

# Chapter 2

## Design for Shear

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### 2.1 Introduction

Shear is the term assigned to forces that act perpendicular to the longitudinal axis of structural elements. Shear forces on beams are largest at the supports, and the shear force at any distance  $x$  from a support decreases by the amount of load between the support and the distance  $x$ . Under uniform loading, the slope of the shear diagram equals the magnitude of the unit uniform load. Shear forces exist only with bending forces. Concrete beams are expected to crack in flexure, with such cracks forming perpendicular to longitudinal tension reinforcement, i.e., perpendicular also to a free edge. Principal tension stresses change direction from horizontal at the longitudinal reinforcement to  $45^\circ$  at the neutral axis and vertical at the location of maximum compression stress. Consequently, cracks in concrete tend to “point” toward the region of maximum compression stress as indicated by the cracks shown in Fig. 4.1. Axial compression force plus bending makes the area of compressed concrete larger than without axial force.

ACI 318-05 permits the evaluation of shear capacity for most beams to be taken as the combination of strength from concrete without shear reinforcement  $V_c$  plus the strength  $V_s$  provided by shear reinforcement. Shear strength of a slab that resists flexural forces in two orthogonal directions around a column (flat plates, footings and pile caps), is evaluated as the shear strength of a prism located at a distance of half the slab depth  $d$  from the faces of the column.

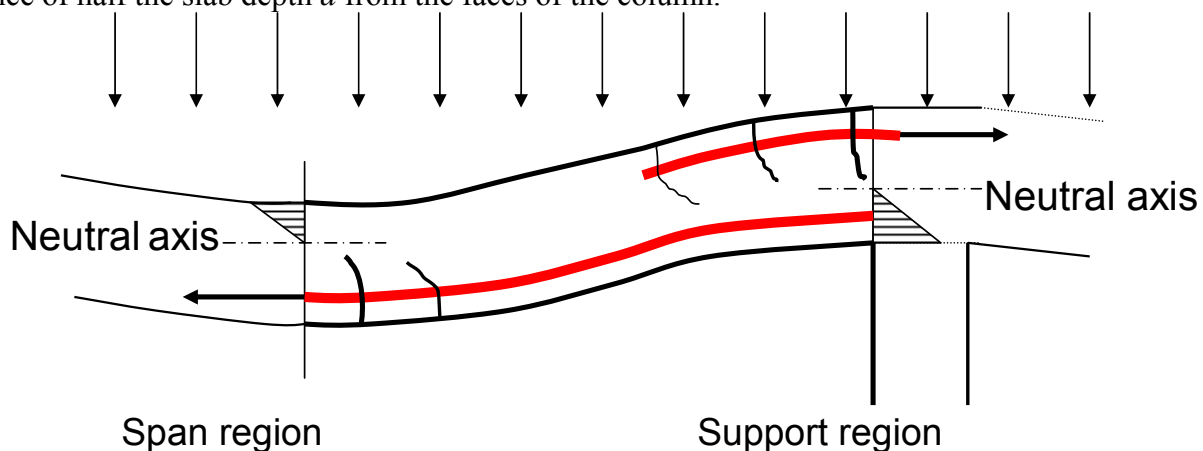


Fig. 4.1 – Reinforced concrete beam in bending

## 2.2 Shear strength of beams

Equation (11-3) of ACI 318-05, Section 11.3.1.1 permits the shear strength  $V_c$  of a beam without shear reinforcement to be taken as the product of an index limit stress of  $2\sqrt{f'_c}$  times a nominal area  $b_w d$ . With  $f'_c$  expressed in  $\text{lb/in}^2$  units and beam dimensions in inches, nominal shear strength  $V_c = 2\sqrt{f'_c} b_w d$  in units of lb. Shear reinforcement is not required for slabs, which can be considered as very wide beams. If the width of a beam is more than twice the thickness  $h$  of the beam, ACI 318-05, Section 11.5.6.1(c) exempts such beams from the requirement of shear reinforcement as long as the shear capacity of the concrete is greater than the required shear force. A more complex method for determining  $V_c$  is given in ACI 318-05, Section 11.3.2.1. The method is demonstrated in SHEAR EXAMPLE 2. A special type of ribbed floor slab known as a joist system can be constructed without any shear reinforcement in the joist ribs. Joist system relative dimensions, slab thickness, rib width and spacing between ribs are specified in ACI 318-05, Section 8.11.

A diagonal crack that might result in shear failure, as suggested in Fig.2.2, can form no closer to the face of the support than the distance  $d$  from the face of the support. Consequently, Section 11.1.3.1 of ACI 318-05 permits the maximum required value of shear  $V_u$  to be determined at a distance  $d$  from the face of such a support when the support provides compression resistance at the face of the beam opposite the loading face. If loads had been suspended from the bottom of the beam, or if the support were no deeper than the beam itself, maximum required shear must be taken as the shear at the face of the support.

The most common form of shear reinforcement is composed of a set of bars bent into U-shaped stirrups as indicated by the vertical bars in Fig. 2.2. The stirrups act as tension hangers with concrete performing as compression struts.

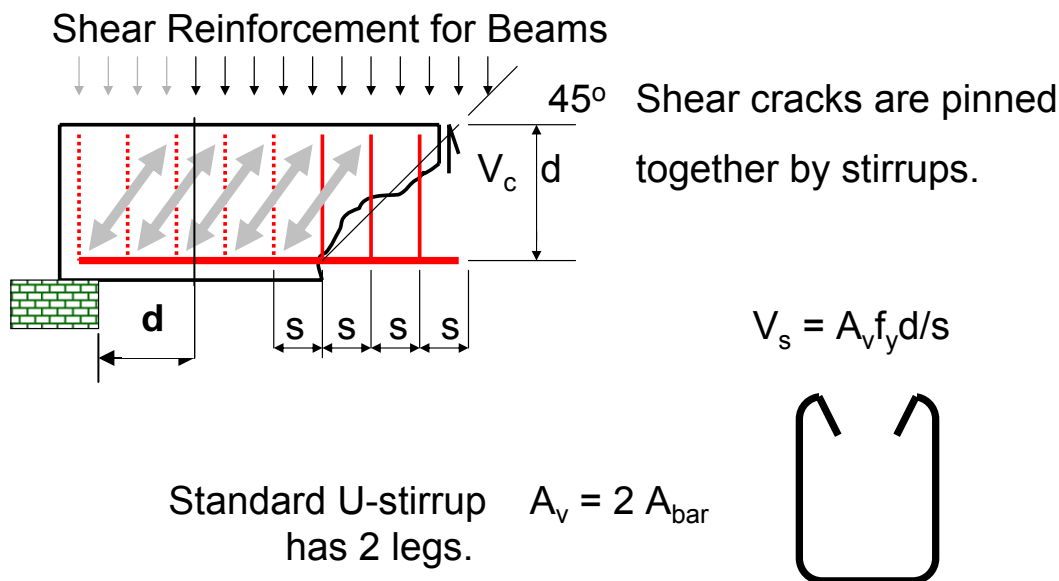


Fig. 2.2 – Shear reinforcement

Each vertical leg of a stirrup has a tension capacity equal to its yield strength, and the most common stirrup has 2 vertical legs. The shear capacity of vertical stirrups is the tension strength of one stirrup times the number of stirrups that interrupt potential cracks on a 45-degree angle from the tension steel. Thus,  $V_s = A_v f_y d / s$ . A U-stirrup has an area  $A_v = 2(\text{area of one stirrup leg})$ . Shear capacity at any location along a beam  $V_n = V_c$  plus  $V_s$ .

## 2.3 Designing stirrup reinforcement for beams

Shear reinforcement  $A_v$  must provide the strength required in addition to the strength of concrete  $V_c$ . Thus, the required amount of  $A_v = (V_n - V_c)/(f_y d/s)$ . The strength reduction factor  $\phi$  for shear is 0.75.  $\phi V_n$  must be greater than  $V_u$ . When the quantities  $A_v$ ,  $f_y$  and  $d$  are known, stirrup spacing  $s$  can be computed as

$$s = (\phi A_v f_y d) / (V_u - \phi V_c) \quad (2.1)$$

ACI 318, Section 11.5.6.1 requires the placement of shear reinforcement in all beams for which the required strength is more than half the value of  $\phi V_c$ . The full development of a critical shear crack between stirrups is prevented by ACI 318-05, Section 11.5.5, which sets the maximum spacing of stirrups at  $d/2$  when  $V_u < 6\phi V_c$ , but maximum spacing is  $d/4$  when  $V_u > 6\phi V_c$ . Concrete cannot act effectively as compression struts if the required amount of  $V_s$  exceeds  $8V_c = 8b_w d \sqrt{f_c'}$  regardless of shear reinforcement. Thus, a beam section must be made larger if  $V_n > 10b_w d \sqrt{f_c'}$ .

A graph given in design aid **SHEAR 1** displays limits of nominal shear stress values of  $V_n/(b_w d)$  for concrete strength  $f_c'$  from 3000 psi to 10,000 psi. The graph is not intended for precise evaluation of member capacity, as precise strength values are given in other design aids. Rather, the graphs clearly show stress ranges for which design requirements change. No shear reinforcement (stirrups) are required if  $V_n/(b_w d)$  is less than  $1.0\sqrt{f_c'}$ . The capacity  $V_c$  of concrete in sections reinforced for shear is  $2.0b_w d \sqrt{f_c'}$ . The strength of stirrups can be added to the concrete strength  $V_c$  to determine the total strength of a section. Required stirrups must be spaced no more than  $d/2$  apart where  $V_n/(b_w d) < 6.0\sqrt{f_c'}$ . Where  $V_n/(b_w d) > 6.0\sqrt{f_c'}$ , maximum stirrup spacing becomes  $d/4$ . The compressive strut capacity of concrete is reached if  $V_n/(b_w d) = 10.0\sqrt{f_c'}$ . Additional stirrups cannot increase section shear strength, as the concrete strength is considered exhausted when  $V_n/(b_w d) > 10\sqrt{f_c'}$ .

Design aid **SHEAR 2** consists of 3 tables that may be used to determine shear capacity for rectangular sections of width  $b$  or  $b_w$  from 10 in to 32 in and thickness  $h$  from 10 in to 48 in. It is assumed that depth  $d$  is 2.5 inches less than thickness for  $h < 30$  in, but that larger longitudinal bars would make  $d \approx h - 3$  in for deeper beams.

Table 2a gives values  $K_{fc} = \sqrt{f_c'/4000}$  to be used as modifiers of  $K_{vc}$  when members are made with concrete strength different from  $f_c' = 4000$  psi. In conjunction with required stirrups, the nominal shear strength of concrete  $V_c = K_{fc}K_{vc}$ .

Table 2b contains values  $K_{vs}$  for determining nominal stirrup capacity  $V_s = K_{vs}(A_v/s)$ .

Table 2c gives values  $K_{vc}$  in kips.  $K_{vc}$  is the shear strength of concrete when required stirrups are used in members made with  $f_c' = 4000$  psi concrete.

The nominal strength of a rectangular section is the sum of concrete strength  $V_c$  and reinforcement strength  $V_s$  to give  $V_n = K_{fc}K_{vc} + K_{vs}(A_v/s)$ .

**SHEAR 3** is a design aid for use if Grade 60 stirrups larger than #5 are to be used, and sections must be deep enough for tension strength bar development of larger stirrups or closed ties. Required thickness of section values are tabulated for concrete strengths from 3000 psi to 10,000 psi and for #6, #7 and #8 stirrups. It should be noted that ACI 318-05, Section 11.5.2 limits the yield strength of reinforcing bar stirrups to no more than 60,000 psi.

ACI 318-05, Section 11.5.6.3 sets lower limits on the amount of shear reinforcement used when such reinforcement is required for strength. These limits are intended to prevent stirrups from yielding upon

the formation of a shear crack. The limit amount of  $A_v$  must exceed  $50b_w s/f_y > 0.75 \sqrt{f_c'} b_w s/f_y$ . The second quantity governs when  $f_c'$  is greater than 4444 lb/in<sup>2</sup>.

The design of shear reinforcement includes the selection of stirrup size and the spacing of stirrups along the beam. Design aids **SHEAR 4.1** and **SHEAR 4.2** give strength values  $V_s$  of #3 U stirrups and #4 U stirrups (two vertical legs) as shear reinforcement tabulated for depth values  $d$  from 8 in to 40 in and stirrup spacing  $s$  from 2 in to maximum permitted spacing  $s = d/2$ . Each table also lists the maximum section width for which each stirrup size may be used without violating the required minimum amount of shear reinforcement. SHEAR 4.1 applies for Grade 40 stirrups, and SHEAR 4.2 applies for Grade 60 stirrups.

## 2.4 Shear strength of two-way slabs

Loads applied to a relatively small area of slabs create shear stress perpendicular to the edge(s) of the area of load application. Columns that support flat plate slabs and columns that are supported by footings are the most common examples. ACI 318-05, Section 11.12.2.1 provides expressions for determining shear strength in such conditions for which shear failure is assumed to occur near the face(s) of the columns. Failure is assumed to occur on the face(s) of a prism located at a distance of  $d/2$  from each column face. The perimeter  $b_o$  of the prism multiplied by the slab depth  $d$  is taken as the area of the failure surface.

Three expressions are given for computing a critical stress on the failure surface. A coefficient  $\alpha_s = 40$  for interior columns,  $\alpha_s = 30$  for edge columns and  $\alpha_s = 20$  for corner columns is used to accommodate columns located along the perimeter of slabs. The critical (failure) stress may be taken as the least value of either  $4 \sqrt{f_c'}$ ,  $(2 + 4/\beta) \sqrt{f_c'}$ , or  $(\alpha_s d/b_o + 2) \sqrt{f_c'}$ . The quantity  $\beta$  is the ratio of long side to short side of the column. The first expression governs for centrally loaded footings and for interior columns unless the ratio  $\beta$  exceeds 2 or the quantity  $40d/b_o$  is less than 2. Shear strength at edge columns and corner columns that support flat plates must be adequate not only for the direct force at the column but also for additional shear forces associated with moment transfer at such columns.

Diagrams for the prism at slab sections for columns are shown with SHEAR EXAMPLES 5, 7 and 8.

Design aid **SHEAR 5.1** gives shear strength values of two-way slabs at columns as limited by potential failure around the column perimeter.

Table 5.1a gives values of K1 as a function of slab  $d$  and column size  $b$  and  $h$ .

Table 5.1b gives values of the shear stress factor K2 as a function of the ratio  $\beta_c$  between the longer side and the shorter side of rectangular column sections.

Table 5.1c gives values of nominal strength  $V_c$  as a function of the product K1K2 and the nominal compressive strength of slab concrete  $f_c'$ .

Design aid **SHEAR 5.2** is similar to SHEAR 5.1 for determining slab shear capacity at round columns. For circular columns, there is no influence of an aspect ratio as for rectangular columns, and the design aid is less complex.

Table 5.2a gives, for slab  $d$  and column diameter  $h$ , values of a shape parameter K3 in sq in units.

Table 5.2b gives, for K3 and slab concrete  $f_c'$ , the value of nominal shear capacity  $V_c$  in kip units.

## 2.5 Shear strength with torsion plus flexural shear

Torsion or twisting of a beam creates shear stress that is greatest at the perimeter of sections. The shear stress due to torsion adds to flexural shear stress on one vertical face, but it subtracts from flexural shear on the opposite vertical face. Shear stress due to torsion is negligibly small near the center of sections. ACI 318-05, Section 11.6 provides empirical expressions for torsion strength. It is assumed that significant torsion stress occurs only around the perimeter of sections, and no torsion resistance is attributed to concrete. The definitions of section properties are displayed in Fig. 2.3.

### Definitions

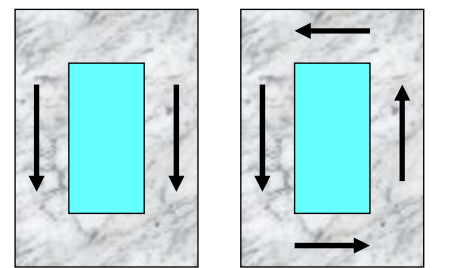
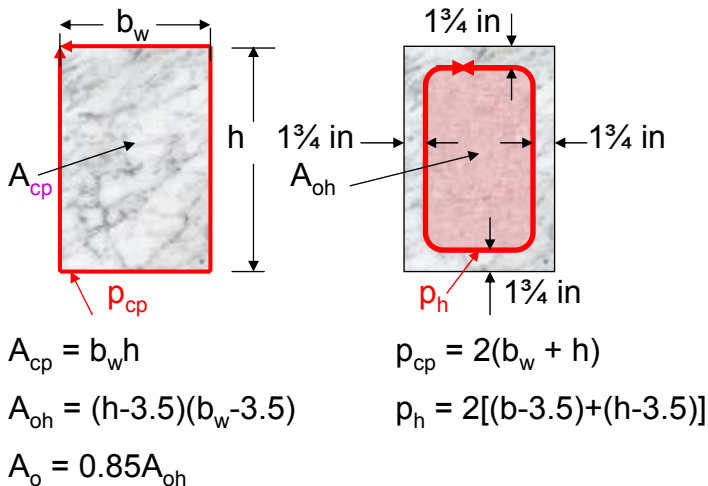
$A_{cp}$  = area enclosed by outside perimeter of section.

$A_o$  = gross area enclosed by shear flow path.

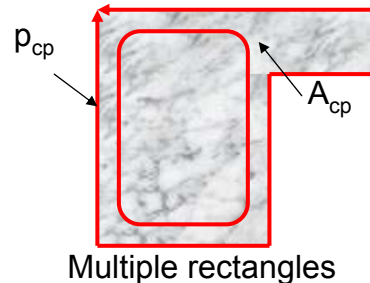
$A_{oh}$  = area enclosed by centerline of closed tie.

$p_{cp}$  = outside perimeter of concrete section.

$p_h$  = perimeter of centerline of closed tie.



