{p0}$

1st Law for real process: $-w = C_{p0}(T_e - T_1) = 1.004(200-20)$

$-w = 180.72$

Ideal process: $-w_s = -w\,\eta_s = 180.72(0.70) = 126.5$

1st Law for ideal process: $126.5 = C_{p0}(T_{e,s} - T_1)$

$= 1.004(T_{es} - 293.2)$

$T_{es} = 419.2\ K$

(ideal process is isentropic)

$\therefore\ P_e = P_1\left(\frac{T_e}{T_1}\right)^{\frac{k}{k-1}} = 100\left(\frac{419.2}{293.2}\right)^{3.5}$

$\boxed{P_e = 349\ kPa}$

$-\dot{W}_{real} = \dot{m}(-w) = 0.1(180.72$

$\boxed{\dot{W}_{real} = -18.07\ kW}$

9.116/ C.V Nozzle, Steady 1 inlet and 1 exit flows.
- No heat transfer, no work.

1st Law: $h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2}$.

2nd Law: $s_i + s_{gen} = s_e$

Ideal nozzle $s_{gen} = 0$ and assume same exit pressure as actual nozzle. Instead of using standart entropy from table A.7 and eqn. 8.28, use constant heat capacity + avg T and eqn 8.32.

From table A.7.1, $C_{p1150} = \frac{1277.81 - 1161.18}{1200 - 1100}$
$= 1.166 \text{ kJ/kg K}$.

$C_v = C_{p1150} - R$.

$C_v = 1.166 - 0.287 = 0.8793$

$k = \frac{C_{p1150}}{C_v} = 1.326$.

(Notice how they differ from table A.5 values at really high temp).

ideal Process (* isentropic): $T_{e,s} = T_i\left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}} = 1200\left(\frac{650}{1000}\right)^{0.24585} = 1079.4\text{K}$.

$\therefore \frac{V_{es}^2}{2} = \frac{V_i^2}{2} + C(T_i - T_{es}) = \frac{1}{2}(15)^2 + 1.166(1200 - 1079.4)1000$

$= 112.5 + 140619.6 = 140732 \text{ J/kg}$

$V_{es} = 530.5 \text{ m/s}$.

actual nozzle with given temp at exit.

$$\frac{V_{e(ac)}^2}{2} = \frac{V_i^2}{2} + h_i - h_{e(ac)} = 112.5 + 1.166\,(1200-1100)1000$$

$$= 116712.5 \text{ J/kg}$$

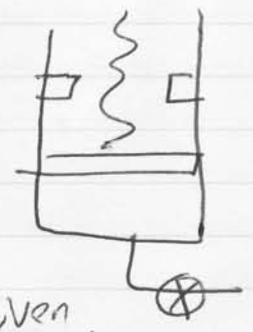
$$\boxed{V_{e,actual} = 483 \text{ m/s}}$$

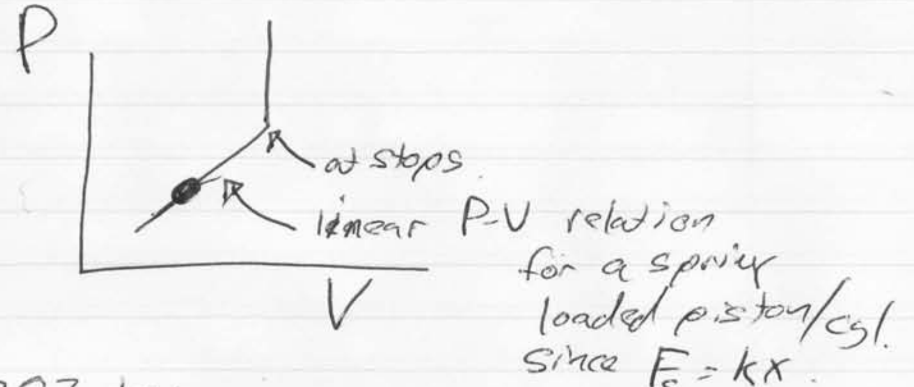
$$\eta_{nozzle} = \frac{\left(\frac{V_{e\,act}^2}{2} - \frac{V_i^2}{2}\right)}{\left(\frac{V_{es}^2}{2} - \frac{V_i^2}{2}\right)}$$

