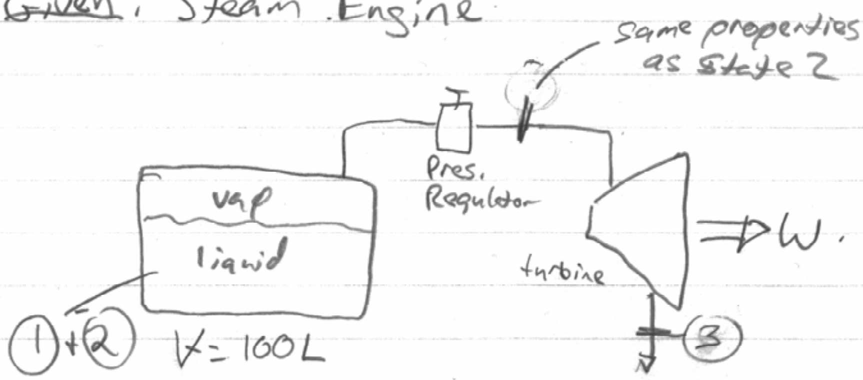
WEEK 8 SOLUTIONS

6.131/ Transient Process.

Given: Steam Engine



CV. tank

State 1: $X_1 \approx 0$
$P_1 = 100\,kPa$

State 2: $P_2 = 700\,kPa$
$X_2 = 1$

CV. turbine

State 3: $X_3 = 1$
$P_3 = 100\,kPa$

Find: $W_{Turbine}$, $Q_{Boiler}(Q_{12})$

Soln: CV. Boiler tank (1st consider tank).

- Assume Rigid ∴ No work $W_{12} = 0$
- Continuity: $m_2 - m_1 = -m_e$
- Conservation of energy

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_i$$ (neglecting ke + pe)

integrating gives:

$$\Delta E = \Delta U = m_2 u_2 - m_1 u_1 = Q_{12} - m_e h_i$$

state 1

from table B.1.2 @ 100 kPa ⇒ $v_1 = 0.001043\ m^3/kg$
$u_1 = 417.36\ kJ/kg$

$$m_1 = \frac{V}{v_1} = \frac{0.1 m^3}{0.001043 m^3/kg} \Rightarrow 95.877\ kg$$

State 2 from table B.1.2 @ 700 kPa $\Rightarrow v_2 = v_g = 0.2729\ m^3/kg$
$u_2 = 2572.5\ kJ/kg$

$$m_2 = \frac{V}{v_2} = \frac{0.1\ m^3}{0.2729\ m^3/kg} = 0.366\ kg$$

exit state, from table B.1.2 @ 700 kPa
$h_e = 2763.5\ kJ/kg$

$m_e = m_1 - m_2 = 95.511\ kg$

$\therefore Q_{Boiler} = m_2 u_2 - m_1 u_1 + m_e h_e$
$= (0.366)(2572.5) - 95.877(417.36) + 95.5(2763.5)$

$$\boxed{Q_{Boiler} = 224.9\ MJ}$$

Now conside CV as the turbine.
Assume steady state + continuity
$\therefore \dot{m}_{in} = \dot{m}_{out} = m_e$ from Boiler $= 95.511\ kg$

$$\frac{dE}{dt} = 0 = \dot{Q} - \dot{W} + \Sigma \dot{m}_i h_i - \Sigma \dot{m}_e h_e$$

or, $\Delta E = 0 = Q - W + m_i h_i - m_e h_e$
- Assume adiabatic $\therefore Q = 0$

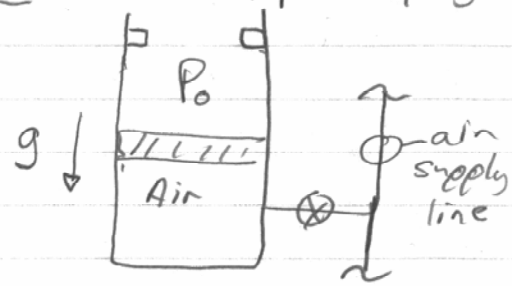
$\therefore W = m_i h_i - m_e h_e$

$h_i = h_e$ from boiler
$\therefore h_i = 2763.5\ kJ/kg$
h_e from table B.1.2
@ 100 kPa
$h_e = 2675.5\ kJ/kg$