Flexural shear    Torsional shear



Multiple rectangles

**Fig. 2.3 – Torsion strength definitions of section properties**

Concrete beams properly reinforced for torsion display considerable ductility, continuing to twist without failure after reinforcement yields. Consequently, ACI 318-05, Section 11.6.2.2 permits design for torsion in indeterminate beams to be made for the torsion force that causes cracking. A member is determinate if torsion forces can be determined from the equations of statics without considering compatibility relationships in the structural analysis. A member is indeterminate if torsion forces must be estimated with consideration of compatibility conditions, i.e., there exists more than one load path for resisting torsion. The illustrations in Fig. 2.4 show two conditions of a spandrel beam supporting a brick ledge. The determinate beam in the upper sketch must transfer to columns all of the eccentric load on the ledge only through the twisting resistance (torsion) of the beam. In contrast, the indeterminate beam in the lower sketch supports a slab that extends outward to receive the eccentric load on the ledge. The eccentric load can be transferred to columns both by torsion of the beam and by flexure of the cantilevered slab.

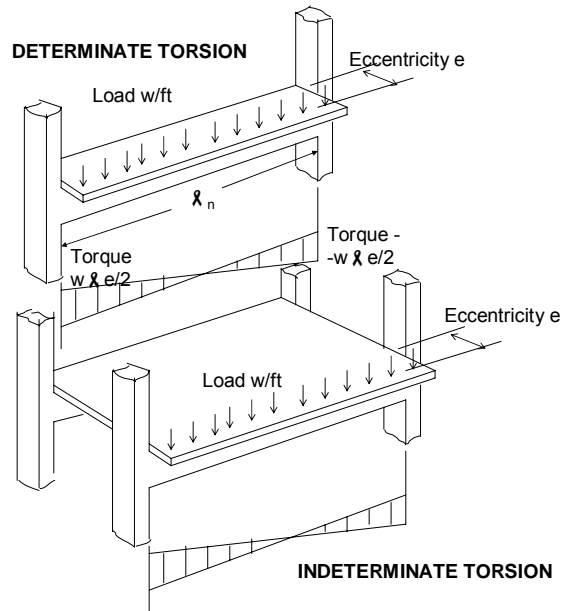


Fig. 2,4 – Determinate torsion versus Indeterminate torsion

Cracking torque  $T_{cr}$  is to be computed without consideration of torsion reinforcement.

$$T_{cr} = 4 \sqrt{f'_c} (A_{cp})^2 / p_{cp} \quad (2.2)$$

Torques smaller than one-quarter of the cracking torque  $T_{cr}$  will not cause any structurally significant reduction in either the flexural or shear strength and can be ignored. An upper limit to the torque resistance of concrete functioning as compression struts is taken from ACI 318-05, Eq. (11-18) as:

$$T_{max} = 17(A_{oh})^2 \sqrt{f'_c} / p_h . \quad (2.3)$$

Torsion reinforcement requires both closed ties and longitudinal bars that are located in the periphery of the section. With torsion cracks assumed at an angle  $\theta$  from the axis of the member, torsion strength from closed ties is computed as

$$T_n = (2A_o A_t f_{yt} \cot \theta) / s \quad (2.4)$$

The angle  $\theta$  must be greater than 30 degrees and less than 60 degrees. A value  $\theta = 45^\circ$  has been used for design aids in this chapter. The size of solid concrete sections must be large enough to resist both flexural shear  $V_u$  and torsion shear  $T_u$  within the upper limits established for each. ACI 318-05, Eq. (11-18) gives

$$\sqrt{[V_u / (b_w d)]^2 + [T_u p_h / (1.7 A_{oh}^2)]^2} \leq \phi [V_c / (b_w d) + 8 \sqrt{f'_c}] . \quad (2.5)$$

In addition, ACI 318, Eq (11-22) requires that longitudinal bars with an area  $A_\ell$  be placed around the periphery of sections.

$$A_\ell = A_t p_h / s . \quad (2.6)$$

Longitudinal spacing of transverse closed ties must be no greater than  $p_h/8$  or 12 in. The spacing between longitudinal bars in the periphery of sections must be no greater than 12 in. Where torsion reinforcement is required, the area of 2 legs of closed tie ( $A_v + 2A_t$ ) must be greater than  $0.75(b_w s/f_{yt}) \sqrt{f'_c}$  but be not less than  $50b_w s/f_y$ .

Design aid **SHEAR 6.1** displays critical values of torsion strength for rectangular sections made with concrete strength  $f'_c = 4000$  psi. If concrete strength  $f'_c$  is different from 4000 psi, the correction factor  $K_{fc}$  from SHEAR 2, Table 2a must be multiplied by torque values  $T_n$  from Table 6.1a and  $T_{cr}$  from Table 6.1b.

Table 6.1a displays values of  $K_t$ , the maximum torque  $lim T_n$  a section can resist as a function of section thickness  $h$  and width  $b$ . It is assumed that the distance from section surface to the center of closed ties is 1.75 in.

Table 6.1b displays values  $K_{tcr}$  of torque  $T_{cr}$  that will cause sections to crack as a function of section dimensions  $b$  and  $h$ .

Design aid **SHEAR 6.2** can be used to determine the torsion strength of closed ties. Numbers  $K_{ts}$  for width  $b$  and thickness  $h$  listed in the charts are multiplied by the ratio between tie area  $A_t$  and tie spacing  $s$  in order to compute the nominal torque  $T_s$  resisted by closed ties. The distance from section surface to tie centerline is taken to be 1.75 in.

Table 6.2a applies for Grade 40 ties. Table 6.2b applies for Grade 60 ties.

## 2.6 Deep beams

The definition of deep beams is found in ACI 318-05 Section 11.8.1. Deep beams have a span-to-depth ratio not greater than 4 or a concentrated force applied to one face within a distance less than  $2d$  from the supported opposite face. If a non-linear analysis is not used for deep beams, the beams can be designed by the strut-and-tie method given in Appendix A of ACI 318-05. Shear reinforcement must include both horizontal bars and vertical bars. Beams more than 8 in thick must have two reinforcement grids, one in each face. A maximum shear limit  $V_n$ , and minimum shear reinforcement values  $A_v$  for vertical bars and  $A_{vh}$  for horizontal bars for deep beams are given in Fig. 2.5.

Deep beams may be designed using Appendix A (Strut & Tie model).

$$V_n < (10 \sqrt{f'_c})(b_w d).$$

$$A_v > 0.0025b_w s \text{ with } s < d/5 \text{ or } 12 \text{ in}$$

$$A_{vh} > 0.0015b_w s_2 \text{ with } s_2 < d/5 \text{ or } 12 \text{ in}$$

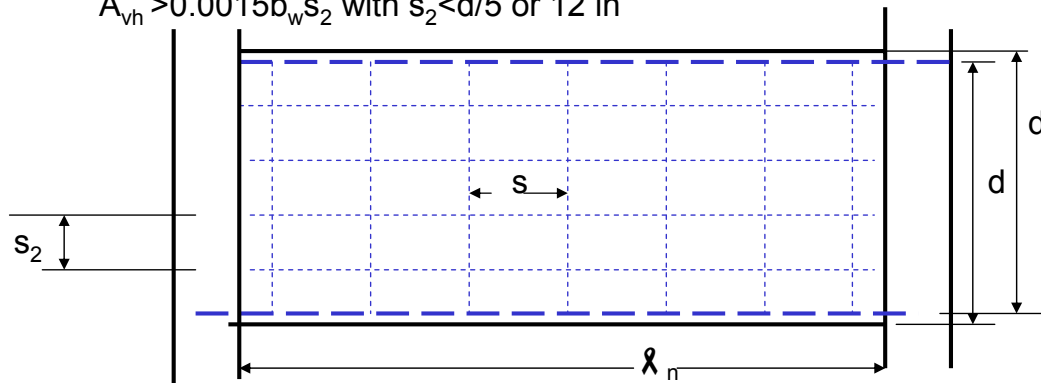


Fig. 2.5 – Deep beam limits in ACI 318-05, Section 11.8

ACI 318-05, Appendix A presents rules for analysis of forces on a truss composed of nodal points connected by concrete compression struts and reinforcing bar tension members. Diagonal concrete compression struts may cross lines of vertical (tension strut) reinforcement, and the angle between any reinforcement and the axis of the diagonal compression strut cannot be less than 25 degrees. Concrete strut area  $A_{cs}$  has a width  $b$  and a thickness  $A_{cs}/b$  that may be considered to increase at a rate equal to the distance along the strut from the node to the center of the strut. A nodal point at which the 3 force components act toward the joint is termed a CCC joint. If two nodal point forces act toward the joint and one force is (tension) away from the joint, the nodal point is designated as CCT. Nodal points with two tensile forces and one compression force is designated CTT, and if all force components act away from the node, the designation becomes TTT. A prismatic strut has the same thickness throughout its length, and a strut wider at the center than at the ends of its length is called a bottle-shaped strut. ACI 318, Section A.3 specifies the nominal strength  $F_n$  of compression struts without longitudinal reinforcement. Two coefficients,  $\beta_s$  for strut shape and  $\beta_n$  for nature of nodal points are used. For struts of uniform cross section in which strut area  $A_{cs}$  can be taken as the same as the nodal bearing area  $A_{nn}$ , then  $A_{nn} = A_{cs}$  and

$$F_n = \beta_n f_{cs} A_{cs} = 0.85 \beta_n \beta_s f_c' A_{cs}. \quad (2.7)$$

for which

- $\beta_s = 1$  for a strut of uniform cross section.
- $\beta_s = 0.75$  for a bottle-shaped strut.
- $\beta_s = 0.40$  for a strut that could be required to resist tension.
- $\beta_s = 0.60$  for all other cases
- $\beta_n = 1$  for struts at CCC nodal points
- $\beta_n = 0.80$  for struts at CCT nodal points
- $\beta_n = 0.60$  for struts at CTT nodal points.

The capacity of prismatic (constant size) concrete struts can be based on strength at its nodal points. The capacity reduction factor for shear,  $\phi = 0.75$ , must be applied to computed values of nominal strength. Concrete compressed struts must be “confined” laterally by reinforcement with a density that satisfies the minimum reinforcement relationship of Equation (A4) from ACI318-05, Section A3.3.1

$$\sum A_{vi}(\sin \alpha_i)/(bs_i) \geq 0.003 \quad (2.8)$$

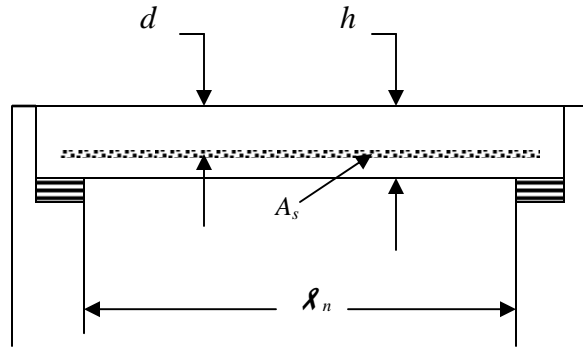
The subscripted index  $i$  refers to the 2 directions, horizontal and vertical, for the sum of shear reinforcement densities. The angle  $\alpha$  is the angle between the diagonal and the direction of tension reinforcement, and  $\alpha$  must be greater than 25 degrees and less than 65 degrees. Minimum requirements for placement of shear reinforcement specified in ACI 318, Section 11.8.4 and Section 11.8.5 will satisfy Eq. (2.8).

Design aid **SHEAR 7** gives solutions to Equation (2.8) for angles  $\gamma$  between a vertical line and the compression strut, with  $\gamma = 25^\circ$  to  $65^\circ$  in increments of  $15^\circ$ . In each chart, solutions to Eq. (A4) are tabulated for bars #3 to #6, and the product of beam width  $b$  and bar spacing  $s$ . For a given angle  $\gamma$  the sum of values for vertical bars and for horizontal bars must be at least 0.003. The sine of a vertical angle  $\gamma$  applies for vertical bars, and the cosine of  $\gamma$  applies for horizontal bars. Reinforcement limits specified in ACI 318-05, Sections 11.8.4 and 11.8.5 limit the maximum product of width and spacing permitted for any tie bar area. Each table shows the value  $(As_i \sin \gamma)/(bs_i)$  when the bar size and spacing limit is reached.



## SHEAR EXAMPLE 1 – Determine stirrups required for simply supported beam

Determine the required shear  $V_n$  for which this beam should be designed. Use the simplified method ACI 318-05 Section 11.3.1.1 to determine the strength  $\phi V_c$  with normal weight concrete. If stirrups are needed, specify a spacing from face of support to the #3 U stirrups that may be required.



Given:

Live load = 1.5 k/ft  
 Superimposed Dead load = 1.4 k/ft  
 $l_n = 20.0$  ft  
 $f'_c = 3000$  psi  
 Stirrups are Grade 60 ( $f_y = 60,000$  psi)

$b_w = 14$  in  
 $d = 19.5$  in (taken as  $h - 2.5$  in)  
 $h = 22$  in

$A_s = 3.16$  sq in (4 #8 longitudinal bars)

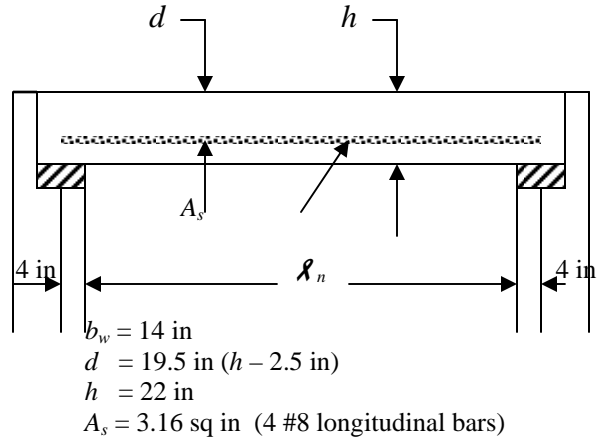
ACI 318-05 Section	Procedure	Calculation	Design Aid
9.2.1	Step 1 - Determine factored (required) load $w_u$ .  Compute beam weight Compute total dead load = beam self weight. + superimposed DL  Compute $w_u = 1.2D + 1.6L$	Self weight $= 14\text{in}(22\text{in})(0.15\text{k/ft}^3)/144\text{in}^2/\text{ft}^2$ $= 0.32$ k/ft DL = 0.32 + 1.40 = 1.72 k/ft  $w_u = 1.2(1.72) + 1.6(1.50) = 4.47$ k/ft	
11.1.2.1	Step 2 – Determine $V_u$ at distance $d$ from face of support.  Compute $V_u = w_u (l_n / 2 - d)$	$V_u = (4.47\text{k/ft})(20.0\text{ft}/2 - 19.5\text{in}/12\text{in/ft})$ $= 37.4$ k	
11.2.1.1	Step 3 – Determine the strength of concrete in shear $V_c$ using the simplified method.  Compute $V_c = 2(\sqrt{f'_c}) b_w d$  Alternate procedure using Design Aids with $f'_c$ find $\phi V_c = \phi K_{fc} K_{vc}$	$V_c = 2(\sqrt{3000\text{psi}})14\text{in}(19.5\text{in})$ $= 29,900$ lbs = 29.9 k  For $f'_c = 3000\text{psi}$ , $K_{fc} = 0.866$ For $b=14\text{in}$ & $h=22\text{in}$ , $K_{vc} = 34.5$ k	SHEAR 2 Table 2a Table 2c
9.2.2.3	Compute $V_c = K_{fc} K_{vc}$	$V_c = (0.866)34.5\text{k} = 29.9$ k	
11.5.5.1	Step 4 – If $V_u > 0.5\phi V_c$ , stirrups are req'd. Compute $0.5\phi V_c$ Compare $V_u$ and $0.5\phi V_c$	$0.5\phi V_c = 0.5(0.75)22.5\text{k} = 11.2$ k $V_u = 37.4\text{k} > 11.2\text{k}$ , stirrups are required	
11.1.1	Step 5 – Compute $\max V_s = V_u / \phi - V_c$	$\max V_s = 37.4\text{k}/0.75 - 29.9\text{k} = 20.0$ k	
11.5.6.9	Step 6 – Note that section is large enough if $V_s < 4V_c$ Section size is adequate	$4V_c = 4(29.9\text{k}) = 119.6$ k $> V_s = 20$ k	

**SHEAR EXAMPLE 1 - Continued**

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.5.6.2  11.5.4.1  11.5.5.3	<p>Step 7 – Determine stirrup spacing for maximum <math>V_s = 20</math> k            Compute <math>A_v</math> for #3 U stirrup            Compute <math>s = A_v f_y d / V_s</math>            Maximum spacing = <math>d/2</math></p> <p>Alternate procedure using Design Aid            #3 Grade 60 stirrups and <math>s = d/2 = 10</math> in            Since max <math>b_w = 26</math> in and <math>b_w = 14</math> in,</p>	<p>Two legs give <math>A_v = 2(0.11 \text{ in}^2) = 0.22 \text{ in}^2</math>  <math>s = (0.22 \text{ in}^2)(60 \text{ k/in}^2)(19.5 \text{ in}) / 20 \text{ k} = 12.9 \text{ in}</math>            max. <math>s = 19.5 \text{ in} / 2 = 9.75 \text{ in}</math>, use <math>s = 10</math> in</p> <p>Shear strength <math>V_s = 26</math> k  <math>A_v f_y &gt; 50 b_w s</math></p>	SHEAR 4.2 Table 4.2a
11.5.5.1	<p>Step 8 – Determine position beyond which no stirrups are required.            No stirrups req'd if <math>V_u &lt; 1/2 \phi V_c</math>.            With zero shear at midspan, the distance <math>z</math> from mid-span to <math>V_u = 1/2 V_c</math> becomes <math>z = 1/2 \phi V_c / w_u</math></p> <p>Stirrups are required in the space <math>(10.0 \text{ ft} - 2.51 \text{ ft}) = 7.49 \text{ ft}</math> from face of each support. Compute in inches</p> <p>Begin with a half space = 5 in, and compute <math>n</math> = number of stirrup spaces required</p> <p>Use 10 #3 U stirrups spaced</p>	<p><math>z = 0.5(0.75)29.9 \text{ k} / 4.47 \text{ k/ft} = 2.51 \text{ ft}</math></p> <p><math>7.49 \text{ ft} = 7.49 \text{ ft}(12 \text{ in/ft}) = 90 \text{ in}</math></p> <p><math>n = (90 \text{ in} - 5 \text{ in}) / 10 = 8.5</math> Use 9 spaces.</p> <p>5 in, 9 @ 10 in from each support.</p>	

## SHEAR EXAMPLE 2 – Determine beam shear strength of concrete by method of ACI 318-05, Section 11.3.2.1

Use the detailed method of ACI 318-05, Section 11.3.2.1 to determine the value of  $\phi V_c$  attributable to normal weight concrete using the detailed method to determine the strength  $\phi V_c$ . Assume normal weight concrete is used.



