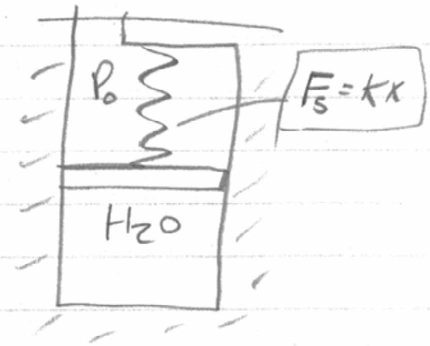
Problem Set #4

4.42/ Given: State 1: $m_1 = 1\,kg$ liquid water
$T_1 = 20°C$
$P_1 = 300\,kPa$.



State 2: $P_2 = 3\,MPa$
$V_2 = 0.1\,m^3$.

Find: a) T_2 b) Plot P-v diagram c) W_{12}.

Assumptions: 1, conservation of mass $m_1 = m_2 = m$.

Sol'n: a) State 1: $v_1 \approx v_{f@T=20} = 0.001002\ m^3/kg$
from table B1.1

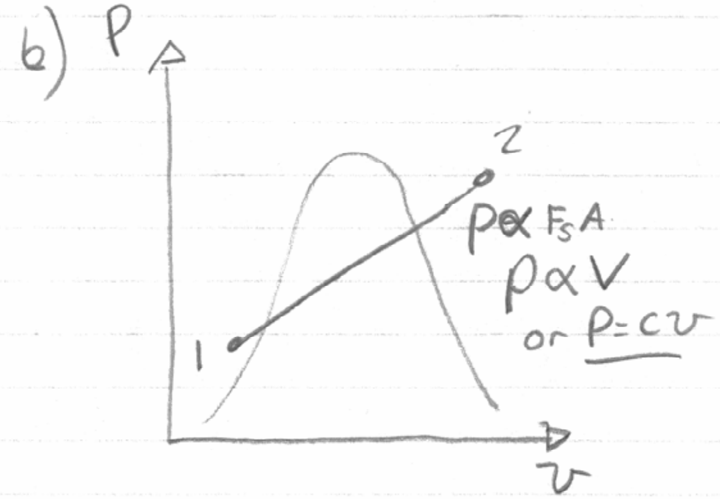
State 2: $v_2 = \frac{V_2}{m} = \frac{0.1\,m^3}{1\,kg} = 0.1\ m^3/kg$

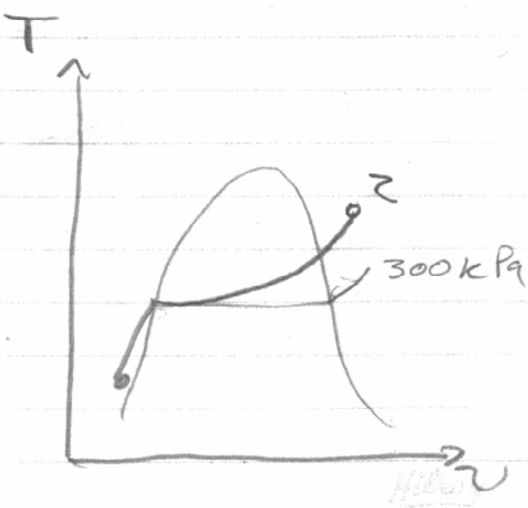
∴ superheated vapour at $P_2 = 3\,MPa$
interpolate to find T_2 (close to 400°C)
from table B.1.3:

$$T_2 = 400°C + (450-400)\left(\frac{0.1-0.09936}{0.10787-0.09936}\right)$$

$$\boxed{T_2 = 404°C}$$

b)





c) work $W_{12} = \int P dV = P_{avg}(V_2 - V_1)$

$$W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - m v_1)$$

$$W_{12} = \frac{1}{2}(300\,kPa + 3000\,kPa)(0.1 - (1)(0.001))$$

$$\boxed{W_{12} = 163.35\,kJ}$$

Ammonia in a balloon

4.54/ Given: State 1: $m_1 = 2\,kg$ state 2: $P_2 = 600\,kPa$

$T_1 = 0°C$

$x_1 = 0.60$

also, $P \propto D^2$ ← Pressure varies with diameter squared

Find: work W_{12}

Assumption: 1. conservation of mass $m_1 = m_2 = m$

Solution: $P \propto D^2$ but volume V varies with D^3

so $V \propto D^3$

$$\therefore P \propto V^{2/3} \quad \text{or} \quad P = cV^{2/3}$$

first we need to find the volume at state 1

from table B.2.1 $v_1 = v_f + x\, v_{fg}$

$$v_1 = 0.00156 + (0.6)(0.28920)$$

$$v_1 = 0.17508\ m^3/kg$$

then $V_1 = m v_1 = (2\,kg)(0.17508\ m^3/kg)$

$$V_1 = 0.3485\ m^3$$

then $V_2 = V_1 \left(\frac{P_2}{P_1}\right)^{3/2} = (0.3485)\left(\frac{600}{429.3}\right)^{3/2}$.

