Problem Set #3

3.73/ **Given:** Propane

Initial State

Tank A: $P_{A1} = 100\,kPa$

$T_{A1} = 300\,K$

$V_{A1} = 1\,m^3$ (Rigid tank) $= V_{A2}$

Tank B: $P_{B1} = 250\,kPa$

$T_{B1} = 400\,K$

$V_{B1} = 0.5\,m^3$ (Rigid tank) $= V_{B2}$

Final State

$T_{A2} = 325\,K = T_{B2} = T_2$

$P_{B1} = P_{B2} = P_2$

(Uniform state between the tanks)

Find: P_2

Assumptions: 1. Conservation of mass

$$m_{A1} + m_{B1} = m_{B2} + m_{A2} = m_{tot}$$

2. Ideal Gas $\therefore$ $PV = mRT$ applies

Sol'n: from table A.5 $R = 0.1886\ kJ/kg\,K$

$$m_{A1} = \frac{P_{A1} V_A}{R T_{A1}} = \frac{(100)(1)}{(0.1886)(300)} = 1.7674\ kg$$

$$m_{B1} = \frac{P_{B1} V_B}{R T_{B1}} = \frac{(250)(0.5)}{(0.1886)(400)} = 1.6564\ kg$$

$$m_{tot} = m_{A1} + m_{B1} = 3.4243\ kg$$

$$V_{tot} = V_A + V_B = 1.5\ m^3$$

$$\therefore P_2 = \frac{m_{tot} R T_2}{V_{tot}} = \frac{(3.4243)(0.1886)(325)}{1.5}$$

$$\boxed{P_2 = 139.9\ kPa}$$

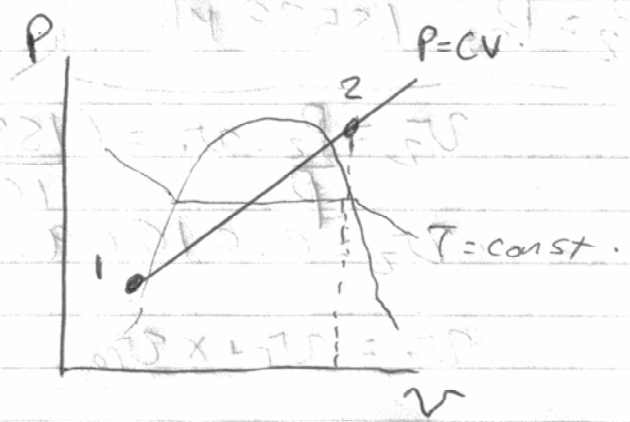
QUIZ #2

3.102/ **Given:** State 1
$T_1 = 90°C$
$P_1 = 100 kPa = CV_1$ since $P = CV$

State 2
$T_2 = 200°C$

Find: P_2 and X (if it exists)

Soln:



$P > P_{sat}$ ∴ State 1 is compressed liquid.

State 1: $v_1 \approx v_f = 0.001036\ m^3/kg$

State 2: since $P = CV$ ⇒ $\frac{P_1}{V_1} = \frac{P_2}{V_2}$.

mass is constant ∴ $\boxed{P_2 = \frac{P_1}{v_1} \cdot v_2}$

– Start by finding out if it is two phase or superheated vapour

from B.1.1 $v_g @ T_2 = 0.12736$
$P_{sat} @ T_2 = 1.5538\ MPa$

if $v_2 \geq v_g$

$$P_2 = \frac{(100\,kPa)}{(0.001036)}(0.12736) = 12.3\ MPa$$

$P_2 > P_{sat}$ — this is not possible for super heated vapour

∴ must be a mixture

$$\boxed{\text{if } P_2 = P_{sat} = 1.5538\ MPa}$$

$v_f = 0.001156$
$v_{fg} = 0.12620$
$v_g = 0.12736$

$$v_2 = \frac{P_2}{P_1} \cdot v_1 = \left(\frac{1553.8}{100}\right) \cdot 0.001036$$

$$v_2 = 0.01609$$

$$v_2 = v_f + x\,v_{fg}$$

$$x = \frac{v_2 - v_f}{v_{fg}} = \frac{0.01609 - 0.001156}{0.12620}$$

$$\boxed{x = 0.118}$$

3.107/ Given: Air

Initial State: $V_1 = 1m^3$ (Rigid tank) $= V_2 = V$

$P_1 = 1MPa$

$T_1 = 400\ K$

Final State: $P_2 = 5\ MPa$

$T_2 = 450\ K$

Find: a) m_1 and m_2

b) if $T_3 = 300k$ what is P_3

Assumption: Ideal gas ∴ $PV = mRT$ applies.