$\therefore W = (95.511)(2763.5 - 2675.5)$

$$\boxed{W_{turb} = 8405\ kJ}$$

6.133/ Given: Air in piston/cylinder (Transient Process)



State 1: $P_1 = 300$ kPa
$T_1 = 17°C = 290K$
$V_1 = 0.25 m^3$

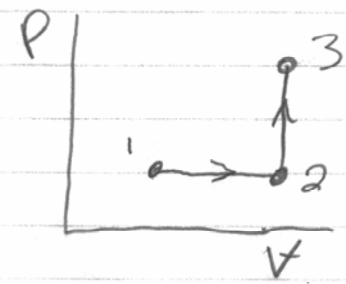
State 2 $V_2 = 1 m^3$

State 3: $P_3 = 400$ kPa
$T_3 = 350 K$

Air Supply $P_i = 500$ kPa
$T_i = 600$ K

Find: m_i, W, Q

Sol'n: C.V is the cylinder volume
Assume: 1. Continuity: $m_3 - m_1 = m_i$
2. cons. of energy
$\Delta E \cong \Delta U = m_3 u_3 - m_1 u_1 = Q - W + m_i h_i$
3. Process is constant P from state 1 to 2
then constant V from 2 to 3



(state 2 is at the stops)

4. Ideal Gas, $PV = mRT$

State 1 $$m_1 = \frac{P_1 V_1}{R T_1} = \frac{300 \times 0.25}{0.287 \times 290} = 0.90 \text{ kg}$$

State 3 $$m_3 = \frac{P_3 V_3}{R T_3} = \frac{400 \times 1}{0.287 \times 350} = 3.982 \text{ kg}$$

$$\therefore m_i = m_3 - m_1 = 3.982 - 0.90$$

$$\boxed{m_i = 3.082 \text{ kg}}$$

- Only work from state $1 \rightarrow 2$

$$\therefore W_{12} = \int_1^2 P dV = P_1(V_2 - V_1) = 300(1 - 0.25)$$

$$\boxed{W_{12} = 225 \text{ kJ}} = W_{13}$$

from energy eqn.

$$Q = m_3 u_3 - m_1 u_1 + W - m_i h_i$$

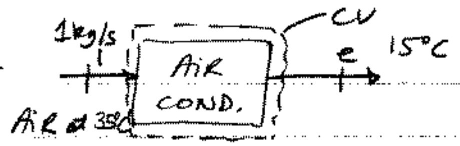
$$Q = m_3 C_v T_3 - m_1 C_v T_1 + W - m_i C_p T_i$$

$$Q = (3.982)(0.717)(350) - (0.90)(0.717)(290) + 225$$
$$- (3.082)(1.004)(600)$$

$$\boxed{Q = -819.2 \text{ kJ}}$$

- Can also use table A.7.1 to find u's and h_i

7.47



Find power needed.

Assumptions (1) Steady state, (2) $\dot{W}_{CV}=0$, (3) $\Delta PE = \Delta KE = 0$
(4) Perfect Gas.

Mass Conservations: $\cancel{\frac{dM_{CV}}{dt}}_{(1)} = \sum \dot{m}_i - \sum \dot{m}_e \Rightarrow \dot{m}_i = \dot{m}_e = 1\ \text{kg/s}$

1st Law: $\dot{Q}_{CV} - \cancel{\dot{W}_{CV}} = \left[\cancel{\frac{m_2(u_2+V_2^2/2+gz_2) - m_1(u_1+V_1^2/2+gz_1)}{\Delta t}}\right]_{CV}$