$$= \frac{h_i - h_{e,(ac)}}{h_i - h_{e,s}} = \frac{116600}{140619.6}$$

$$\boxed{\eta = 0.829.}$$

9.122/






Given

$T_0 = 10°C = 283.15K$

$A_p = 0.1 m^2$

$V_{stop} = 50L$

Assume: Ideal Gas

Air from table A.5 $\Rightarrow$ $R = 0.287$

$C_p = 1.004$

$C_v = 0.717$ kJ/kgK

State 1 $T_1 = 10°C$, $P_1 = 200$ kPa, $V_1 = 20L = 0.02 m^3$

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{200(0.02)}{0.287(283.15)} = 0.0492 \text{ kg}$$

State 2 $T_2 = 80°C$, $P_2 = 800$ kPa

inlet $T_i = 50°C$ $P_i = 800$ kPa

a) $P_{stop} = P_1 + \frac{k_s}{A_p^2}(V_{stop} - V_1) = 500$ kPa.

$P_2 > P_{stop}$ $\therefore$ piston hits stops.

$V_2 = V_{stop} = 50L$, $m_2 = \frac{PV}{RT} = 0.3946$ kg

b) 1st Law: $Q_{12} + m_i h_i = m_2 u_2 - m_1 u_1 + m_e h_e + W_{12}$

Continuity: $m_e = 0 \quad \therefore \quad m_i = m_2 - m_1$

$$W_{12} = \int P\,dV = (P_1 + P_{stop})\frac{(V_{stop} - V_1)}{2} = 10.5\ kJ$$

Assuming constant specific heats

$$Q_{12} = m_2 C_v T_2 - m_1 C_v T_1 - (m_2 - m_1) C_p T_i + W_{12}$$

$$\boxed{Q_{12} = -11.6\ kJ}$$

c) 2nd Law: $\Delta S_{net} = m_2 s_2 - m_1 s_1 - m_i s_i - \frac{Q_{cv}}{T_0}$

$$\Delta S_{net} = m_2(s_2 - s_i) - m_1(s_1 - s_i) - \frac{Q_{cv}}{T_0}$$

$$s_2 - s_i = C_p \ln\left(\frac{T_2}{T_i}\right) - R \ln\left(\frac{P_2}{P_i}\right) = 0.08907\ kJ/kg\,K$$

$(P_2 = P_i)$

$$s_1 - s_i = C_p \ln\left(\frac{T_1}{T_i}\right) - R \ln\left(\frac{P_1}{P_i}\right) = 0.26529\ kJ/kg\,K.$$

$$\boxed{\Delta S_{net} = 0.663\ kJ/K}$$

9.124. C.V. turbine, $\eta_s = 0.7$, insulated.

Air: $C_p = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$

Inlet: $T_i = 50°C$, $\dot{V}_i = 20$ L/s $= 0.02$ m$^3$/s.

Exit $T_e = -30°C$, $P_e = 100$ kPa

a) 1st Law steady flow: $q + h_i = h_e + w_T$
adiabatic $\therefore$ $q = 0$
Assume constant specific heat

$\therefore$ $$w_T = h_i - h_e = C_p(T_i - T_e) = 80.3 \text{ kJ/kg}$$

$$w_{Ts} = \frac{w}{\eta} = 114.7 \text{ kJ/kg} \quad w_{Ts} = C_p(T_i - T_{es})$$

$\therefore$ $T_{es} = 208.9$ K.

for an isentropic process: $P_e = P_i\left(\frac{T_e}{T_i}\right)^{\frac{k}{k-1}}$

$$\boxed{P_i = 461 \text{ kPa}}$$

b) $\dot{W}_T = \dot{m} w_T$

$$\dot{m} = \frac{P\dot{V}}{RT} = 0.099 \text{ kg/s}$$

$\therefore$ $$\boxed{\dot{W}_T = 7.98 \text{ kW}}$$