Sol'n: a) from table A.5 ⇒ $R = 0.287\ kJ/kg\,K$

$$\boxed{\begin{aligned} \therefore m_1 &= \frac{P_1 V_1}{R T_1} = \frac{1000 \times 1}{0.287(400)} = 8.711\ kg \\ m_2 &= \frac{P_2 V}{R T_2} = \frac{5000 \times 1}{0.287(450)} = 38.715\ kg \end{aligned}}$$

b) System cools from T_2 to T_3 at constant mass (valve is closed) ∴ $m_3 = m_2$.

$$\therefore \boxed{P_3 = \frac{P_2 T_3}{T_2} = \frac{5000(300)}{450} = 3.33\ MPa}$$

3.111/ Given: Air

Initial State: $P_1 = 250$ kPa

$T_1 = 300°C = 573.K$

$V_1 = \frac{\pi d^2}{4} h_1 = \frac{\pi (0.1)^2 (0.25)}{4} = 1.96L$

Atmosphere: $P_0 = 100$ kPa

$T_0 = 20°C = 293K$

mass of piston $= 50$ kg $= m_p$

Area of piston $A_p = \frac{\pi d^2}{4} = \frac{\pi (0.1)^2}{4} = 0.00785 m^2$.

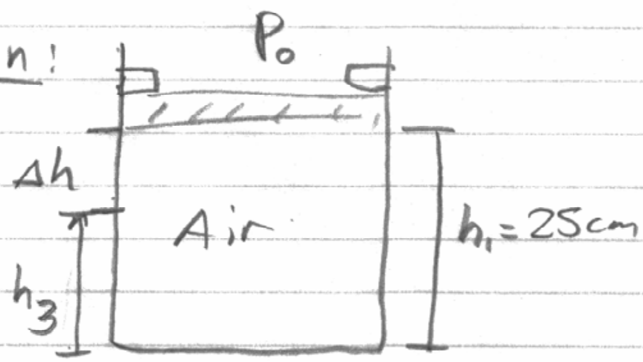
Find: a) at what T does piston begin to move, T_2

b) how far piston moves when $T_3 = 293K$, Δh

Assumption: conservation of mass $m_1 = m_2 = m_3 = m$

and Ideal Gas. $PV = mRT$

Sol'n:



State 2: $\rightarrow P_2 = P$ required to float the piston.

and $V_2 = V_1$

Force balance:

$$\Sigma F_y = 0 = P_2 A_p - P_0 A_p - m_p g$$

$$P_2 = P_0 + \frac{m_p g}{A_p} = 100 + \frac{(50)(9.807)}{0.00785}$$

$$P_2 = 162.5 \text{ kPa} = P_3$$

$$\boxed{T_2 = \frac{T_1 P_2}{P_1} = \frac{(573)(162.5)}{250} = 372.5 K}$$

b) Process $2 \rightarrow 3$ is constant Pressure

$$\therefore V_3 = \frac{V_2 T_3}{T_2} = \frac{(1.96)(293)}{372.5} = 1.54 L$$

$$\Delta h = h_1 - h_2 = \frac{(V_1 - V_3)}{A_p} = \frac{(1.96 - 1.54) \times 0.001\ m^3/L}{0.00785}$$

$$\boxed{\Delta h = 0.053 m = 5.3 cm}$$

3.112/ Given: Air

	Tank	Balloon
State 1:	$P_{T1} = 1 MPa$ $T_{T1} = 20°C = 293 K$	$V_{B1} = 0 m^3$
State 2:		$P_{B2} = 200 kPa$ $T_{B2} = 20°C = 293 K$ $r_2 = 2 m$ $P = Cr$

constant temp, T

Find: m_{B2} and $V_{T\,min}$

Assumptions: 1. Tank is rigid ∴ const vol in tank, V_T
2. cons. of mass $m_{T1} + m_{B1} = m_{T2} + m_{B2} = m$
3. Ideal gas ∴ $PV = mRT$