$- \sum \dot{m}_i (h_i + \cancel{V_i^2/2} + \cancel{gz_i}) + \sum \dot{m}_e (h_e + \cancel{V_e^2/2} + \cancel{gz_e})$

$\dot{Q}_{CV} = \dot{m}_e h_e - \dot{m}_i h_i = \dot{m}\,(h_e - h_i)$

$= \dot{m}\, C_p (T_e - T_i)$

$= 1\,(1.004)(15-35)$

↳ Table A.5

$\dot{Q}_{CV} = -20.08\ \text{kW}$

∴ 20 kW needs to be removed from the air

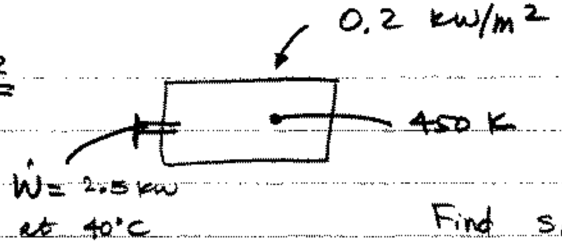
Need to calculate β: use the Carnot Cycle:

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{273+15}{35-15}$$

$= 14.4$

$$\therefore\ \dot{W} = \frac{\dot{Q}_L}{\beta} = \frac{20}{14.4} = 1.39\ \text{kW}$$

7.62



HEAT ENGINE

Find size of solar collector.

For a heat engine:

$$\eta_{HE} = \frac{W_{HE}}{Q_H} = 1 - \frac{T_L}{T_H}$$

$T_H = 450\ K$ & $T_L = 40°C = 313.15\ K$

$$\therefore \eta_{HE} = 1 - \frac{313.15}{450} = 0.304$$

$$\therefore Q_H = \frac{W_{HE}}{\eta_{HE}} = \frac{2.5\ kW}{0.304}$$

$$Q_H = 8.22\ kW$$

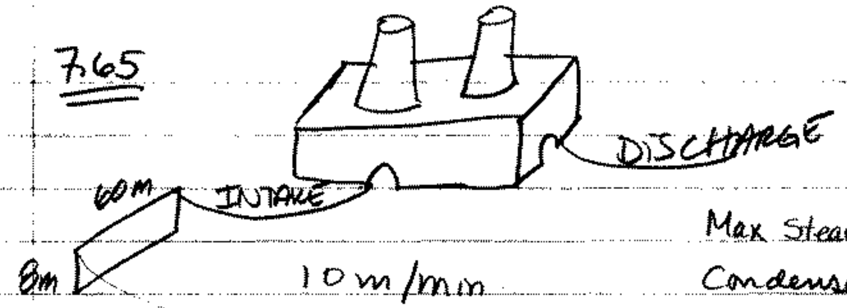
and $Q_H = 0.2 \times AREA$

$$AREA = \frac{Q_H}{0.2} = \frac{8.22\ kW}{0.2\ kW/m^2}$$

$$AREA = 41\ m^2$$

45.81°C

7.65



$W_{Plant} = 1000$ MW

Max Steam Temp = 550°C

Condenser Pressure = 10 kPa

Estimate the temperature rise of the river.

$$\eta = \frac{W_{HE}}{Q_H} = 1 - \frac{T_L}{T_H}$$

$T_H = 550°C = 823.15\ K$

$T_L = 45.81°C = 318.96\ K$ (T_{sat} at $P = 10$ kPa)

$$\eta = 1 - \frac{318.96}{823.15} = 0.6125$$

$$\therefore Q_H = \frac{W}{\eta} = \frac{1000\ MW}{0.6125} = 1632\ MW$$

But we need Q_L : $Q_L = Q_H - W = 1632 - 1000 = 632\ MW$

Apply mass conservation & first Law to the river:

$$\cancel{\frac{dm_{cv}}{dt}} = \Sigma \dot{m}_i - \Sigma \dot{m}_e \Rightarrow \dot{m}_i = \dot{m}_e = \dot{m}$$