Given:

$$w_u = 4.47 \text{ kips/ft}$$

$$l_n = 20.0 \text{ ft}$$

$$f_c' = 3000 \text{ psi}$$

Stirrups are Grade 60 ( $f_y = 60,000 \text{ psi}$ )

$$d = 19.5 \text{ in } (h - 2.5 \text{ in})$$

$$h = 22 \text{ in}$$

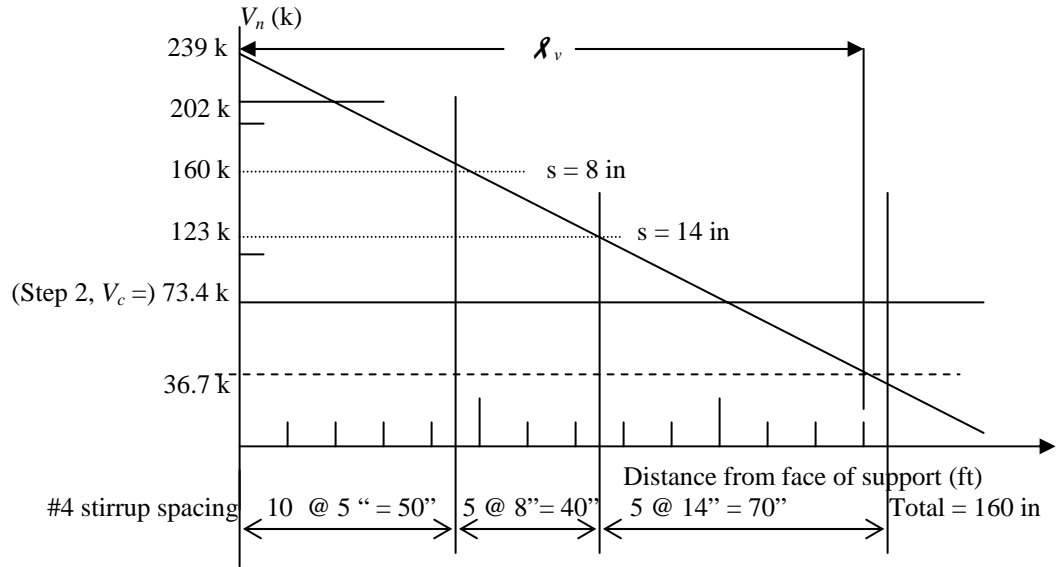
$$A_s = 3.16 \text{ sq in } (4 \text{ #8 longitudinal bars})$$

ACI 318-02 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 – Calculate the moment $M_u$ at d from the face of support. Distance d from the face of support is d + 4 in from column centerline. Compute $l = l_n + \text{column thickness}$ $M_u = (w_u l/2)(d+4) - w_u(d+4)^2/24$	$d + 4\text{in} = 19.5\text{in} + 4\text{in} = 23.5 \text{ in}$ $l = 20.0\text{ft} + 2(4\text{in})/12\text{in/ft} = 20.67 \text{ ft}$ $M_u = 4.47\text{k/ft}(20.67\text{ft})(19.5\text{in}+4\text{in})/2$ $- 4.47\text{k/ft}(23.5\text{ft})^2/24 = 983 \text{ in-k}$	
	Step 2 – Compute $\rho_w = A_s/(b_w d)$	$\rho_w = 3.16\text{in}^2/(14\text{in} \times 19.5\text{in}) = 0.012$	
	Step 3 – Compute at d from support $V_u = w_u (l/2 - 21.5/12)$	$V_u = 4.47\text{k/ft}(20.67\text{ft}/2 - 23.5\text{in}/12\text{in/ft}) = 37.4 \text{ k}$	
	Step 4 – Compute $\rho_w V_u d/M_u$	$\rho_w V_u d/M_u = 0.012(37.4\text{k})23.5\text{in}/983\text{in-k}$ $= 0.011$	
11.3.2.1	Step 5 – Compute $\phi V_c/(b_w d)$ $= \phi[1.9\sqrt{f_c'} + 2500\rho_w V_u d/M_u]$ Compute $\phi V_c = 98.7(b_w d)$	$= 0.75[1.9\sqrt{3000\text{psi}} + 2500(0.010)] = 98.7 \text{ lb/in}^2$ $\phi V_c = 98.7\text{lb/in}^2(14\text{in} \times 19.5\text{in}) = 26,900 \text{ lbs}$ $= 26.9 \text{ k}$	

Compare with SHEAR EXAMPLE 1 for which  $\phi V_c = 22.5 \text{ k}$   
Frequently the more complex calculation for  $V_c$  will indicate values 10% to 15% higher than those from the simpler procedure.

### SHEAR EXAMPLE 3 – Vertical U-stirrups for beam with triangular shear diagram

Determine the size and spacing of stirrups for a beam if  $b_w = 20$  in,  $d = 29$  in ( $h = 32$  in),  $f_y = 60,000$  psi,  $f_c' = 4000$  psi,  $V_u = 177$  k,  $w_u = 11.6$  k/ft. Assume normal weight concrete is used.



ACI 318-05 Section	Procedure	Calculation	Design Aid
9.3.2.3 11.1.3.1	Step 1 – Determine $V_n = \max V_u / \phi$ ( $\phi = 0.75$ ) Compute $w_u / \phi$ Compute $V_u / \phi$ at $d$ from face of support. At $d$ from face, $V_u / \phi = V_u / \phi - (w_u / \phi)_u d$ $V_n$ must exceed 199 k at support.	$V_u / \phi = 179 / (0.75) = 239$ k $w_u / \phi = 11.6 / (0.75) = 15.5$ k/ft $V_u / \phi$ at $d = 239 - 15.5(29/12) = 202$ k	
11.3.1.1	Step 2 – Determine $V_c = K_{fc} K_{vc}$  For $b = 20$ in and $h = 32$ in Show this $V_c$ line on graph above	with $f_c' = 4000$ psi $K_{fc} = 1$ $K_{vc} = 73.4$ k $V_c = (1)(73.4k) = 73.4$ k	SHEAR 2 Table 2a Table 2c
11.5.5.1	Step 3 – Compute distance $\lambda_v$ over which stirrups are required, $\lambda_v = \frac{(V_n - 0.5V_c)}{(w_u / \phi)}$	$\lambda_v = \frac{239k - 0.5(73.4k)}{15.5k/ft} = 13.0$ ft = 156 in	
11.5.6.2 11.5.4.3	Step 4 – Select stirrup size for max $V_s$ Compute max $V_s = (V_u$ at $d - V_c)$ Read stirrup spacing for $d = 29$ in and $V_n = 126$ k  Select #4 stirrups, and use 5-in spacing from face of support  Note, if $V_s > 2V_c$ , $s$ must be $< d/4 = 7.25$ in Since $s = 5$ in is $< 7.25$ in, spacing is OK	max $V_s = 202 - 73.4 = 128.6$ kips  With #3 stirrups, $s$ must be $< 3$ in With #4 stirrups, $s$ can be 5 in  compute $2V_c = 2(73.4) = 147$ k	SHEAR 4.2 Table 4.2a Table 4.2b
11.5.4.1 11.1.1	Step 5 – Determine $V_n$ with maximum stirrup spacing of $s = 14$ in for $d = 29$ in $V_n = V_c + V_s$  Show this line on graph above	$V_s = 48$ k $V_n = 73.4k + 49.5k = 123$ k	SHEAR 4.2 Table 4.2b

### SHEAR EXAMPLE 3 –Vertical U-stirrups for beam with triangular shear diagram (continued)

ACI 318-05 Section	Procedure	Calculation	Design Aid
	<p>Step 6 – Stirrup spacing can be selected for convenience of placement.</p> <p>With <math>s = 5</math> in, compute <math>V_{n5} = V_c + V_{s5}</math></p> <p>With <math>s = 8</math> in, compute <math>V_{n8} = V_c + V_{s8}</math></p> <p>Construct these lines on graph above</p>	$V_{n5} = 73.4k + 129.5k = 203 \text{ k}$ $V_{n8} = 73.4k + 87k = 160 \text{ k}$	<p>SHEAR 4.2 Table 4.2b Table 4.2c</p>
	<p>Step 7 -Determine distances from face of support to point at which each selected spacing is adequate.</p> <p>Use graph to see that strength is adequate at each position for which spacing changes.</p> <p>10 spaces @ 5 in = 50 in      <math>V_n = 203 \text{ k}</math></p> <p>Plus 5 spaces @ 8 in = 40 in      <math>V_n = 160 \text{ k}</math></p> <p>Plus 5 spaces @ 14 in = 70 in      <math>V_n = 123 \text{ k}</math></p>	$50 + 40 = 90 \text{ in}$ $90 + 70 = 160 \text{ in} > \lambda_v$	

Editor's Note:

Generally, beams of the same material and section dimensions will be used in continuous frames. Each end of the various spans will have a triangular shear diagram that differs from other shear diagrams. The same chart constructed for the section selected can be used with the other shear diagrams simply by sketching the diagram superimposed on the chart already prepared. New sets of spacings can be read such that the strength associated with a spacing exceeds the ordinate on the diagram of required shear strength  $V_n = V_u / \phi$ .

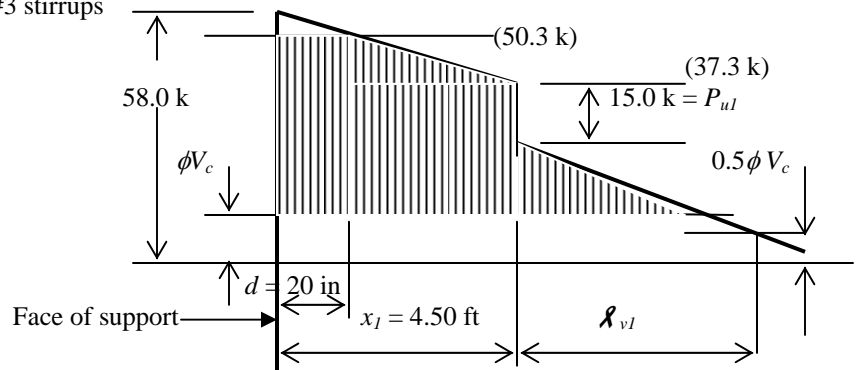
## SHEAR EXAMPLE 4 – Vertical U-stirrups for beam with trapezoidal and triangular shear diagram

Determine the required spacing of vertical #3 stirrups for the shear diagram shown.

Given: Normal weight concrete

$b = 13$  in                       $V_u = 58.0$  kips  
 $d = 20$  in                       $w_u = 4.6$  k/ft  
 $f'_c = 4000$  psi                   $x_l = 4.50$  ft  
 $f_y = 60,000$  psi                 $P_{ul} = 15.0$  kips

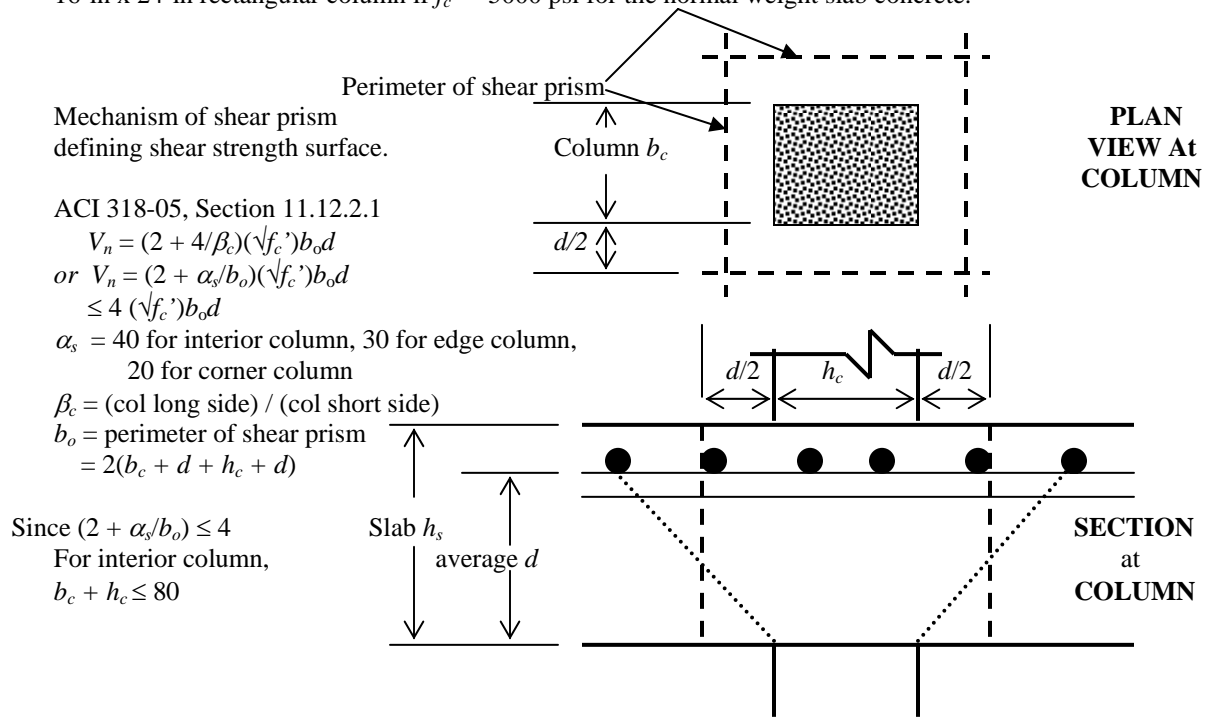
$\phi = 0.75$  for shear



ACI 318-05 Section	Procedure	Calculation	Design Aid
11.1.3.1	Step 1 – Determine at $d$ from face of support the value of $\max V_u = V_u - w_u d/12$	$\max V_u = 58.0\text{k} - 4.6\text{k/ft}(20\text{in})/12\text{in/ft} = 50.3$ k	
11.3.1.1 9.3.2.5	Step 2 – Determine the value $\phi V_c = 2\phi(\sqrt{f'_c})bd$ OR use Design Aid for $b=13\text{in}$ & $h=22.5\text{in}$ , $K_{vc} = 32.9$ k	$\phi V_c = 2(0.75)(\sqrt{4000}\text{lb/in}^2)13\text{in}(20\text{in}) = 24,700\text{lb}$ $\phi V_c = 0.75K_{fc}K_{vc} = 0.75(1.00)32.9\text{k} = 24.7$ k	SHEAR 2 Table 2c
	Step 3 – Determine required $V_u$ each side of $P_{ul}$ Left of $P_{ul}$ , $V_u = V_u - w_u x_l$ Right of $P_{ul}$ , change in $V_u = P_{ul}$	Left $P_{ul}$ , $V_u = 58.0\text{k} - 4.6\text{k/ft}(4.5\text{ft}) = 37.3$ k Right $P_{ul}$ , $V_u = 37.3\text{k} - 15\text{k} = 22.3$ k	
11.1.1 11.5.6.2	Step 4 – Determine spacing $s_l$ required for #3 U-stirrups at face of support. $A_v = 2(0.11) = 0.22$ sq in $s_l = \phi A_v f_y d / (\max V_u - \phi V_c)$	$s_l = (0.75)0.22\text{in}^2(60.0\text{k/in}^2)20\text{in} / (50.3\text{k} - 24.7\text{k}) = 7.7$ in	
11.5.4.1 11.5.6.2 11.1.1	Step 5 – Since maximum spacing $s = d/2$ with $d = 10$ in, determine value of $\phi V_u = \phi V_c + \phi A_v f_y d/s$ OR – Use Design Aid for $V_s$ when $s = 10$ in $\phi V_u = \phi V_c + \phi V_s$	$\max$ spacing $s_{\max} = 20\text{in}/2 = 10$ in $\phi V_u = 24.7\text{k} + 0.75(0.22\text{in}^2)60\text{k/in}^2(20\text{in})/10\text{in} = 44.5$ k $V_s = 26$ k for #3 stirrups @ 10 in spacing $\phi V_u = 24.7\text{k} + 0.75(26\text{k}) = 44.2$ kips	SHEAR 4.2
	Step 6 – Determine distance $x$ from face of support to point at which $V_u = 44.5$ kips. $x = (\text{Change in shear})/w_u$	$x = (58.0\text{k} - 44.5\text{k})/4.6\text{k/ft} = 2.93$ ft = 35 in	
11.5.4.1	Step 7 – Determine distance $\lambda_{vl}$ , distance beyond $x_l$ at which no stirrups are required. Find $\lambda_{vl} = (V_u - V_c/2)/w_u$ Compute $x_l + \lambda_{vl}$  Conclude: use $s = 7$ in until $\phi V_u < 44.5$ k and use $s = 10$ in until $\phi V_u < 0.5\phi V_c$	$\lambda_{vl} = (22.3\text{k} - 24.7\text{k}/2)/4.6\text{k/ft} = 2.16$ ft $x_l + \lambda_{vl} = 4.50\text{ft} + 2.16\text{ft} = 6.76$ ft = 81 in  5 spaces @ 7 in (35 in) 5 spaces @ 10 in (50 in) 85 in > 81 in OK	

## SHEAR EXAMPLE 5 – Determination of perimeter shear strength at an interior column supporting a flat slab ( $\alpha_s = 40$ )

Determine the shear capacity  $V_n$  of a 10-in thick two-way slab based on perimeter shear strength at an interior 16-in x 24-in rectangular column if  $f_c' = 5000$  psi for the normal weight slab concrete.



[Note that the uniform load acting within the shear perimeter of the prism does not contribute to the magnitude of required load  $V_c$ . The area within the shear perimeter is negligibly small with respect to the area of a flat plate around an interior column, usually only one to two percent. In contrast for footings, the area within the shear perimeter may be 15% or more of the bearing area of the footing. Computation of  $V_c$  for footing “slabs” must reflect that influence.]

ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 – Estimate $d$ keeping clear cover 0.75 in $d = h_s - 0.75 - \text{bar diameter}$	$d = 10\text{in} - 0.75\text{in} - 0.75_{\text{est}}\text{in} \approx 8.5$ in	
11.12.2.1	Step 2 – Use $b_o = 2(b_c + d + h_c + d)$	$b_o = 2(16\text{in} + 8.5\text{in} + 24\text{in} + 8.5) = 114$ in	
	Step 3 - Compute $\beta_c = h_c / b_c$ Since $\beta_c < 2$ , Compute $V_n = 4(\sqrt{f_c'})b_o d$	$\beta_c = 24\text{in} / 16\text{in} = 1.50$ $V_n = 4(\sqrt{5000}\text{lb/in}^2)114\text{in}(8.5\text{in})$ $= 274,000$ lbs = 274 k	
	ALTERNATE METHOD with Design Aid		
	Step 1 – Compute $b_c + h_c$ Use $d \approx 8.5$ in	$16\text{in} + 24\text{in} = 40$ in Interpolate $K1 = (3.58\text{ksi} + 4.18\text{ksi})/2$ $= 3.88$ ksi	SHEAR 5.1 Table 5.1a
	Step 2 – Compute $\beta_c = h_c / b_c$ Since $b_c < 2$ ,	$\beta_c = 24\text{in}/16\text{in} = 1.5$ $K2 = 1$	SHEAR 5.1 Table 5.1b
	Step 3 – With $f_c' = 5000$ psi, and $K1 * K2 = 1(3.88\text{ksi})$ , interpolate	$V_n = 212\text{k} + (3.88\text{ksi} - 3.00\text{ksi})(283\text{k} - 212\text{k})$ $= 275$ kips	SHEAR 5.1 Table 5.1c

## SHEAR EXAMPLE 6 – Thickness required for perimeter shear strength of a flat slab at an interior rectangular column

Given:  $f'_c = 5000$  psi    See SHEAR EXAMPLE 5 for diagram of shear perimeter and Code clauses.  
 $h_c = 24$  in  
 $b_c = 16$  in  
 $V_u = 178$  k  
 Assume normal weight concrete

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.12.2.1 9.3.2.3	Step 1 – Set up expression for $\phi V_c$ $\phi V_c = 4\phi(\sqrt{f'_c})b_c d$ $= 4\phi(\sqrt{f'_c})2(h_c + d + b_c + d)d$	$\phi V_c = 4(0.75)(\sqrt{5000\text{psi}})2(24\text{in} + d + 16\text{in} + d)d$ in $= (424.3\text{lb/in}^2)[(40\text{in})d\text{in} + 2d^2\text{in}^2]$	
9.1.1	Step 2 – Equate $V_u$ to $\phi V_c$ and solve for d	$178\text{k}(1000\text{ lb/k}) = 424.3\text{lb/in}^2(40d + 2d^2)\text{in}^2$ $419.5\text{ in}^2 = (40d + 2d^2)\text{in}^2$ $209.8 + 100 = 100 + 20d + d^2$ $d = (\sqrt{309.8}) - 10 = 17.6 - 10 = 7.6$ in	
7.7.1	Step 3 – Allow for 0.75 in clear cover of tension bars to make $h_s = d + 0.75 + \text{bar diameter (estimated)}$	$h_s = 7.6\text{in} + 0.75\text{in} + 0.625\text{in} = 8.98$ in	
ALTERNATE METHOD using Design Aid			SHEAR 5.1
9.3.2.1	Step 1 – Compute minimum $V_n = V_u/\phi$ and compute $(h_c + b_c)$	$V_n = 178\text{k}/0.75 = 237$ k $(h_c + b_c) = (24\text{in} + 16\text{in}) = 40$ in	
11.12.1.2	Step 2 – With $f'_c = 5000$ psi and $V_n = 237$ k Use $(h_c + b_c) = 40$ in, interpolate K1K2	237 k is between $V_n = 212$ k and $V_n = 283$ k $K1K2 = 3.00\text{ksi} + 1.00\text{ksi} \frac{(237\text{k} - 212\text{k})}{(283\text{k} - 212\text{k})} = 3.35$ ksi	Table 5.1c
11.12.2.1	Step 3 – Compute $\beta_c = h_c/b_c$	$\beta_c = 24\text{in}/16\text{in} = 1.5 < 2$ , so $K2 = 1$ and $K1 = K1K2$ ksi / $K2 = 3.35\text{ksi}$	Table 5.1b
	Step 4 – Table 5.1a with $(h_c + b_c) = 40$ and $K1 = 3.35$ ksi Interpolate for d	$d = 7\text{in} +$ $(8\text{in} - 7\text{in})(3.35\text{ksi} - 3.02\text{ksi}) / (3.58\text{ksi} - 3.02\text{ksi})$ $d = 7\text{ in} + 0.59\text{ in} = 7.59$ in	Table 5.1a
7.7.1	Step 5 – Allow for 0.75 in clear cover of tension bars to make $h_s = d + 0.75 + \text{bar diameter (estimated)}$	$h_s = 7.59\text{in} + 0.75\text{in} + 0.625\text{in} = 9.0$ in slab	



## SHEAR EXAMPLE 7 – Determination of perimeter shear strength at an interior rectangular column supporting a flat slab ( $\beta_c > 4$ )

Determine the shear capacity  $V_n$  of a 12-in thick two-way slab based on perimeter shear strength at an interior 12in x 44-in rectangular column.

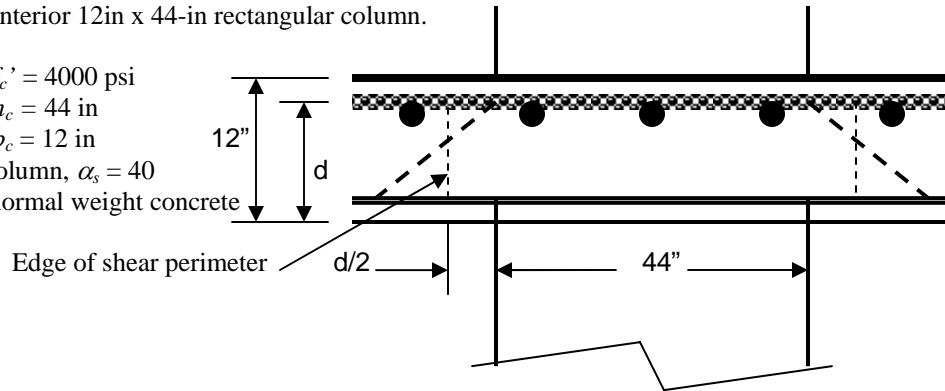
Given:  $f'_c = 4000$  psi

$h_c = 44$  in

$b_c = 12$  in

Interior column,  $\alpha_s = 40$

Assume normal weight concrete



ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 Estimate $d$ keeping clear cover 0.75 in $d \approx h_s - 0.75\text{in} - \text{bar thickness (est. 1 in)}$	$d \approx 12\text{in} - 0.75\text{in} - 1.0\text{in}$ , use $d = 10.2$ in	
11.12.2.1	Step 2 - Compute $b_c + h_c$ With $d = 10.2$ , find $K1$ from Table 5.1a	$b_c + h_c = 12\text{in} + 44\text{in} = 56$ in $K1 = 6.08\text{ksi} + (7.68\text{ksi} - 6.08\text{ksi})(10.2\text{in} - 10\text{in}) / (12\text{in} - 10\text{in}) = 6.24$ ksi	SHEAR 5.1 Table 5.1a
11.12.2.1	Step 3 - Compute $\beta_c = h_c / b_c$ With $\beta_c = 3.67$ , interpolate for $K2$	$\beta_c = 44\text{in} / 12\text{in} = 3.67$ $K2 = 0.778 + (0.763 - 0.778)(3.67 - 3.60) / (3.8 - 3.6) = 0.773$	SHEAR 5.1 Table 5.1b
	Step 4 - Compute $K1(K2)$ With $K1K2 = 4.82$ and $f'_c = 4000$ find $V_n$	$K1K2 = 6.24\text{ksi}(0.773) = 4.82$ ksi $V_n = 253\text{k} + (316\text{k} - 253\text{k})(4.82\text{ksi} - 4.00\text{ksi}) = 305$ k	SHEAR 5.1 Table 5.1c

ALTERNATE METHOD – Compute strength directly using ACI 318-05, Eqn. (11-33)

7.7.1c	Step 1: Estimate $d$ keeping clear cover 0.75 in $d \approx h_s - 0.75 - \text{estimate of bar thickness}$	$d \approx 12\text{in} - 0.75\text{in} - 1.0\text{in}$ , use $d = 10.2$ in	
11.12.1.2	Step 2 - Compute $b_o = 2(b_c + d + h_c + d)$	$b_o = 2(12\text{in} + 10.2\text{in} + 44\text{in} + 10.2\text{in}) = 153$ in	
	Step 3 - Compute $\beta_c = h_c / b_c$	$\beta_c = 44\text{in} / 12\text{in} = 3.67$	
11.12.2.1	Step 4 - Compute $V_n = (2 + 4/\beta_c)(\sqrt{f'_c})b_o d$	$V_n = (2 + 4/3.67)(\sqrt{4000\text{lb/in}^2})153\text{in}(10.2\text{in}) = 305,000$ lbs = 305 k	

## SHEAR EXAMPLE 8 – Determine required thickness of a footing to satisfy perimeter shear strength at a rectangular column

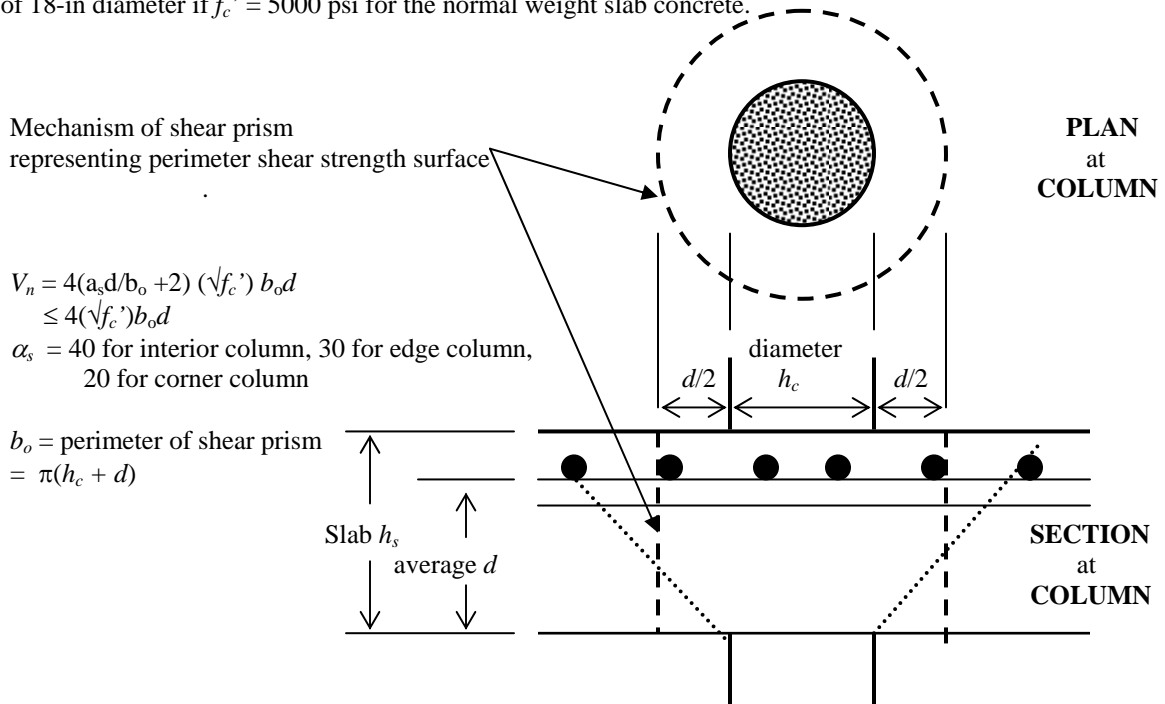
Given:  $P_u = 262$  k  
 Column  $b_c = h_c = 16$  in  
 Footing size = 7 ft x 7 ft

Footing size = 7 ft x 7 ft  
 $f'_c = 3000$  psi normal weight concrete  
 $f_y = 60,000$  psi

ACI 318-05 Section	Procedure	Calculation	Design Aid
	Step 1 – Determine net bearing pressure under factored load $P_u$ $f_{br} = P_u / (\text{footing area})$	$f_{br} = 262\text{k} / (7.0\text{ft} \times 7.0\text{ft}) = 5.35 \text{ k/ft}^2$	
	Step 2 – Express $V_u = f_{br}(\text{footing area} - \text{prism area})$ $= f_{br}[7.0 \times 7.0 - (16+d)^2/144]$	$V_u = 5.35\text{k/ft}^2 [49\text{ft}^2 - (16\text{in} + d\text{in})^2/144\text{in}^2/\text{ft}^2]$ $= 252.6\text{k} - (1.188\text{k/in})d - (0.0372\text{k/in}^2)d^2$	
11.12.2.1 9.3.2.3	Step 3 – Express $\phi V_c = \phi[4(\sqrt{f'_c})b_o d]$ $= \phi[4(\sqrt{f'_c})^4(16 + d)d]/1000\text{lb/k}$	$\phi V_c = 0.75[4(\sqrt{3000}\text{lb/in}^2)$ $4(16+d)\text{in}(d\text{in})/1000\text{lb/k}$ $= [0.657\text{k/in}^2(16d + d^2)\text{in}^2]$	
	Step 4 – Equate $V_u = \phi V_c$ and solve for $d$	$(252.6 - 1.188d - 0.0372d^2)\text{k}$ $= 0.657(16d + d^2)\text{k}$ $0.694d^2 + 11.70d = 252.6$ $d^2 + 16.86d + 8.43^2 = 364.0 + 71.1$ $d = (\sqrt{435.1}) - 8.43 = 12.4 \text{ in}$	
7.7.1	Step 5 – Allow 4 in clear cover below steel plus one bottom bar diameter to make $h \approx d + 4$	Use footing $h = 12.4 + 4$ $= \text{Make } h = 17 \text{ in}$	
ALTERNATE METHOD using Design Aid SHEAR 5.1			
	Step 1 – Determine net bearing pressure under factored load $P_u$ $f_{br} = P_u / (\text{footing area})$	$f_{br} = 262\text{k} / (7.0\text{ft} \times 7.0\text{ft}) = 5.35 \text{ k/sq ft}$	
9.3.2.3	Step 2- Estimate that bearing area of shear prism is 10% of footing area Compute $V_u = f_{br} (1 - 0.10) A_{fg}$ Compute $V_n = V_u / \phi$	$V_u = 5.35\text{k/ft}^2(0.90)7.0\text{ft}(7.0\text{ft}) = 236 \text{ k}$ $V_n = 236/0.75 = 315 \text{ k}$	
11.12.2.1	Step 3 – Find K1K2 with $V_n = 315$ k and $f'_c = 3000$ psi  Note that since $h_c/b_c < 2$ , $K2 = 1$ . Thus,	$K1K2 = 5.00\text{ksi} +$ $(6.0\text{ksi} - 5.0\text{ksi})(315\text{k} - 274\text{k})$ $(329\text{k} - 274\text{k})$  $K1K2 = 5.75 \text{ ksi}$ $K1 = 5.75 \text{ ksi}$	Table 5.1c  Table 5.1b
11.12.2.1	Step 4 – Compute $h_c + b_c$ With $K1 = 5.75$ ksi and $h_c + b_c = 32$ in, find $d$	$h_c + b_c = 16\text{in} + 16\text{in} = 32\text{in}$ $d = 12\text{in} + (14\text{in} - 12\text{in})(5.75\text{ksi} - 5.38\text{ksi})$ $(6.72\text{ksi} - 5.38\text{ksi})$  $d = 12.6 \text{ in}$ As above, make footing $h = 17 \text{ in}$	Table 5.1a
In ALTERNATE METHOD, Check assumed Step 2 proportion of shear prism area to footing area..			
	$\% \text{ footing area} = 100[(d + b_c)/12]^2 / (A_{fg})$	$= 100[(16\text{in} + 12.6\text{in})/(12\text{in}/\text{ft})]^2 / (7\text{ft} \times 7\text{ft})$ $= 11.6\% \text{ (estimate was 10\%)}$	
	$V_u$ should have been $(1 - 0.116)262 = 232 \text{ k}$	instead of estimated 236 k	

## SHEAR EXAMPLE 9 – Determination capacity of a flat slab based on required perimeter shear strength at an interior round column

Determine the shear capacity  $V_n$  of a 9-in thick two-way slab based on perimeter shear strength at an interior circular column of 18-in diameter if  $f_c' = 5000$  psi for the normal weight slab concrete.



$$V_n = 4(\alpha_s d/b_o + 2)(\sqrt{f_c'}) b_o d$$

$$\leq 4(\sqrt{f_c'}) b_o d$$

$$\alpha_s = 40 \text{ for interior column, } 30 \text{ for edge column, } 20 \text{ for corner column}$$