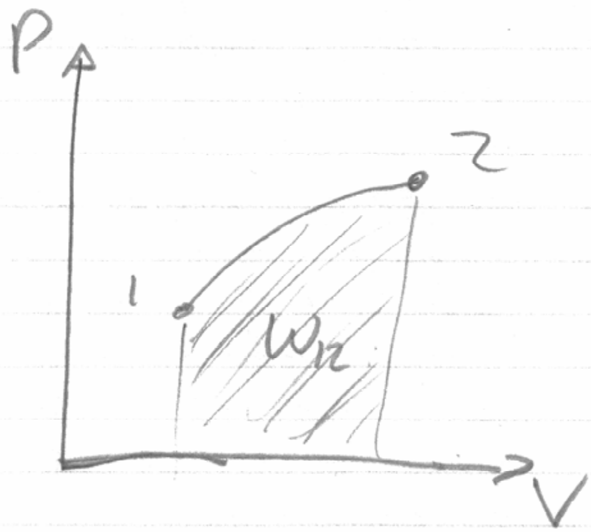
$$V_2 = 0.5758\ m^3$$

$\therefore$ work $W_{12} = \int P dV = \frac{P_2V_2 - P_1V_1}{1-n}$

where $n = -2/3$.
for this polytropic process
$C = PV^{-2/3}$
(Equation 4.4 in text).

$$\therefore W_{12} = \frac{(600)(0.5758) - (429.3)(0.3485)}{1-(-2/3)}$$

$$\boxed{W_{12} = 117.5\ kJ}$$



4.55/ Given: R-134a

State 1: $m_1 = 0.5\,kg$

Sat. vapour $\therefore X_1 = 1$

$T_1 = -10°C$

State 2: $P_2 = 500\,kPa$.

Polytropic process with $n = 1.5$ $(C = PV^{1.5})$

Find: V_2, T_2, W_{12}

Assumption: Conservation of mass $m_1 = m_2 = m$

Soln:

at state 1: $v_1 = 0.09921\,m^3/kg$, $P_1 = P_{sat} = 201.7\,kPa$ } from table B.5.1

at state 2: $V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/1.5}$ But $m_1 = m_2$

$$\therefore v_2 = v_1 \left(\frac{P_1}{P_2}\right)^{1/1.5}$$

$$v_2 = (0.09921)\left(\frac{201.7}{500}\right)^{1/1.5} = 0.05416$$

$$\therefore V_2 = m v_2 = (0.5)(0.05416)$$

$$\boxed{V_2 = 0.02708\,m^3}$$

State 2 is a superheated vapour, given $P_2 + v_2$

from table B.5.2 $\Rightarrow$ $\boxed{T_2 = 79°C}$

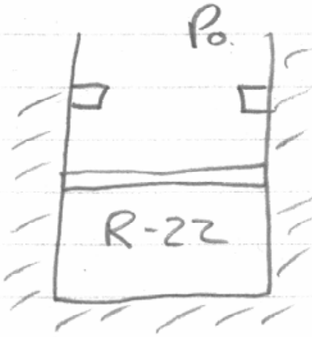
finally work $W_{12} = \int P dV = \frac{m(P_2 v_2 - P_1 v_1)}{1 - 1.5}$

$$W_{12} = \frac{0.5}{-0.5}(500 \times 0.05416 - 201.7 \times 0.09921)$$

$$\boxed{W_{12} = -7.07\,kJ}$$

4.64/ Given: R-22

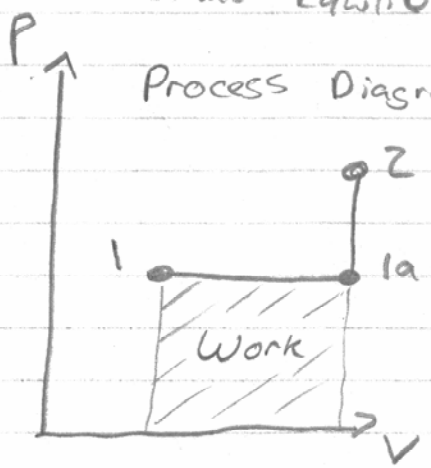
State 1: $T_1 = 30°C$
$P_1 = 150 kPa$
$V_1 = 10 L$
State 1a: $V_{1a} = 11 L$
State 2: $T_2 = 15°C$