(steady)

$$\therefore \dot{m} = \rho V A$$

$$= \left(1000 \frac{kg}{m^3}\right)\left(10 \frac{m}{min}\right)\left(\frac{min}{60 sec}\right)(60m)(8m)$$

$$= 80\ 000\ kg/s$$

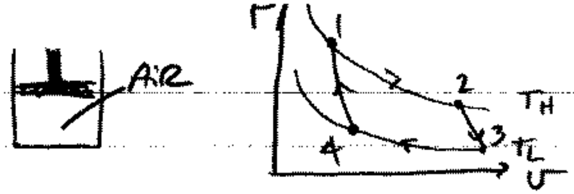
$$\dot{Q}_{cv} - \cancel{\dot{W}_{cv}} = \left[\cancel{\frac{m_2(u_2+V_2^2/2+gz_2) - m_1(u_1+V_1^2/2+gz_1)}{dt}}\right]_{cv} - \Sigma \dot{m}_i (h_i + \cancel{V_i^2/2} + \cancel{gz_i})$$

$$+ \Sigma \dot{m}_e (h_e + \cancel{V_e^2/2} + \cancel{gz_e})$$

$$\dot{Q}_{cv} = \dot{m}_e h_e - \dot{m}_i h_i = \dot{m}(h_e - h_i) \simeq \dot{m} C_p \Delta T$$

$$\Delta T = \frac{\dot{Q}_{cv}}{\dot{m} C_p} = \frac{632 \times 10^6}{(80\ 000)(4180)} = \underline{\underline{1.89\ °C}}$$

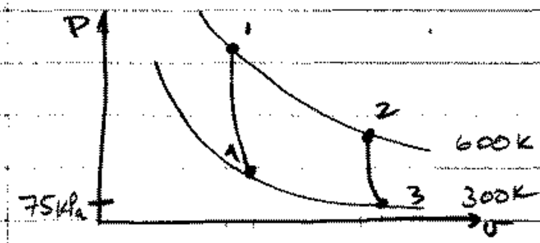
7.79



$T_H = 600K$ & $T_L = 300K$

Heat added at T_H is 250 kJ/kg & $P_3 = 75$ kPa

Find v & P after heat rejection & network per unit mass



* Assume perfect gas.

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5$$

$$w = \eta q_H = 0.5\,(250\ \text{kJ/kg}) = 125\ \text{kJ/kg}$$

$$q_L = q_H - w = 250 - 125 = 125\ \text{kJ/kg}$$

For heat rejection process 3-4 (Eq 7.12):

$$q_L = RT_L \ln(v_3/v_4)$$

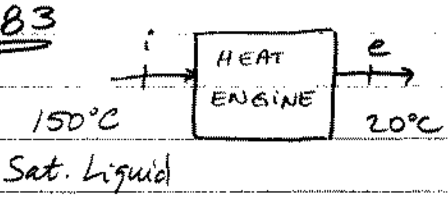
and $v_3 = RT_L/P_3 = (0.287)(300)/(75) = 1.148\ \text{m}^3/\text{kg}$

$$\therefore v_4 = v_3 \exp^{-}(q_L/RT_L)$$

$$v_4 = 1.148 \exp^{-}\left(\frac{125}{(0.287)(300)}\right) = \underline{0.2688\ \text{m}^3/\text{kg}}$$

$$P_4 = \frac{RT_4}{v_4} = \frac{(0.287)(300)}{0.2688} = \underline{320\ \text{kPa}}$$

7.83



Find max thermal efficiency possible.

For a heat engine, the maximum efficiency is obtain with a Carnot Cycle:

$$\eta_{Carnot\ HE} = 1 - \frac{T_L}{T_H}$$

$T_L = 20°C = 293.15K$
$T_H = 150°C = 423.15K$

$$\therefore \eta_{Carnot\ HE} = 1 - \frac{293.15}{423.15} = \underline{\underline{0.307}}$$

It would better to locate a source of saturated vapor at 150°C. The vapor would remain at 150°C at it condenses to liquid, providing a large energy supply at that temperature.