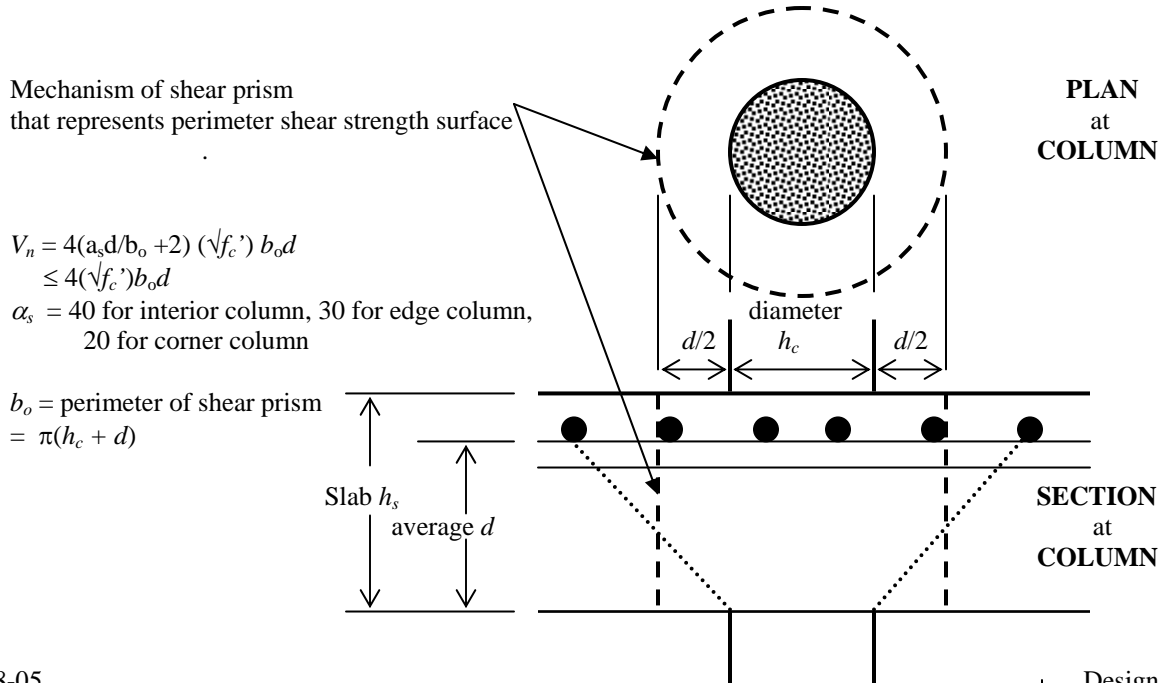
$$b_o = \text{perimeter of shear prism}$$

$$= \pi(h_c + d)$$

ACI 318-05 Section	Procedure	Calculation	Design Aid
7.7.1c	Step 1 – Estimate $d$ keeping clear cover 0.75in $d = h_s - 0.75 - \text{bar diameter (estimate \#6 bar)}$	$d = 9\text{in} - 0.75\text{in} - 0.75\text{in} \approx 7.5$ in	
11.12.2.1 9.3.2.3	Step 2 – Express $V_u = \phi V_n = 4\phi(\sqrt{f_c'}) b_o d$ $= 4\phi(\sqrt{f_c'}) \pi(h_c + d)d$ Solve for $V_u$	$V_u = 4(0.75)(\sqrt{5000\text{lb/in}^2})\pi(18\text{in} + 7.5\text{in})7.5\text{in}$ $= 127,000 \text{ lb} = 127 \text{ k}$	
	ALTERNATE METHOD with Design Aid SHEAR 5.2		SHEAR 5.2
	Step 1 – Estimate $d \approx 7.5$ in as above		
11.12.1.2 &	Step 2 – Find $K_3$ with $h_c=18\text{in}$ and $d=7.5\text{in}$	$K_3 = 2199\text{in}^2 + (2614\text{in}^2 - 2199\text{in}^2) \frac{(7.5\text{in} - 7.0\text{in})}{(8.0\text{in} - 7.0\text{in})}$ $= 2406 \text{ in}^2$	Table 5.2a
11.12.2.1	Step 3 – Find $V_n$ with $f_c' = 5000$ psi, and $K_3 = 2406 \text{ in}^2$ ,	$V_n = 141\text{k} + \frac{(2406\text{in}^2 - 2000\text{in}^2)(212\text{k} - 141\text{k})}{(3000\text{in}^2 - 2000\text{in}^2)}$ $= 170 \text{ k}$	Table 5.2b
9.3.2.3	Step 4 - Compute $V_u = \phi V_n$	$V_u = \phi V_n = (0.75)170 \text{ k} = 127 \text{ k}$	

## SHEAR EXAMPLE 10 – Determine thickness required for a flat slab based on required perimeter shear strength at an interior round column

Determine the thickness required for a two-way slab to resist a shear force of 152 kips, based on perimeter shear strength at an interior circular column of 20-in diameter if  $f'_c = 4000$  psi for the normal weight slab concrete.



$$V_n = 4(\alpha_s d/b_o + 2)(\sqrt{f'_c}) b_o d$$

$$\leq 4(\sqrt{f'_c}) b_o d$$

$\alpha_s = 40$  for interior column, 30 for edge column,  
20 for corner column

$b_o =$  perimeter of shear prism  
 $= \pi(h_c + d)$

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.12.1.2 11.12.2.1 9.3.2.3	Step 1 – Set up equation, $\phi V_n = 4\phi(\sqrt{f'_c}) \pi(h_c + d)d$	$\phi V_n = 4(0.75)(\sqrt{4000 \text{ lb/in}^2}) \pi(20 \text{ in} + d \text{ in})$ 1000 lb/k $= 0.596 \text{ k/in}^2 (20d + d^2) \text{ in}^2$	
11.12.2.1	Step 2 – Equate $V_u$ to $\phi V_n$ and solve for $d$ .	$152 \text{ k} = 0.596 \text{ k/in}^2 (20d + d^2) \text{ in}^2$ $= (11.92 \text{ k/in})d + (0.596 \text{ k/in}^2)d^2$ $255 \text{ in}^2 = (20 \text{ in})d + 1.0 d^2$ $[0.5(20 \text{ in})]^2 + 255 \text{ in}^2 = (d - 10 \text{ in})^2$ $d = \sqrt{(355 \text{ in}^2)} - 10 \text{ in} = 18.84 \text{ in} - 10.0 \text{ in}$ $= 8.84 \text{ in}$	
7.7.1c	Step – 3 Make $h_s$ deep enough for 0.75-in concrete cover plus diameter of top bars. Estimate #7 bars.	$h_s \approx 8.84 \text{ in} + 0.75 \text{ in} + 0.88 \text{ in}$ Use $h_s \approx 10.5 \text{ in}$	
	ALTERNATE METHOD with Design Aid		SHEAR 5.2
9.3.2.3 11.12.1.2	Step 1 – Compute $V_n = V_u/\phi$ With $f'_c = 4000$ psi and $V_n = 203$ k, obtain K3	$V_n = 152 \text{ k}/(0.75) = 203 \text{ k}$ $K3 = 3000 \text{ in}^2 +$ $(4000 \text{ in}^2 - 3000 \text{ in}^2)(203 \text{ k} - 190 \text{ k})$ $(253 \text{ k} - 190 \text{ k})$ $K3 = 3206 \text{ in}^2$	Table 5.2b
11.12.2.1	Step 2 – With $h_c = 20$ in and $K3 = 3206 \text{ in}^2$	Find $d = 8.0 \text{ in} + (9 \text{ in} - 8 \text{ in})(\frac{3206 \text{ in}^2 - 2815 \text{ in}^2}{3280 \text{ in}^2 - 2815 \text{ in}^2})$ $d = 8.84 \text{ in}$	Table 5.2a
7.7.1c	with 0.75 in cover plus bottom bar diameter	Use $h_s \approx 10.5 \text{ in}$	

**SHEAR EXAMPLE 11 – Determine thickness of a square footing to satisfy perimeter shear strength under a circular column.**

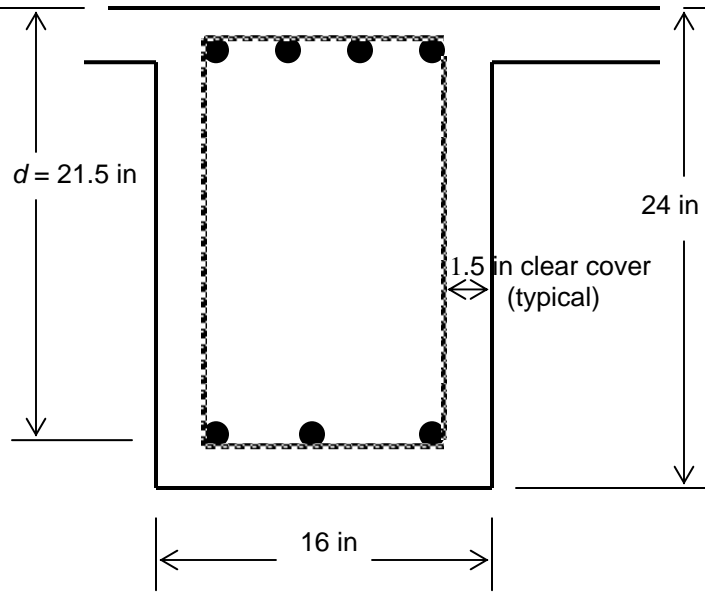
Given:  $P_u = 262$  kips                      Footing size = 7 ft by 7 ft with normal weight concrete  
 $f'_c = 3000$  psi                              Column diameter = 18 in  
 Grade 60 reinforcement

ACI 318-05 Section	Procedure	Calculation	Design Aid
	Step 1 – Compute net bearing pressure Under $P_u$ $f_{net} = P_u/A_{ftg}$	$f_{net} = 262k/(7.0ft \times 7.0ft) = 5.35 \text{ k/ft}^2$	
11.12.2.1 11.12.1.2 9.3.2.3	Step 2 – Express $\phi V_c = \phi[4(\sqrt{f'_c}) b_o d]$ $\phi V_c = \phi[4(\sqrt{f'_c})\pi(h_c + d)d]$	$\phi V_c = (0.75)[4(\sqrt{3000lb/in^2})\pi(18in+d)d]/1000lb/k$ $= 0.5162k/in^2(18d + d^2)in^2$	
	Step 3 – Express $V_u = f_{net}(A_{ftg} - A_{prism})$	$V_u = 5.35k/ft^2[7ft(7ft) - (\pi/4)(18in+d)^2/(12in/ft)^2]$ $= 5.35[49 - 1.77 - 0.196d - 0.00545d^2]$ $= 5.35[47.23 - 0.196d - 0.0055d^2]$ kips	
	Step 4 – Equate $\phi V_c = V_u$ and solve for $d$	$0.5162(18d + d^2) =$ $5.35[47.23 - 0.196d - 0.0055d^2]$ $18d + d^2 = 489.7 - 2.031d - 0.0570d^2$ $1.057d^2 + 20.31d = 489.7$ $d^2 + 19.21d + 9.61^2 = 463.3 + 92.35$ $d = \sqrt{555.6 - 9.61} = 13.96 \text{ in}$	
7.7.1a	Step 5 – Allow for 3 in clear cover plus a bottom bar diameter to make footing	$h_c = 13.96in + 3in + 0.88in = \text{use 18-in footing}$	
ALTERNATE METHOD using Design Aid			SHEAR 5.2
	Step 1 – Estimate that area beneath shear prism will be 10% of area beneath footing	estimate $V_u = (1.0 - 0.10)262 = 236$ kips	
11.12.2.1 11.12.1.2 9.3.2.3	Step 2 – Find K3 with $f'_c = 3000$ psi and $V_n = V_u/\phi = 236/0.75 = 314$ kips	$K3 = 5000in^2 + (6000in^2 - 5000in^2)(314k - 274k)$ $(329k - 274k)$ $K3 = 5747 \text{ in}^2$	Table 5.2b
	Step 3 – For $K3 = 5747 \text{ in}^2$ and col $h_c = 18$ in, Find footing $d$	$ftg \ d = 14.0in + 2in(5745 - 5630)in^2$ $(6836 - 5630)in^2$ $= 14.1 \text{ in}$	Table 5.2a
7.7.1a	Step 4 – Allow for 3 in clear cover plus one bar diameter to make	$h_f = 14.1in + 3in + .88in = \text{use 18 in thick}$	
	Step 4 – Check Step 1 estimate, $A_{prism}/A_{ftg}$  Since $0.115 > 10\%$ 14.1 in for $d$ may be higher than needed, but conclude that it is OK to use 18-in thick footing.	$A_{prism}/A_{ftg} = [(\pi/4)(18+14.2)^2/144in^2/ft^2]/49ft^2$ $= 0.115$	

## SHEAR EXAMPLE 12 – Determine closed ties required for the beam shown to resist flexural shear and determinate torque

Given:  $f'_c = 5000$  psi with normal weight concrete  
Grade 60 reinforcement

$V_u = 61$  kips  
 $T_u = 53$  kip-ft determinate



ACI 318-05 Section	Procedure	Calculation	Design Aid
11.0	Step 1 - Determine section properties for torsion, allowing 0.25 in as radius of ties $A_{cp} = b_w h$ $A_{oh} = (b_w - 3.5)(h - 3.5)$ $A_o = 0.85A_{oh}$ $p_{cp} = 2(b_w + h)$ $p_h = 2(b_w - 3.5 + h - 3.5)$	$A_{cp} = 16\text{in}(24\text{in}) = 384 \text{ in}^2$ $A_{oh} = (16\text{in} - 3.5\text{in})(24\text{in} - 3.5\text{in}) = 256 \text{ in}^2$ $A_o = 0.85(256\text{in}^2) = 218 \text{ in}^2$ $p_{cp} = 2(16\text{in} + 24\text{in}) = 80 \text{ in}$ $p_h = 2(16\text{in} - 3.5\text{in} + 24\text{in} - 3.5\text{in}) = 66 \text{ in}$	
9.3.2.3	Step 2 – Compute cracking torsion $T_{cr}$ $T_{cr} = 4\phi(\sqrt{f'_c})A_{cp}^2/p_{cp}$	$T_{cr} = 4(0.75)(\sqrt{5000\text{lb/in}^2})(384\text{in})^2/80$ $= 391,000 \text{ in lb}$ $T_{cr} = 391,000\text{in lb}/12000\text{in k/in lb} = 32.6 \text{ k-ft}$ Threshold torsion = $0.25(32.6 \text{ k-ft}) = 8.2 \text{ k-ft}$	
11.6.1	Compute threshold torsion = $0.25T_{cr}$ Since $T_u = 29 \text{ k-ft} > 8.2 \text{ k-ft}$ , ties for torsion are required.		
11.6.3.1	Step 3 – Is section large enough? Compute $f_v = V_u/(b_w d)$ Compute $f_{vt} = T_u p_h / (1.7A_{oh}^2)$  Compute limit = $\phi[2\sqrt{f'_c} + 8\sqrt{f'_c}]/1000$  Is $\sqrt{f_v^2 + f_{vt}^2} < \text{Limit}$ Therefore, section is large enough	$V_u = 61\text{k}/(16\text{in} \times 21.5\text{in}) = 0.177 \text{ k/in}^2$ $f_{vt} = 53\text{ft-k}(12\text{in/ft})66\text{in}/[1.7(256\text{in}^2)^2]$ $= 0.377 \text{ k/in}^2$ Limit = $0.75[2+8](\sqrt{5000\text{lb/in}^2})1000\text{lb/k}$ $= 0.530 \text{ k/in}^2$ $\sqrt{[(0.177)^2 + 0.377^2]} = 0.416 < \text{limit } 0.53$	
11.5.6.2	Step 4 Compute $A_v/s = [V_u - 2\phi\sqrt{f'_c}(b_w d)]/(\phi f_y d)$	$A_v/s = \frac{61\text{k} - [2(0.75)(\sqrt{5000\text{k/in}^2})(16\text{in})21.5\text{in}]}{1000\text{lb/in}^2[0.75(60\text{k/in}^2)21.5\text{in}]}$ $= [61 - 36.4]/967.5 = 0.0253 \text{ in}^2/\text{in}$	
11.6.3.6	Compute $A_t/s = T_u/[2\phi A_o f_y \cot \theta]$  Compute $(A_v/s + 2A_t/s)$ Use #4 ties for which $((A_v + 2A_t)/s) = 0.40 \text{ in}$ , and compute $s = 0.40/(A_v/s + 2A_t/s)$	$A_t/s = \frac{53\text{ft k}(12\text{in/ft})}{[2(0.75)218\text{in}^2(60\text{k/in}^2)\cot 45]}$ $= 0.0324 \text{ in}^2/\text{in}$ $(A_v/s + 2A_t/s) = 0.0253 + 2(0.0324) = 0.0900 \text{ in}$ $s = 0.40/(0.0900) = 4.44 \text{ in}$ Use 4 in	
11.6.5.2	Is $0.75(\sqrt{f'_c})b_w/f_y < (A_v/s + 2A_t/s)$	$0.75(\sqrt{5000})16/60,000 = 0.0141 < 0.0900$	YES

## SHEAR EXAMPLE 12 – continued

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.6.3.7 11.6.5.3	Step 5 – Compute $A_s = (A_f/s)(p_h \cot^2 45)$ Is $A_{s,min} = 5(\sqrt{f_c'})A_{cp}/f_y < A_s$ ?	$A_s = (0.20 \text{ in}^2/\text{in}/4.0 \text{ in})66 \text{ in}(1.00) = 3.30 \text{ in}^2$ $A_{s,min} = 5(\sqrt{5000 \text{ lb/in}^2})384 \text{ in}^2/60,000 \text{ lb/in}^2$ $= 2.26 \text{ in}^2 < 3.30 \text{ in}^2$ YES	
	In 8 positions, use #6 for bottom corners and bottom center and use #6 at mid-height in each vertical face. Excess area from flexural bars in top of section is adequate to replace the 3 #6 bars of $A_s$ in top of section.		
ALTERNATE METHOD using design aid			
11.2.1.1 11.5.6.2	Step 1 – Look up parameters for $f_c' = 5000$ psi, Grade 60 reinforcement, $b_w = 16$ in, $h = 24$ in	$K_{fc}K_{vc} = (1.118)43.5 \text{ k} = 48.6 \text{ k}$	SHEAR 2 Table 2a Table 2b
11.6.2.2a 11.6.3.1 11.6.3.6		$K_{vs} = 1290$ ksi $K_{fc}K_t = (1.118)89.1 \text{ k-ft} = 99.6 \text{ k-ft}$ $K_{fc}K_{tcr} = (1.118)38.9 \text{ k-ft} = 43.5 \text{ k-ft}$ $K_{ts} = 1089 \text{ k-ft/in}$	SHEAR 6.1a SHEAR 6.1b SHEAR 6.2b
11.6.1a	Step 2 – If $T_u = 53 > 0.25K_{fc}K_{tcr}$ , ties are required	$53 \text{ k-ft} > 0.25(43.5 \text{ k-ft}) = 10.9 \text{ k-ft}$ Therefore, ties are required.	
	Step 3 – Section is large enough if $\sqrt{[(V_u/(5\phi K_{fc}K_{vc}))]^2 + [T_u/(\phi K_{fc}K_t)]^2} < 1$	$\sqrt{\{61 \text{ k}/[5(0.75)48.6 \text{ k}]\}^2 + \{53 \text{ k-ft}/[0.75(99.6 \text{ k-ft})]\}^2}$ $= \sqrt{\{0.335\}^2 + \{0.710\}^2} = 0.785 < 1$ Therefore, section is large enough.	
	Step 4 – Compute $(A_v/s + 2A_f/s) = (V_u/\phi K_{fc}K_{vc})/K_{vs} + T_u/(\phi K_{ts})$  Compute $s < 0.40/(A_v/s + 2A_f/s)$	$(A_v/s + 2A_f/s) = (61 \text{ k}/0.75 - 48.6 \text{ k})/1290 \text{ k/in}^2 + 53.0 \text{ k-ft}/[(0.75)1089 \text{ k-ft}]$ $= 0.0254 + 0.0649 = 0.0903 \text{ in}^2/\text{in}$ One #4 tie provides $A_v = 0.40 \text{ in}^2/\text{in}$ $s < 0.40/0.0903 = 4.4$ in. Use 4 in spacing	
11.6.3.7 11.6.5.3	Step 5 – Compute $A_s = (A_f/s)(p_h \cot^2 45)$ Is $A_{s,min} = 5(\sqrt{f_c'})A_{cp}/f_y < A_s$ ?	$A_s = (0.20/4)66 \text{ in}(1.00) = 3.30 \text{ in}^2$ $A_{s,min} = 5(\sqrt{5000 \text{ lb/in}^2})384 \text{ in}^2/60,000 \text{ lb/in}^2$ $= 2.26 \text{ in}^2 < 3.30 \text{ in}^2$ YES	
11.6.6.2	In 8 positions, use #6 for bottom corners and bottom center and use #6 at mid-height in each vertical face. Excess area from flexural bars in top of section is adequate to replace the 3 #6 bars of $A_s$ in top of section.		