Find: a) Is the piston at the stops, does $V_2 = V_{1a}$
b) Work, W_{12}

Assumptions: 1. Cons. of mass $m_1 = m_2 = m$
2. frictionless piston/cylinder
3. Constant pressure for process $1 \rightarrow 1a$
Quasi-Equilibrium

Sol'n: Process Diagram if it hits the stops



from table B.4.2: $v_1 = 0.1487 \ m^3/kg$
(superheated vapour)

$$\therefore m = \frac{V_1}{v_1} = \frac{0.010 \ m^3}{0.1487 \ m^3/kg} = 0.06725 \ kg$$

a) to check if state 2 is against the stops.
find if $V_{2a} > V_{1a}$
at $T = 15°C$ and $P = 150 kPa$ from table B.4.2
$v_{2a} = 0.18011$

$\therefore V_{2a} = m v_{2a} = (0.06725)(0.18011) = 0.012 \ m^3 = 12 L$

$V_{2a} > V_{1a} \ \therefore \ V_2 = V_{1a} = 11 L$ (Piston is at stops)

b) $\therefore$ at state 1a, $P_{1a} = 150\,kPa$

and $v_{1a} = \dfrac{V_{1a}}{m} = \dfrac{0.011\,m^3}{0.06725\,kg} = 0.16357\,m^3/kg$

from state $1a \rightarrow 2$ process is constant volume

$\therefore v_2 = v_{1a}$

work done is then: $W_{12} = \int P_{ext}\,dV = P_{ext}(V_2 - V_1)$

$= P_{ext}(V_{1a} - V_1)$

$= (150\,kPa)(0.011 - 0.010)$

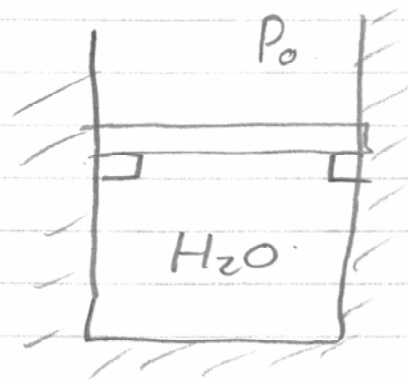
$$\boxed{W_{12} = 0.15\,kJ}$$

4.68/ Given: Water

State 1: $m_1 = 10\ kg$
$X_1 = 0.5$
$P_1 = 100\ kPa$

State 1a: $P_{1a} = 200\ kPa$

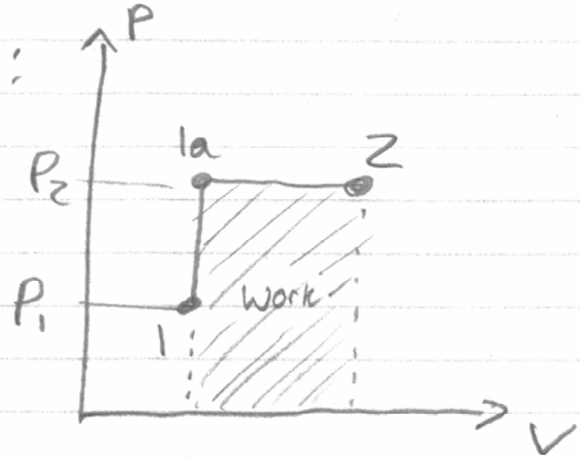
State 2: $V_2 = 3V_1$



Find: a) T_2 and V_2
b) W_{12}

Assumptions: 1. Cons. of mass $m_1 = m_2 = m$ $\therefore v_2 = 3v_1$
2. frictionless piston/cylinder
3. Process from $1a \rightarrow 2$ is in Quasi-Equilibrium (constant P) $\therefore P_2 = P_{1a}$

Sol'n process:



a) process $1 \rightarrow 1a$ is const vol, $\therefore V_{1a} = V_1$
and $v_{1a} = v_1$

at state 1: $v_1 = v_f + x v_{fg}$ when $P_1 = 100\ kPa$
$v_1 = 0.001043 + (0.5)(1.69296)$
$v_1 = 0.8475\ m^3/kg$ from table B.1.2

at state 2: $v_2 = 3(0.8475) = 2.5425\ m^3/kg$

∴ from table B.1.3 (super heated Vapour)

for $v_2, P_2 \Rightarrow \boxed{T_2 = 829°C}$

Volume $V_2 = m v_2 = (10kg)(2.5425 m^3/kg)$

$$\boxed{V_2 = 25.425 m^3}$$

b) $W_{12} = \int P dV = P_{lift}(V_2 - V_1) = P_{lift}(V_2 - m v_1)$

$= (200 kPa)(25.425 - (10)(0.8475))$

$$\boxed{W_{12} = 3390 kJ}$$