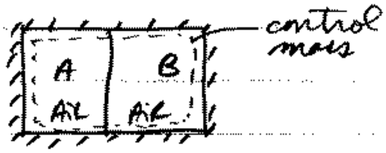
MCG 2135 HOMEWORK 7 (5.90, 5.95, 5.96, 5.102, 5.112)

5.90



$V_A = 0.5\,m^3$ $V_B = 1\,m^3$

$P_A = 250\,kPa$ $P_B = 150\,kPa$

$T_A = 300\,K$ $T_B = 1000\,K$

Find final pressure & temperature (final state is uniform)

* insulated

Apply 1st Law to the control mass:

$${}_1\cancel{Q}_2 - {}_1\cancel{W}_2 = U_2 - U_1 + \cancel{\frac{m(V_2^2 - V_1^2)}{2}} + \cancel{mg(z_2 - z_1)}$$

Assumptions: (1) $\Delta KE = 0$, (2) $\Delta PE = 0$, (3) Perfect Gases

(4) ${}_1W_2 = 0$ (no change in volume) (5) ${}_1Q_2 = 0$ (insulated)

$\therefore$ $U_2 = U_1$ OR $m_A u_A + m_B u_B = m_{total} u_{total}$

$$m_A = \frac{P_A V_A}{R T_A} = \frac{250 \times 10^3 \times 0.5}{287 \times 300} = 1.452\ kg \qquad m_B = \frac{P_B V_B}{R T_B} = \frac{150 \times 10^3 \times 1}{287 \times 1000} = 0.523\ kg$$

Use Table A.7.1 to get u_A & u_B since T_A & T_B are significantly different.

$u_A = 214.36$ kJ/kg & $u_B = 759.19$ kJ/kg

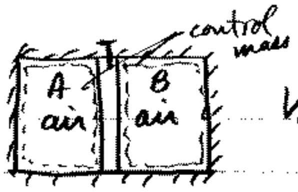
$$\therefore u_{total} = \frac{1.452(214.36) + 0.523(759.19)}{1.452 + 0.523} = 358.6\ \frac{kJ}{kg}$$

$\therefore$ From table A.7.1 at $u = 358.6$ kJ/kg, $\underline{T = 498.4\,K}$

$$P = \frac{m_{total} R T}{V_{total}} = \frac{(1.452 + 0.523)(287)(498.4)}{0.5 + 1} = 188337\ Pa$$

$$\underline{P = 188.3\ kPa}$$

5.95



$V_A = V_B = 1\ m^3$ Find final pressure & temperature.

$P_A = 200\ kPa$ $P_B = 1000\ kPa$ + insulated

$T_A = 300K$ $T_B = 1000K$

Apply 1st Law to the control mass:

$${}_1Q_2 - {}_1W_2 = U_2 - U_1 + \cancel{\frac{m(V_2^2 - V_1^2)}{2}} + \cancel{mg(z_2 - z_1)}$$

Assumptions: (1) $\Delta KE = 0$, (2) $\Delta PE = 0$, (3) Perfect Gas
(4) ${}_1W_2 = 0$ (no change in volume) (5) ${}_1Q_2 = 0$ (insulated)

$U_2 = U_1$ OR $m_A u_A + m_B u_B = m_{total} u_{total}$

$$m_A = \frac{P_A V_A}{R T_A} = \frac{200 \times 10^3 \times 1}{287 \times 300} = 2.323\ kg \qquad m_B = \frac{P_B V_B}{R T_B} = \frac{1000 \times 10^3 \times 1}{287 \times 1000} = 3.484\ kg$$

Use Table A.7.1 to get u_A & u_B since T_A & T_B are significantly different.

$u_A = 214.36\ kJ/kg$ $u_B = 759.19\ kJ/kg$

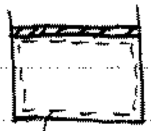
$$\therefore u_{total} = \frac{2.323(214.36) + 3.484(759.19)}{2.323 + 3.484} = 541.2\ \frac{kJ}{kg}$$

$\therefore$ From table A.7.1 at $u = 541.2\ kJ/kg$, $T = 736\ K$

$$P = \frac{m_{total} R T}{V_{total}} = \frac{(2.323 + 3.484)(287)(736)}{(1+1)} = 613\ 312\ Pa$$

$$P = 613\ kPa$$

5.96



control mass

$P_1 = 600\,kPa$ $\quad$ ${}_1W_2 = 54\,kJ$ (from a constant pressure process)

$T_1 = 290\,K$

$V_1 = 0.01\,m^3$ $\quad$ Find T_2 & ${}_1Q_2$

Apply 1st to the control mass: ${}_1Q_2 - {}_1W_2 = U_2 - U_1 + \cancel{\frac{m(V_2^2 - V_1^2)}{2}} + \cancel{mg(z_2 - z_1)}$

Assume: (1) $\Delta KE = 0$, (2) $\Delta PE = 0$ & (3) Perfect Gas.

$$\therefore \; {}_1Q_2 = U_2 - U_1 + {}_1W_2$$

constant pressure process

$$\text{and } {}_1W_2 = \int_1^2 P\,dV = P(V_2 - V_1)$$

$$\therefore \; V_2 = \frac{{}_1W_2}{P} + V_1 = \frac{54 \times 10^3}{600 \times 10^3} + 0.01 = 0.1\,m^3$$

$$\frac{\cancel{P_2} V_2}{T_2} = \frac{\cancel{P_1} V_1}{T_1} \quad \text{OR} \quad T_2 = \frac{V_2}{V_1} \cdot T_1 = \frac{0.1}{0.01} \cdot (290)$$

$$T_2 = 2900\,K$$

$$\text{and } {}_1Q_2 = m_2 u_2 - m_1 u_1 + {}_1W_2$$
$$= m(u_2 - u_1) + {}_1W_2$$