## SHEAR EXAMPLE 13 – Determine closed ties required for the beam of Example 12 to resist flexural shear and indeterminate torque

Given: Use the same data as that for SHEAR EXAMPLE 12, except that the required torsion estimate of 51 k-ft is based on an indeterminate analysis, not an equilibrium requirement.

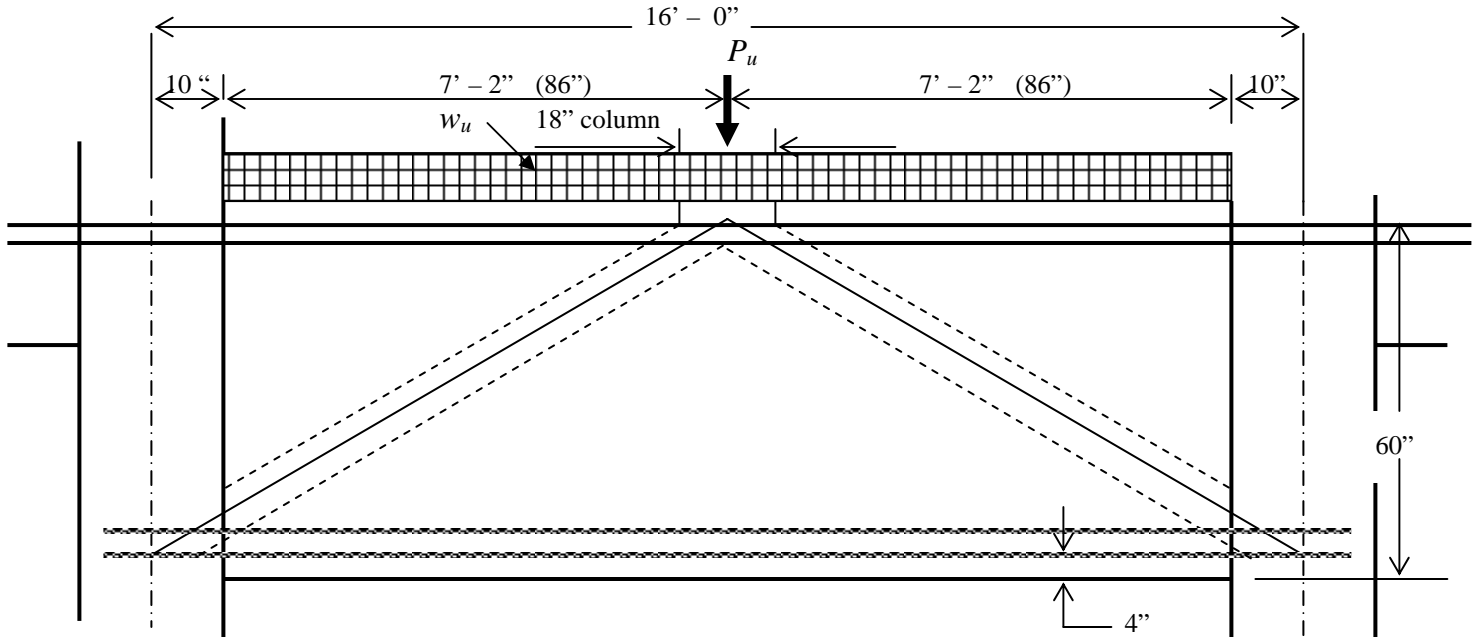
$$\begin{array}{lll}
 f'_c = 5000 \text{ psi} & b_w = 16 \text{ in} & V_u = 61 \text{ k} \\
 f_y = 60,000 \text{ psi} & h = 24 \text{ in} & T_u = 53 \text{ k-ft (based on indeterminate analysis)} \\
 \text{Assume normal weight concrete} & & 
 \end{array}$$

ACI 318-05 Section	Procedure	Calculation	Design Aid
11.2.1.1	Step 1 – Look up parameters for $f'_c = 5000$ psi,	$K_{fc}K_{vc} = (1.118)43.5 = 48.6 \text{ k}$	SHEAR 2 - Table2a & 2c SHEAR 2-2b SHEAR 6.1a SHEAR 6.1b SHEAR 6.2b
11.5.6.2	Grade 60 reinforcement, $b_w = 16$ in, $h = 24$ in	$K_{vs} = 1290 \text{ ksi}$	
11.6.2.2a		$K_{fc}K_t = (1.118)89.1 = 99.6 \text{ k-ft}$	
11.6.3.1		$K_{fc}K_{tcr} = (1.118)38.9 = 43.5 \text{ k-ft}$	
11.6.3.6		$K_{ts} = 1089 \text{ k-ft/in}$	
11.6.2.2a	Step 2 – If indeterminate $T_u > K_{fc}K_{tcr}$ , $K_{fc}K_{tcr}$ can be used as $T_u$ .	$K_{fc}K_{tcr} = 43.5 \text{ k-ft}$ , which is $> 53 \text{ k-ft}$ Use $T_u = 43.5 \text{ k-ft}$	
11.6.1a	Step 3 – If $T_u=43.5 > 0.25K_{fc}K_{tcr}$ , ties are required	$43.5\text{k-ft} > 0.25(43.5\text{k-ft}) = 10.9 \text{ k-ft}$ so ties are required	
	Step 4 – Section is large enough if $\sqrt{[(V_u / (5\phi K_{fc}K_{vc}))^2 + [T_u / (\phi K_{fc}K_t)]^2} < 1$	$\sqrt{\{61\text{k} / [5(0.75)48.6\text{k}]\}^2 + \{43.5\text{k-ft} / [0.75(99.6\text{k-ft})]\}^2} = \sqrt{\{0.335\}^2 + \{0.582\}^2} = 0.671 < 1$ Therefore section is large enough.	
	Step 4 – Compute $(A_v/s + 2A_t/s) = (V_u / \phi K_{fc}K_{vc}) / K_{vs} + T_u / (\phi K_{ts})$  #4 ties provide 0.40 sq in/in Compute $s < 0.40 / (A_v/s + 2A_t/s)$	$= (61\text{k} / 0.75 - 48.6\text{k}) / 1290\text{k/in}^2 + 43.5\text{k-ft} / (0.75)1089\text{k-ft/in}$ $= 0.0254 + 0.0533 = 0.0787 \text{ sq in/in}$  $s < 0.40 / 0.0787 = 5.1 \text{ in}$ . Use 5 in spacing	
11.6.3.7	Step 5 – Compute $A_{\lambda} = (A_v/s)(p_h \cot^2 45)$	$A_{\lambda} = (0.0533 / 5.1)66\text{in}(1.00) = 1.76 \text{ sq in}$	
11.6.5.3	Is $A_{\lambda} > A_{\lambda, \min} = 5(\sqrt{f'_c})A_{cp} / f_y - p_h A_v / s$	$A_{\lambda, \min} = 5(\sqrt{5000\text{lb/in}^2})384\text{in}^2 / 60,000\text{lb/in}^2 = 2.26 \text{ in}^2 > 1.76 \text{ in}^2$ YES	
11.6.6.2	In 6 positions, use #5 in bottom corners and center and #5 in each vertical face. Excess flexural bars in top are adequate for the 3 #5 component of $A_{\lambda}$ in top of section.		



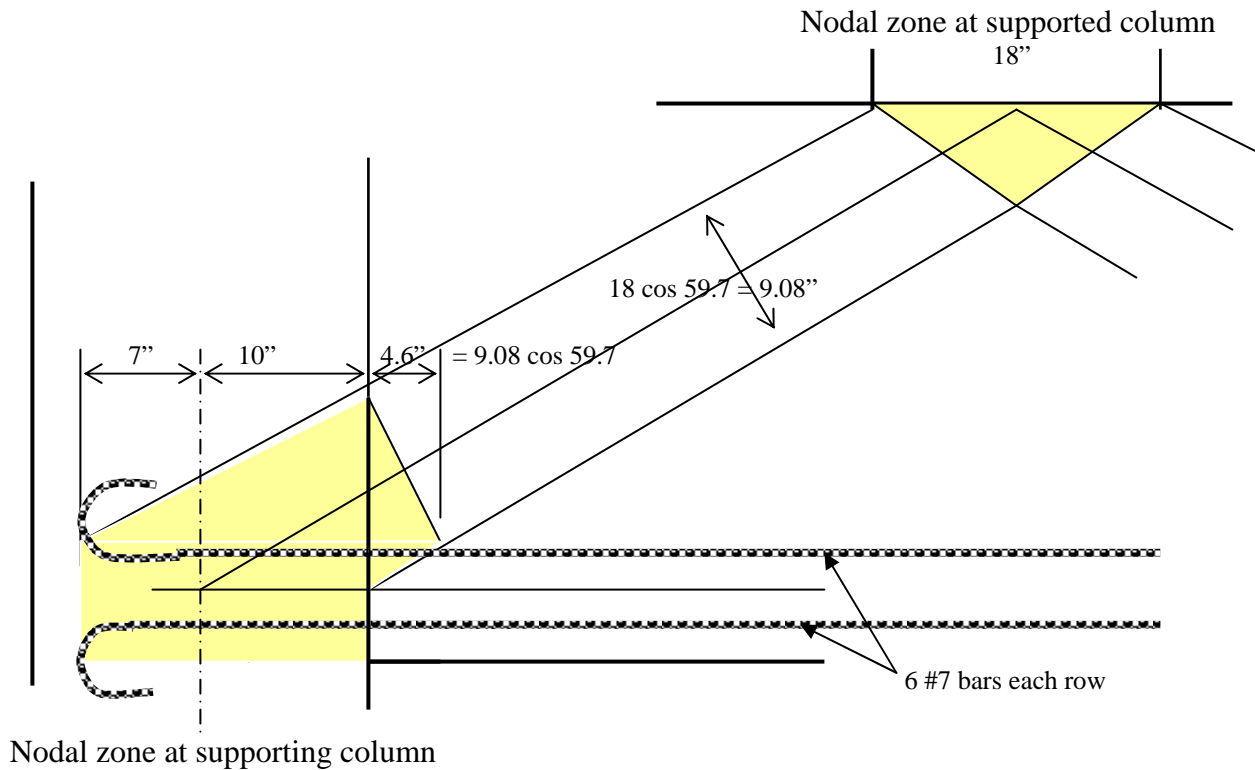
## SHEAR EXAMPLE 14 – Deep transfer beam design by strut-and-tie model

Given:  $P_u = 318$  k.  $f'_c = 4000$  psi normal weight concrete  $b_w = 14$  in  
 $w_u = 6.4$  k/ft Grade 60 reinforcement



ACI 318-05 Reference	Procedure	Calculation	Design Aid
11.8.1 11.8.2 A.2.1	Step 1 – This is a deep beam if $\ell_n/d < 4$ This is a deep beam. Use truss as sketched. Compute strut angle $\gamma = \tan^{-1}(96\text{in}/56\text{in})$	$\ell_n/d = 2(86\text{in})/(60\text{in} - 4\text{in}) = 3.07 < 4$ $\gamma = \tan^{-1}(1.714) = 59.7^\circ$	
A.2.1 A.2.2	Step 2 – Determine strut forces. Compute concentrated load, including $F_v = P_u + 7.17w_u$ Diagonal strut force $F_u = 0.5F_v/\cos \gamma$ Tension strut force $T_u = 0.5F_v \tan \gamma$ Maximum $V_u = 0.5 F_v$	$F_v = 318 + 7.17\text{ft}(6.4\text{k}/\text{ft}) = 364$ k $F_u = 0.5(364\text{k})/\cos 59.7 = 361$ k $T_u = 0.5(364\text{k}) \tan 59.7 = 311$ k Maximum $V_u = 0.5(364\text{k}) = 182$ k	
11.8.3 9.3.2.6	Step 3 – Compute $10\sqrt{f'_c}(b_w d)$ Compute $V_u/\phi$ Since $10\sqrt{f'_c}(b_w d) > V_u/\phi$ , section is adequate	$10\sqrt{f'_c}(b_w d) = 10(\sqrt{4000\text{lb}/\text{in}^2})14\text{in}(56\text{in}) = 496,000$ lb $V_u/\phi = 182\text{k}/0.75 = 243$ k = 243,000 lb	
11.8.4 & 11.8.5	Step 4 – Observe spacing limit $s$ and $s_l < d/5$ Using $s = 10$ in, Compute $\min A_v = 0.0025b_w s$ Try 2 #4 vertical bars at 10-in spacing Compute $\min A_{vh} = 0.0015b_w s_l$ Try 2 #4 horizontal bars at 11-in spacing	limit $s$ and $s_l = 56\text{in}/5 = 11$ in $\min A_v = 0.0025(16\text{in})10\text{in} = 0.400$ sq in $\min A_{vh} = 0.0015(16\text{in})11\text{in} = 0.264$ sq in	
A.3.2.1 A.3.2 A.2.6 A.3.1	Step 5 – Consider strut of uniform width ( $\beta_s = 1$ ) Compute $f_{cu} = 0.85\beta_s f'_c$ Compute $F_{ns} = F_u/\phi$ Compute strut area $A_c = F_{ns}/f_{cu}$ Compute strut width $w_s = A_c/b_w$	$f_{cu} = 0.85(1.0)4000\text{psi} = 3400$ psi = 3.4 ksi $F_{ns} = 361\text{k}/0.75 = 481$ k $A_c = 481\text{k}/3.4\text{k}/\text{in}^2 = 141$ sq in $w_s = 141\text{in}^2/\text{k}/16\text{in} = 8.81$ in	
A.4.1	Step 6 – Tension tie $A_{st} > T_u/(\phi f_y)$ Use 12 #7 bars hooked at columns	$A_{st} > 311\text{k}/[0.75(60\text{k}/\text{in}^2)] = 6.91$ sq in	