Sol'n: Balloon final state:

$$V_{B2} = \frac{4}{3}\pi r^3 \text{ (sphere)} = \frac{4}{3}\pi (2)^3 = 33.51 m^3$$

$$\boxed{m_{B2} = \frac{P_{B2}V_{B2}}{R\,T_{B2}} = \frac{(200)(33.51)}{(0.287)(293)} = 79.66 kg}$$

where $R = 0.287 kJ/kgK$ from table A-5

Tank initial state:

tank must have $P_{T2} \geq P_{B2} = 200 kPa = P_2$

and $m_{T2} \geq P_2 V_T / RT$

from ideal gas law: $\frac{P_{T1} V_T}{RT} = m_{T1} = m_{B2} + m_{T2} = m_{B2} + \frac{P_2 V_T}{RT}$

$$\boxed{V_T = \frac{RT\,m_{2B}}{P_{T1} - P_2} = \frac{0.287(293)(79.66)}{1000 - 200} = 8.377 m^3}$$

min Vol of tank.

3.113 **Given:** Air

piston diameter $d = 10cm = 0.1m$

piston area $A_p = \frac{\pi d^2}{4} = \frac{\pi (0.1)^2}{4} = 0.007854 m^2$

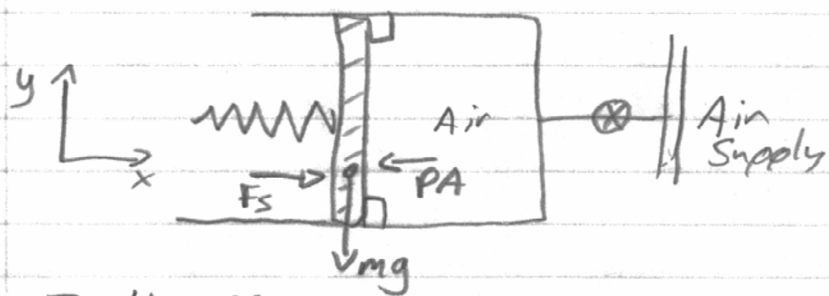
Spring force constant $K_s = 80\ kN/m$

State 1: $V_1 = 1L = 0.001 m^3$

State 2: $P_2 = 150\ kPa$

State 3: $V_3 = 1.5L = 0.0015 m^3$

$T_3 = 80°c = 353K$



Find: Mair.

Assumption 1. cons. of mass $m = m_{air} = m_1 = m_2 = m_3$

2. Ideal gas $PV = mRT$

Sol'n: Force balance on piston at any state:

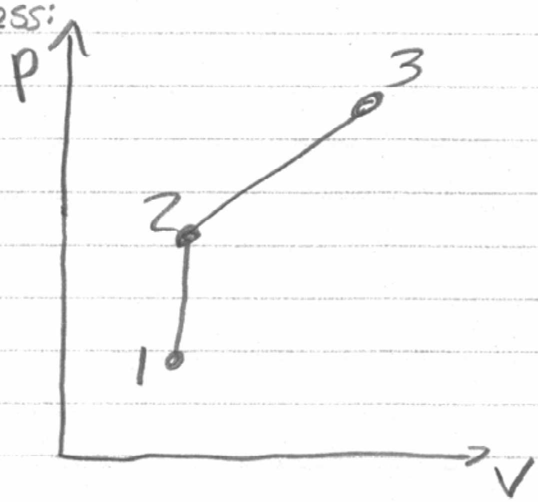
$$\Sigma F_x = 0 = PA_p - \cancel{mg}^{\to 0} - F_s - P_o A_p$$

$$\therefore\ P = P_o + \frac{k_s \Delta V}{A_p^2} \qquad \text{where } F_s = k_s \Delta X = \frac{k_s \Delta V}{A_p}$$

But volume only varies between states 2 → 3

(Vol is constant from 1 → 2)

Process:



Re-arrange pressure (force balance) eqn to get

$$P_0 = P_2 - \frac{k_s V_2}{A_p^2} = P_3 - \frac{k_s V_3}{A_p^2}$$

$$\therefore P_3 = P_2 + \frac{k_s (V_3 - V_2)}{A_p^2}$$

$$= 150 + \frac{(80000)(0.0015 - 0.001)}{(0.007854)^2 (1000\ Pa/kPa)}$$

$$P_3 = 798.5\ kPa$$

$$\therefore \boxed{m = \frac{P_3 V_3}{R T_3} = \frac{(798.5)(0.0015)}{(0.287)(353)} = 0.012\ kg}$$

Where $R = 0.287\ kJ/kgK$ from table A.5