Use u_1 & u_2 from Table A7.1 since T_2 & T_1 are significantly different

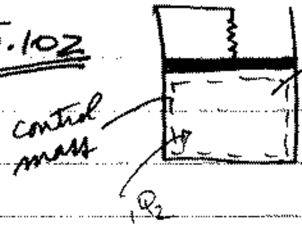
$u_1 = 207.19\,kJ/kg$ & $u_2 = 2563.80\,kJ/kg$

$$m = \frac{P_1 V_1}{R T_1} = \frac{600 \times 10^3 \times 0.01}{287 \times 290} = 0.0721\,kg$$

$${}_1Q_2 = 0.0721(2563.80 - 207.19) + 54$$
$${}_1Q_2 = 223.9\,kJ$$

Hilroy

5.102

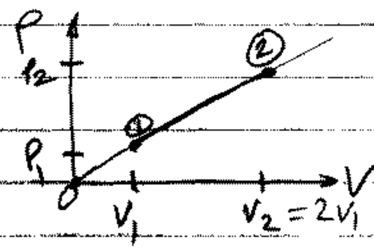


0.25 kg of Ar $\qquad V_2 = 2V_1$

$P_1 = 500$ kPa $\qquad$ (for $V=0$, $P=0$)

$T_1 = 27°C = 300$ K $\qquad$ & linear spring

a) P-V Diagram



b) Final pressure & temperature

Apply 1st to the control mass: $_1Q_2 - {_1W_2} = U_2 - U_1 + \cancel{\frac{m(V_2^2 - V_1^2)}{2}} + \cancel{mg(Z_2 - Z_1)}$

Assume: (1) $\Delta KE = 0$, (2) $\Delta PE = 0$ (3) Perfect Gas.

$$\therefore \; {_1Q_2} = m(u_2 - u_1) + {_1W_2}$$

Since P varies linearly with V: $P = A + BV$

at $V=0$, $P=0$ $\quad \therefore A = 0$ $\quad$ and $\quad P = BV$

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.25(287)(300)}{500 \times 10^3} = 0.04305 \text{ m}^3$$

$$\therefore B = \frac{P_1}{V_1} = \frac{P_2}{V_2} \Rightarrow P_2 = P_1\frac{V_2}{V_1} = 2P_1 = 1000 \text{ kPa} = P_2$$

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \Rightarrow T_2 = \frac{P_2V_2}{P_1V_1} \cdot T_1 = 4T_1 = 1200 \text{ K} = T_2$$

c) Work & Heat Transfer

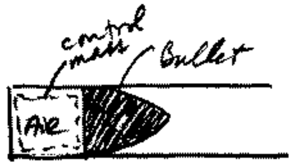
$$_1W_2 = \int_1^2 P\,dV = \frac{P_1 + P_2}{2}(V_2 - V_1) = \frac{500 + 1000}{2}(0.0861 - 0.04305)$$

area under P-V diag. $\qquad = 32.3 \text{ kJ} = {_1W_2}$

$$_1Q_2 = 0.25(933.37 - 214.36) + 32.3 \text{ kJ}$$

$$_1Q_2 = 212.1 \text{ kJ}$$

5.112



$m_{Bullet} = 15\,g$

$V_1 = 1\,cm^3$

$P_1 = 1\,MPa$ $\qquad P_2 = 0.1\,MPa$

$T_1 = 27°C = 300\,K$ $\qquad T_2 = 300\,K$ (isothermal)

a) Find V_2 + mass of air

Take control mass at State 1: a perfect gas

$$m_{air} = \frac{P_1 V_1}{R T_1} = \frac{1\times10^6 \times 1\times10^{-6}}{287 \times 300} = 1.16\times10^{-5}\ kg$$

$$\frac{P_1V_1}{\cancel{T_1}} = \frac{P_2V_2}{\cancel{T_2}} \quad \text{OR} \quad P_1V_1 = P_2V_2 = \text{constant}$$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{1}{0.1} \times 1\times10^{-6}$$

$$= 1\times10^{-6}\,m^3$$

$$V_2 = 10\,cm^3$$

b) ${}_1W_2$ by air + ${}_1W_2$ on atmosphere

$${}_1W_{2\,Air} = \int P\,dV = P_1V_1 \ln(V_2/V_1) = 1\times10^6(1\times10^{-6}) \ln(10/1)$$

$$= 2.30\,J$$

$${}_1W_2 = 2.30\,J$$

$${}_1W_{2\,ATM} = \int P\,dV = P_0(V_2 - V_1)$$ (work is done on the atmosphere due to the atm. pressure P_0)

$$= 0.1\times10^6(10\times10^{-6} - 1\times10^{-6})$$

$$= 0.9\,J \qquad {}_1W_2\,atm = 0.9\,J$$

The work done on the bullet is: ${}_1W_{2\,Bullet} = {}_1W_{2\,Air} - {}_1W_{2\,Atm}$

→ this is (-ve) work

$$= 2.30 - 0.9 = 1.4\,J$$

Apply 1st Law on BULLET:

$${}_1\cancel{Q}_2 - {}_1W_2 = \cancel{U_2 - U_1} + m(V_2^2 - \cancel{V_1^2})/2 + \cancel{mg(z_2 - z_1)}$$

$$\therefore -{}_1W_2 = \frac{m}{2}V_2^2 \qquad \therefore V_2 = \sqrt{\frac{-2(-1.4)}{15\times10^{-3}}} = 13.7\,m/s$$