**SHEAR EXAMPLE 14 – Deep transfer beam design by strut-and-tie model (continued)**



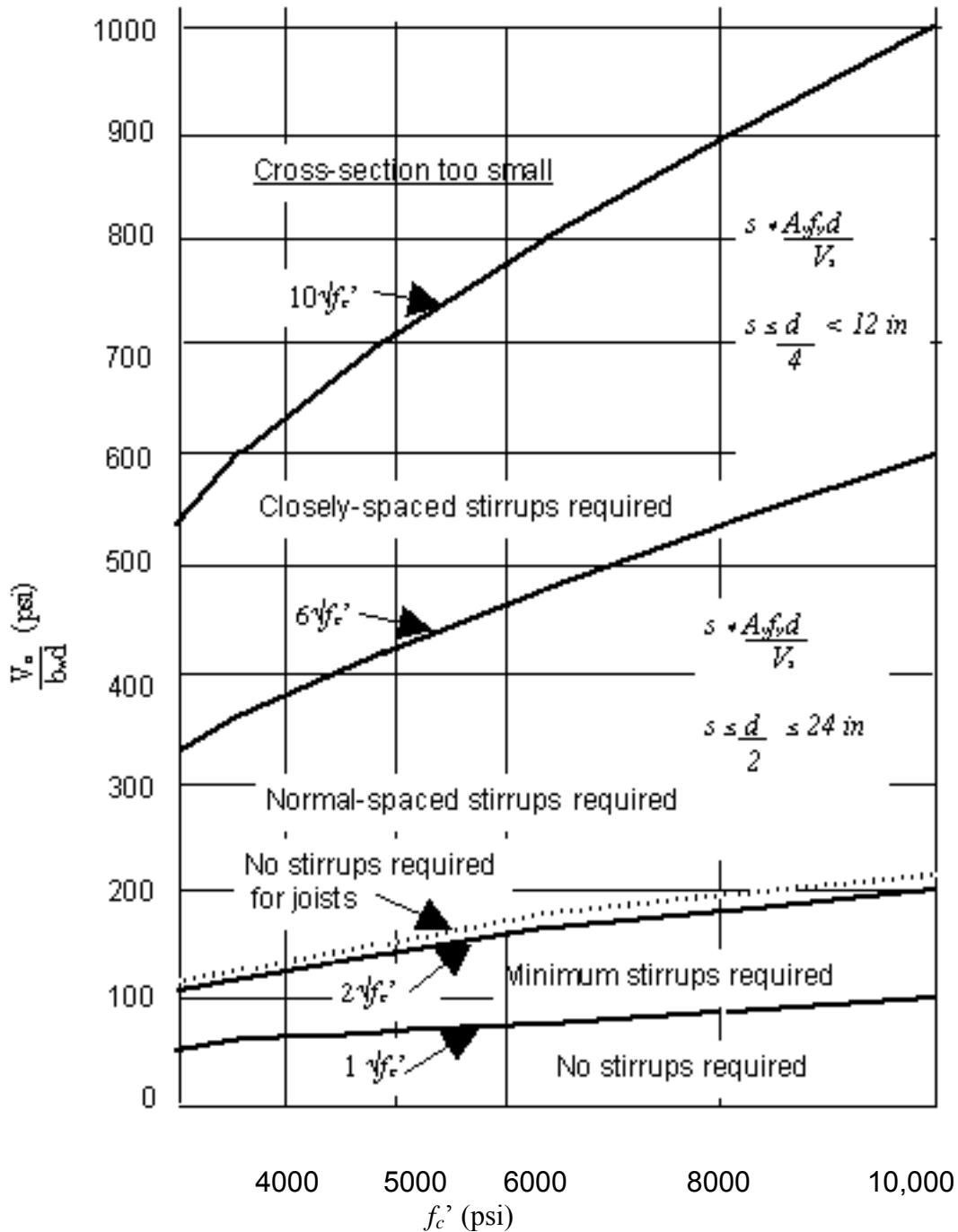
ACI 318-05 Reference	Procedure	Calculation	Design Aid
A.5.1 A.3.2.1	Step 7 – Check width of strut from C-C-C node at base of supported column. $w_s = 18 \cos 59.7$ Since $w_s = 8.81$ in $< 9.08$ in available, strut will be of uniform cross section area.	strut width = $18 \cos 59.7 = 9.08$ in available	
A.4.3.2 REINFORCE-	Step 8 – Is nodal zone at supporting column long enough to develop # 7 bars ?  18 in are available, 60 ksi can be developed.	For #7 hooked bars, $l_{hb} = 16.6$ in	MENT 18.1
A.3.3.1	Step 9 – Check Eq (A-4) $\Sigma A_{vi} \sin \gamma_{vi} / (b_w s_i) > 0.003$ Vertical: $A_v \sin \gamma_v / (b_w s) =$ Horizontal: $A_{vh} \sin (90 - \gamma_v) / (b_w s_l) =$  Alternate, using Design Aid For $\gamma_v = 65^\circ$ , $b_w s = 160$ , with #4 bars For $\gamma_v = 55^\circ$ , $b_w s = 160$ , #4 bars Interpolate for $\gamma_v = 59.7^\circ$ For $\gamma_h = 25^\circ$ , $b_w s_l = 176$ , #4 bars For $\gamma_h = 35^\circ$ , $b_w s_l = 176$ , #4 bars Interpolate for $\gamma_h = 30.3^\circ$	$2(0.20) \sin 59.7 / [16(10)] = 0.00216$ $2(0.20) \sin 30.3 / [16(11)] = 0.00115$ $\Sigma A_{vi} \sin \gamma_{vi} / (b_w s_i) = 0.00331$ OK  $A_v \sin \gamma_v / (b_w s) = 0.00227$ $A_v \sin \gamma_v / (b_w s) = 0.00205$ $A_v \sin \gamma_v / (b_w s) = 0.00216$ $A_{vh} \sin \gamma_h / (b_w s_l) = 0.00099$ $A_{vh} \sin \gamma_h / (b_w s_l) = 0.00134$ $A_v \sin \gamma_v / (b_w s) = 0.00117$ $\Sigma A_{vi} \sin \gamma_{vi} / (b_w s_i) = 0.00333$ OK	SHEAR 7 SHEAR 7 SHEAR 7 SHEAR 7

**SHEAR 1 – Section limits based on required nominal shear stress =  $V_n = V_u / (b_w d)$**

Reference: ACI 318-02, Sections 11.11.1, 11.3.1.1, 11.5.4, 11.5.6.2, 11.2.6.8, and 8.11.8

Section 11.2.1.1 states that when  $f_{ct}$  is specified for lightweight concrete, substitute  $f_{ct}/6.7$  for  $\sqrt{f'_c}$ , but  $f_{ct}/6.7$  must be  $\geq \sqrt{f'_c}$ .

$$V_n \geq \frac{V_u}{\phi}$$

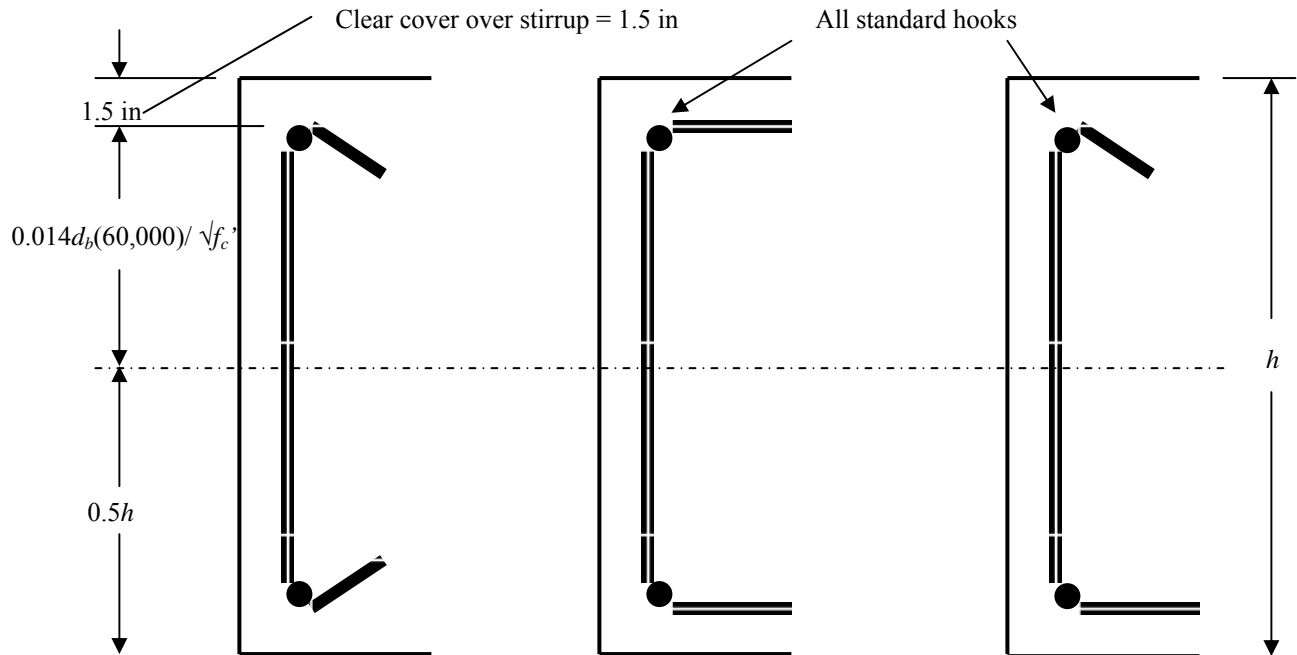




### SHEAR 3 – Minimum beam height to provide development length required for #6, #7 and #8 Grade 60 stirrups

Reference: ACI 318-05, Section 12.13.2.1 and Section 12.13.2.2

ACI 318-05, Section 11.5.2 states “Design yield strength of shear reinforcement (bars) shall not exceed 60,000 psi, ....”



Minimum beam height  $h = 2[0.014d_b(60,000)/\sqrt{f_c'} + 1.5]$  in inches

Minimum beam height h (in)			
Stirrup size	#6	#7	#8
Concrete $f_c'$			
3000	26.0	29.8	33.7
4000	22.9	26.2	29.6
5000	20.8	23.8	26.8
6000	19.3	22.0	24.7
8000	17.1	19.4	21.8
10000	15.6	17.7	19.8

\*Values shown in the table are for 1.5 in clear cover over stirrups. For cover greater than 1.5 in, add  $2(\text{cover}-1.5)$  to tabulated values.

Example: Determine whether a beam 24 in high ( $h = 24$  in) with 5000 psi concrete will provide sufficient development length for #6 Grade 60 vertical stirrups.

Solution: For #6 stirrups, minimum beam height reads 20.8 in for beams with 1.5-in clear cover over stirrups. Since  $h = 24$  in, the beam is deep enough.

### SHEAR 4.1 – Shear strength $V_{sn}$ with Grade 40 stirrups

Reference: ACI 318-05, Sections 11.5.5.3 and 11.5.6.2

$$V_s = V_n - V_c = A_v f_y (d/s)$$

$$\text{Maximum } b_w = A_v f_y / (50s) \text{ if } f_c' < 4440 \text{ psi}$$

$$\text{Maximum } b_w = 4A_v f_y / (3s \sqrt{f_c'}) \text{ if } f_c' > 4440 \text{ psi}$$

**TABLE 4.1a**

Stirrup size	spa. s (in) depth d (in)	Values of $V_s$ (kips)														
		2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
#3 stirrups	8	35	23	18												
	10	44	29	22	18											
	12	53	35	26	21	18										
	14	62	41	31	25	21	18									
	16	70	47	35	28	23	20	18								
	18	79	53	40	32	26	23	20	18							
	20	88	59	44	35	29	25	22	20	18						
	22	97	65	48	39	32	28	24	22	19	18					
	24	106	70	53	42	35	30	26	23	21	19	18				
	26	114	76	57	46	38	33	29	25	23	21	19				
	28	123	82	62	49	41	35	31	27	25	22	21	18			
	30	132	88	66	53	44	38	33	29	26	24	22	19			
	32	141	94	70	56	47	40	35	31	28	26	23	20	18		
	34	150	100	75	60	50	43	37	33	30	27	25	21	19		
	36	158	106	79	63	53	45	40	35	32	29	26	23	20	18	
	38	167	111	84	67	56	48	42	37	33	30	28	24	21	19	
	40	176	117	88	70	59	50	44	39	35	32	29	25	22	20	18
<b>Maximum <math>b_w</math> (in) for <math>f_c' &lt; 4440</math> psi</b>		<b>88</b>	<b>59</b>	<b>44</b>	<b>35</b>	<b>29</b>	<b>25</b>	<b>22</b>	<b>20</b>	<b>18</b>	<b>16</b>	<b>15</b>	<b>13</b>	<b>11</b>	<b>10</b>	<b>9</b>

**TABLE 4.1b**

Stirrup size	spa. s (in) depth d (in)	Values of $V_s$ (kips)														
		2	3	4	5	6	7	8	9	10	11	12	14	16	18	20
#4 stirrups	8	64	43	32												
	10	80	53	40	32											
	12	96	64	48	38	32										
	14	112	75	56	45	37	32									
	16	128	85	64	51	43	37	32								
	18	144	96	72	58	48	41	36	32							
	20	160	107	80	64	53	46	40	36	32						
	22	176	117	88	70	59	50	44	39	35	32					
	24	192	128	96	77	64	55	48	43	38	35	32				
	26	208	139	104	83	69	59	52	46	42	38	35				
	28	224	149	112	90	75	64	56	50	45	41	37	32			
	30	240	160	120	96	80	69	60	53	48	44	40	34			
	32	256	171	128	102	85	73	64	57	51	47	43	37	32		
	34	272	181	136	109	91	78	68	60	54	49	45	39	34		
	36	288	192	144	115	96	82	72	64	58	52	48	41	36	32	
	38	304	203	152	122	101	87	76	68	61	55	51	43	38	34	
	40	320	213	160	128	107	91	80	71	64	58	53	46	40	36	32
<b>Maximum <math>b_w</math> (in) for <math>f_c' &lt; 4440</math> psi</b>		<b>160</b>	<b>107</b>	<b>80</b>	<b>64</b>	<b>53</b>	<b>46</b>	<b>40</b>	<b>36</b>	<b>32</b>	<b>29</b>	<b>27</b>	<b>23</b>	<b>20</b>	<b>18</b>	<b>16</b>

## SHEAR 4.2 – Shear strength $V_{sn}$ with Grade 60 stirrups

Reference: ACI 318-05, Sections 11.5.5.3 and 11.5.6.2

$$V_s = V_n - V_c = A_v f_y (d/s)$$

$$\text{Maximum } b_w = A_v f_y / (50s) \text{ if } f_c' < 4440 \text{ psi}$$

$$\text{Maximum } b_w = 4A_v f_y / (3s \sqrt{f_c'}) \text{ if } f_c' > 4440 \text{ psi}$$

**TABLE 4.2a**

Stirrup size	spa. s (in) depth $d$ (in)	Values of $V_s$ (kips)																													
		2	3	4	5	6	7	8	9	10	11	12	14	16	18	20															
#3 stirrups	8	53	35	26	_____																										
	10	66	44	33	26	_____																									
	12	79	53	40	32	26	_____																								
	14	92	62	46	37	31	26	_____																							
	16	106	70	53	42	35	30	26	_____																						
	18	119	79	59	48	40	34	30	26	_____																					
	20	132	88	66	53	44	38	33	29	26	_____																				
	22	145	97	73	58	48	41	36	32	29	26	_____																			
	24	158	106	79	63	53	45	40	35	32	29	26	_____																		
	26	172	114	86	69	57	49	43	38	34	31	29	_____																		
	28	185	123	92	74	62	53	46	41	37	34	31	26	_____																	
	30	198	132	99	79	66	57	50	44	40	36	33	28	_____																	
	32	211	141	106	84	70	60	53	47	42	38	35	30	26	_____																
	34	224	150	112	90	75	64	56	50	45	41	37	32	28	_____																
	36	238	158	119	95	79	68	59	53	48	43	40	34	30	26	_____															
	38	251	167	125	100	84	72	63	56	50	46	42	36	31	28	_____															
	40	264	176	132	106	88	75	66	59	53	48	44	38	33	29	26	_____														
<b>Maximum <math>b_w</math> (in) for <math>f_c' &lt; 4440</math> psi</b>		<b>132</b>	<b>88</b>	<b>66</b>	<b>53</b>	<b>44</b>	<b>38</b>	<b>33</b>	<b>29</b>	<b>26</b>	<b>24</b>	<b>22</b>	<b>19</b>	<b>17</b>	<b>15</b>	<b>13</b>															

**TABLE 4.2b**

Stirrup size	spa. s (in) depth $d$ (in)	Values of $V_s$ (kips)																													
		2	3	4	5	6	7	8	9	10	11	12	14	16	18	20															
#4 stirrups	8	96	64	48	_____																										
	10	120	80	60	48	_____																									
	12	144	96	72	58	48	_____																								
	14	168	112	84	67	56	48	_____																							
	16	192	128	96	77	64	55	48	_____																						
	18	216	144	108	86	72	62	54	48	_____																					
	20	240	160	120	96	80	69	60	53	48	_____																				
	22	264	176	132	106	88	75	66	59	53	48	_____																			
	24	288	192	144	115	96	82	72	64	58	52	48	_____																		
	26	312	208	156	125	104	89	78	69	62	57	52	_____																		
	28	336	224	168	134	112	96	84	75	67	61	56	48	_____																	
	30	360	240	180	144	120	103	90	80	72	65	60	51	_____																	
	32	384	256	192	154	128	110	96	85	77	70	64	55	48	_____																
	34	408	272	204	163	136	117	102	91	82	74	68	58	51	_____																
	36	432	288	216	173	144	123	108	96	86	79	72	62	54	48	_____															
	38	456	304	228	182	152	130	114	101	91	83	76	65	57	51	_____															
	40	480	320	240	192	160	137	120	107	96	87	80	69	60	53	48	_____														
<b>Maximum <math>b_w</math> (in) for <math>f_c' &lt; 4440</math> psi</b>		<b>240</b>	<b>160</b>	<b>120</b>	<b>96</b>	<b>80</b>	<b>69</b>	<b>60</b>	<b>53</b>	<b>48</b>	<b>44</b>	<b>40</b>	<b>34</b>	<b>30</b>	<b>27</b>	<b>24</b>															

**SHEAR 5.1 – Shear capacity of slabs based on perimeter shear at interior rectangular columns ( $\alpha_s = 40$ )**

Reference: ACI 318-05, Sections 11.12.1.2 and 11.12.2.1

$$V_c = (\mathbf{K1})(\mathbf{K2})\sqrt{f'_c} \geq V_u/\phi \quad \mathbf{K1} = 8(b+h+2d)d/1000 \quad \mathbf{K2} = (2 + 4/\beta_c)/4$$

$$V_c = (2 + 4/\beta_c)(\sqrt{f'_c})b_o d \quad (11-33) \quad \beta_c = \frac{\text{longer dimension of column section}}{\text{shorter dimension of column section}}$$

$$V_c = (2 + \alpha_s d/b_o)(\sqrt{f'_c})b_o d \quad (11-34)$$

$$V_c = 4(\sqrt{f'_c})b_o d \quad (11-35)$$

**Note:** Eq. (11-35) governs if  $8d > b + h$ , or if  $40d/b_o \geq 2$ , or if  $\beta_c < 2$

**TABLE 5.1a** Values K1 (ksi)

$d$ (in)	3	4	5	6	7	8	9	10	12	14	16	18	20
$b+h$ (in)													
16	0.53	0.77	1.04	1.34	1.68	2.05	2.45	2.88	3.84	4.93	6.14	7.49	8.96
20	0.62	0.90	1.20	1.54	1.90	2.30	2.74	3.20	4.22	5.38	6.66	8.06	9.60
24	0.72	1.02	1.36	1.73	2.13	2.56	3.02	3.52	4.61	5.82	7.17	8.64	10.24
28		1.15	1.52	1.92	2.35	2.82	3.31	3.84	4.99	6.27	7.68	9.22	10.88
32		1.28	1.68	2.11	2.58	3.07	3.60	4.16	5.38	6.72	8.19	9.79	11.52
36			1.84	2.30	2.80	3.33	3.89	4.48	5.76	7.17	8.70	10.37	12.16
40			2.00	2.50	3.02	3.58	4.18	4.80	6.14	7.62	9.22	10.94	12.80
44				2.69	3.25	3.84	4.46	5.12	6.53	8.06	9.73	11.52	13.44
48				2.88	3.47	4.10	4.75	5.44	6.91	8.51	10.24	12.10	14.08
52					3.70	4.35	5.04	5.76	7.30	8.96	10.75	12.67	14.72
56					3.92	4.61	5.33	6.08	7.68	9.41	11.26	13.25	15.36
60						4.86	5.62	6.40	8.06	9.86	11.78	13.82	16.00
64						5.12	5.90	6.72	8.45	10.30	12.29	14.40	16.64
68							6.19	7.04	8.83	10.75	12.80	14.98	17.28
72							6.48	7.36	9.22	11.20	13.31	15.55	17.92
76								7.68	9.60	11.65	13.82	16.13	18.56
80								8.00	9.98	12.10	14.34	16.70	19.20

**TABLE 5.1b** Values K2

$\beta_c$	$\leq 2$	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.5	5.0
K2	1.000	0.955	0.917	0.885	0.857	0.833	0.813	0.794	0.778	0.763	0.750	0.722	0.700

**TABLE 5.1c** Values  $V_c$  (kips)

$K1 \cdot K2$ (ksi)	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	10.00	12.00	16.00	20.00
$f'_c$ (psi)												
3000	55	110	164	219	274	329	383	438	548	657	876	1095
4000	63	126	190	253	316	379	443	506	632	759	1012	1265
5000	71	141	212	283	354	424	495	566	707	849	1131	1414
6000	77	155	232	310	387	465	542	620	775	930	1239	1549
8000	89	179	268	358	447	537	626	716	894	1073	1431	1789
10000	100	200	300	400	500	600	700	800	1000	1200	1600	2000



## SHEAR 5.2 – Shear Capacity of Slabs Based on Perimeter Shear on Interior Round Columns

Reference: ACI 318-05, Sections 11.12.1.2 and 11.12.2.1

$$V_n = V_c = (K3) \sqrt{f_c'} \quad (\text{kips})$$

with  $h_c$  = column diameter (in)       $d$  = slab depth (in)

$$K3 = 4\pi d(d + h_c) \text{ if } h_c < 5.37d$$

$$K3 = 2\pi d(h_c + 7.37d) \text{ if } h_c > 5.37d \text{ for which Table 5.2a values are in italics}$$

**TABLE 5.2a** Values K3 (in<sup>2</sup>)

d (in)	3	4	5	6	7	8	9	10	12	14	16	18	20
Col h (in)													
8	415	603	817	1056	1319	1608	1923	2262	3016	3870	4825	5881	7037
10	490	704	942	1206	1495	1810	2149	2513	3317	4222	5227	6333	7540
12	565	804	1068	1357	1671	2011	2375	2765	3619	4574	5630	6786	8042
14	641	905	1194	1508	1847	2212	2601	3016	3921	4926	6032	7238	8545
16	716	1005	1319	1659	2023	2413	2827	3267	4222	5278	6434	7690	9048
18	756	1106	1445	1810	2199	2614	3054	3518	4524	5630	6836	8143	9550
20	794	1206	1571	1960	2375	2815	3280	3770	4825	5981	7238	8595	10053
22	<i>831</i>	<i>1294</i>	1696	2111	2551	3016	3506	4021	5127	6333	7640	9048	10555
24	<i>869</i>	<i>1344</i>	1822	2262	2727	3217	3732	4272	5429	6685	8042	9500	11058
26	<i>907</i>	<i>1394</i>	1948	2413	2903	3418	3958	4524	5730	7037	8444	9952	11561
28	<i>945</i>	<i>1445</i>	2037	2563	3079	3619	4184	4775	6032	7389	8846	10405	12063
30	<i>982</i>	<i>1495</i>	2100	2714	3255	3820	4411	5026	6333	7741	9249	10857	12566
32	<i>1020</i>	<i>1545</i>	2163	2865	3431	4021	4637	5278	6635	8093	9651	11309	13069
34	<i>1058</i>	<i>1595</i>	2226	2949	3606	4222	4863	5529	6936	8444	10053	11762	13571
36	<i>1095</i>	<i>1646</i>	2289	3024	3781	4421	5087	5778	7235	8792	10450	12208	14067
38	<i>1133</i>	<i>1696</i>	2351	3100	3940	4624	5315	6032	7540	9148	10857	12667	14577
40	<i>1171</i>	<i>1746</i>	2414	3175	4028	4825	5542	6283	7841	9500	11259	13119	15079

**TABLE 5.2b** Values of  $V_n$  (kips)

Fc' (psi)	3000	4000	5000	6000	8000	10000
K3 (sq.in)						
700	38	44	49	54	63	70
1000	55	63	71	77	89	100
2000	110	127	141	155	179	200
3000	164	190	212	232	268	300
4000	219	253	283	310	358	400
5000	274	316	354	387	447	500
6000	329	380	424	465	537	600
7000	383	443	495	542	626	700
8000	438	506	566	620	716	800
9000	493	569	636	697	805	900
10000	548	633	707	775	894	1000
12000	657	759	849	930	1073	1200
14000	767	886	990	1084	1252	1400
16000	876	1012	1131	1239	1431	1600
18000	986	1139	1273	1394	1610	1800
20000	1095	1265	1414	1549	1789	2000
22000	1205	1392	1556	1704	1968	2200
24000	1314	1518	1697	1859	2147	2400
26000	1424	1645	1838	2014	2325	2600
28000	1534	1771	1980	2169	2504	2800

## SHEAR 6.1 – Shear and Torsion Coefficients $K_t$ and $K_{tcr}$

Reference: ACI 318-05, Sections 11.6.2.2a and 11.6.3.1

Use SHEAR 2, Table 2a for values of  $K_{fc}$

$$\text{Limit } T_n = K_{fc}K_t = 17(\sqrt{f_c'}) (A_{oh}^2)/p_h \quad \text{Cracking } T_{cr} = K_{fc}K_{tcr} = 4(\sqrt{f_c'}) (A_{cp}^2)/p_{cp}$$

with  $A_{oh} = (b-3.5)(h-3.5)$        $p_h = 2(b + h - 7)$        $p_{cp} = 2(b + h)$

	Values $K_t$ (k-ft)										
Beam $b$ (in)	10	12	14	16	18	20	22	24	26	28	30
Beam $h$ (in)											
10	6.2	9.1	12.3	15.6	19.0	22.4	25.9	29.5	33.0	36.7	40.3
12	9.1	13.8	18.8	24.1	29.6	35.2	41.0	46.9	52.9	58.9	64.9
14	12.3	18.8	25.9	33.6	41.5	49.8	58.3	67.0	75.8	84.7	93.7
16	15.6	24.1	33.6	43.8	54.5	65.7	77.3	89.1	101.3	113.6	126.0
18	19.0	29.6	41.5	54.5	68.3	82.7	97.7	113.1	128.9	145.0	161.3
20	22.4	35.2	49.8	65.7	82.7	100.6	119.3	138.5	158.3	178.6	199.2
22	25.9	41.0	58.3	77.3	97.7	119.3	141.8	165.2	189.3	214.0	239.3
24	29.5	46.9	67.0	89.1	113.1	138.5	165.2	193.0	221.7	251.1	281.3
26	33.0	52.9	75.8	101.3	128.9	158.3	189.3	221.7	255.2	289.7	325.0
28	36.7	58.9	84.7	113.6	145.0	178.6	214.0	251.1	289.7	329.4	370.3
30	40.3	64.9	93.7	126.0	161.3	199.2	239.3	281.3	325.0	370.3	416.9
32	43.9	71.1	102.9	138.7	177.9	220.2	265.0	312.1	361.2	412.1	464.6
34	47.6	77.2	112.1	151.4	194.7	241.4	291.1	343.4	398.1	454.8	513.4
36	51.3	83.4	121.3	164.3	211.7	262.9	317.6	375.2	435.6	498.3	563.2
38	54.9	89.6	130.6	177.3	228.8	284.7	344.3	407.4	473.6	542.5	613.9
40	58.6	95.8	140.0	190.3	246.1	306.6	371.4	440.0	512.1	587.3	665.3

	Values $K_{tcr}$ (k-ft)										
Beam $b$ (in)	10	12	14	16	18	20	22	24	26	28	30
Beam $h$ (in)											
10	5.3	6.9	8.6	10.4	12.2	14.1	15.9	17.9	19.8	23.3	23.7
12	6.9	9.1	11.4	13.9	16.4	19.0	21.6	24.3	27.0	31.9	32.5
14	8.6	11.4	14.5	17.6	20.9	24.3	27.8	31.3	34.9	41.4	42.3
16	10.4	13.9	17.6	21.6	25.7	30.0	34.4	38.9	43.4	51.6	52.8
18	12.2	16.4	20.9	25.7	30.7	35.9	41.3	46.8	52.5	62.4	64.0
20	14.1	19.0	24.3	30.0	35.9	42.2	48.6	55.2	62.0	73.9	75.9
22	15.9	21.6	27.8	34.4	41.3	48.6	56.1	63.9	71.8	85.8	88.3
24	17.9	24.3	31.3	38.9	46.8	55.2	63.9	72.9	82.1	98.2	101.2
26	19.8	27.0	34.9	43.4	52.5	62.0	71.8	82.1	92.6	111.0	114.5
28	21.7	29.7	38.6	48.1	58.2	68.9	80.0	91.5	103.4	124.1	128.2
30	23.7	32.5	42.3	52.8	64.0	75.9	88.3	101.2	114.5	137.5	142.3
32	25.7	35.3	46.0	57.6	69.9	83.0	96.7	111.0	125.8	151.3	156.7
34	27.7	38.1	49.8	62.4	75.9	90.3	105.3	121.0	137.3	165.3	171.3
36	29.7	41.0	53.5	67.2	82.0	97.6	114.0	131.1	148.9	179.5	186.3
38	31.7	43.8	57.4	72.2	88.1	105.0	122.8	141.4	160.8	193.9	201.4
40	33.7	46.7	61.2	77.1	94.2	112.4	131.6	151.8	172.7	208.6	216.8

## SHEAR 6.2 – Shear and Torsion Coefficients $K_{ts}$

Reference: ACI 318-05, Section 11.6.3.6

$$T_n = (2A_o A_f f_y / s) \cot \theta = K_{ts} (A_f / s) \text{ k-ft}$$

with  $A_o = 0.85(h-3.5)(b-3.5)$  and  $\theta = 45$  degrees

TABLE 6.2a		Values		$K_{ts}$ (ft-k/in)		with		Grade		40 ties	
Beam $b$ (in)	10	12	14	16	18	20	22	24	26	28	30
Beam $h$ (in)											
10	120	157	193	230	267	304	341	377	414	451	488
12	157	205	253	301	349	397	445	494	542	590	638
14	193	253	312	372	431	491	550	610	669	729	788
16	230	301	372	443	513	584	655	726	797	868	938
18	267	349	431	513	596	678	760	842	924	1006	1089
20	304	397	491	584	678	771	865	958	1052	1145	1239
22	341	445	550	655	760	865	970	1074	1179	1284	1389
24	377	494	610	726	842	958	1074	1191	1307	1423	1539
26	414	542	669	797	924	1052	1179	1307	1434	1562	1689
28	451	590	729	868	1006	1145	1284	1423	1562	1701	1839
30	488	638	788	938	1089	1239	1389	1539	1689	1839	1989
32	525	686	848	1009	1171	1332	1494	1655	1817	1978	2140
34	562	734	907	1080	1253	1426	1599	1771	1944	2117	2290
36	598	783	967	1151	1335	1519	1703	1887	2072	2256	2440
38	635	831	1026	1222	1417	1613	1808	2004	2199	2395	2590
40	672	879	1086	1293	1499	1706	1913	2120	2327	2533	2740

TABLE 6.2b		Values		$K_{ts}$ (ft-k/in)		with		Grade		60 ties	
Beam $b$ (in)	10	12	14	16	18	20	22	24	26	28	30
Beam $h$ (in)											
10	180	235	290	345	401	456	511	566	622	677	732
12	235	307	379	452	524	596	668	741	813	885	957
14	290	379	469	558	647	736	826	915	1004	1093	1183
16	345	452	558	664	770	877	983	1089	1195	1302	1408
18	401	524	647	770	894	1017	1140	1263	1387	1510	1633
20	456	596	736	877	1017	1157	1297	1438	1578	1718	1858
22	511	668	826	983	1140	1297	1455	1612	1769	1926	2084
24	566	741	915	1089	1263	1438	1612	1786	1960	2135	2309
26	622	813	1004	1195	1387	1578	1769	1960	2152	2343	2534
28	677	885	1093	1302	1510	1718	1926	2135	2343	2551	2759
30	732	957	1183	1408	1633	1858	2084	2309	2534	2759	2985
32	787	1030	1272	1514	1756	1999	2241	2483	2725	2968	3210
34	843	1102	1361	1620	1880	2139	2398	2657	2917	3176	3435
36	898	1174	1450	1727	2003	2279	2555	2832	3108	3384	3660
38	953	1246	1540	1833	2126	2419	2713	3006	3299	3592	3886
40	1008	1319	1629	1939	2249	2560	2870	3180	3490	3801	4111

## SHEAR 7 – Horizontal and vertical shear reinforcement for strut and tie method

Reference:

- ACI 318-05 Section A.3.3.1 requires  $\sum[(A_s \sin \gamma) / (bs_i)] \geq 0.003$   $\gamma$  is strut angle with reinforcement
- Section 11.8.4 requires  $A_v > 0.0025 bs_1$   $A_v$  = area of vertical bars at spacing  $s_1$
- Section 11.8.5 requires  $A_h > 0.0015 bs_2$   $A_h$  = area of horizontal bars at spacing  $s_2$

### Values of $\sum[(A_s \sin \gamma) / (bs_i)]$

<b>Strut Angle <math>\gamma</math> with vertical = 25°</b>									
Vertical $A_v$	2#3	2#4	2#5	2#6	Horizontal $A_v$	2#3	2#4	2#5	2#6
$bs_1$ (sq in)					$bs_2$ (sq in)				
50	0.00186	0.00338	0.00524	0.00744	50	0.00399	0.00725	0.01124	0.01595
100	max $bs_1$	0.00169	0.00262	0.00372	100	0.00199	0.00363	0.00562	0.00798
150	= 88	0.00113	0.00175	0.00248	150	max $bs_2$	0.00242	0.00375	0.00532
200		max $bs_1$	0.00131	0.00186	200	= 147	0.00181	0.00281	0.00399
250		= 160	max $bs_1$	0.00149	250		0.00145	0.00225	0.00319
300			= 248	0.00124	300		max $bs_2$	0.00187	0.00266
350	0.00106 at max $bs_1$			0.00106	350		= 267	0.00161	0.00228
400				Max $bs_1$	400			0.00140	0.00199
				= 352	500	.00136@	max $bs_2$	max $bs_2$	0.00160
					600			= 413	max 586
<b>Strut Angle <math>\gamma</math> with vertical = 35°</b>									
Vertical $A_v$	2#3	2#4	2#5	2#6	Horizontal $A_v$	2#3	2#4	2#5	2#6
$bs_1$ (sq in)					$bs_2$ (sq in)				
50	0.00252	0.00459	0.00711	0.01009	50	0.00360	0.00655	0.01016	0.01442
100	max $bs_1$	0.00229	0.00356	0.00505	100	0.00180	0.00328	0.00508	0.00721
150	= 88	0.00153	0.00237	0.00336	150	max $bs_2$	0.00218	0.00339	0.00481
200		max $bs_1$	0.00178	0.00252	200	= 147	0.00164	0.00254	0.00360
250		= 160	max $bs_1$	0.00202	250		0.00131	0.00203	0.00288
300			= 248	0.00168	300		max $bs_2$	0.00169	0.00240
350	0.00143 at max $bs_1$			0.00144	350		= 267	0.00145	0.00206
400				Max $bs_1$	400			0.00127	0.00180
				= 352	500	00122@	max $bs_2$	max $bs_2$	0.00144
					600			= 413	max 586
<b>Strut Angle <math>\gamma</math> with vertical = 45°</b>									
Vertical $A_v$	2#3	2#4	2#5	2#6	Horizontal $A_v$	2#3	2#4	2#5	2#6
$bs_1$ (sq in)					$bs_2$ (sq in)				
50	0.00252	0.00459	0.00711	0.01009	50	0.00360	0.00655	0.01016	0.01442
100	max $bs_1$	0.00229	0.00356	0.00505	100	0.00180	0.00328	0.00508	0.00721
150	= 88	0.00153	0.00237	0.00336	150	max $bs_2$	0.00218	0.00339	0.00481
200		Max $bs_1$	0.00178	0.00252	200	= 147	0.00164	0.00254	0.00360
250		= 160	max $bs_1$	0.00202	250		0.00131	0.00203	0.00288
300			= 248	0.00168	300		max $bs_2$	0.00169	0.00240
350	0.00177 at max $bs_1$			0.00144	350		= 267	0.00145	0.00206
400				Max $bs_1$	400			0.00127	0.00180
				= 352	500	0.00106 at	max $bs_2$	max $bs_2$	0.00144
					600			= 413	max 586

**SHEAR 7 – Horizontal and vertical shear reinforcement for strut and tie method**  
(continued)

Values of  $\sum[(As_i \sin \gamma_i) / (bs_i)]$

Strut Angle $\gamma$ with vertical = 55°									
Vertical Av bs1(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88	Horizontal Av bs2(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88
50	0.00360	0.00655	0.01016	0.01442	50	0.002524	0.004588	0.007112	0.010094
100	max bs1	0.00328	0.00508	0.00721	100	0.001262	0.002294	0.003556	0.005047
150	= 88	0.00218	0.00339	0.00481	150	max bs2	0.001529	0.002371	0.003365
200		max bs1	0.00254	0.00360	200	= 147	0.001147	0.001778	0.002524
250		= 153	max bs1	0.00288	250		0.000918	0.001422	0.002019
300			= 248	0.00240	300		max bs2	0.001185	0.001682
350	0.00205 at max bs1			0.00206	350		= 267	0.001016	0.001442
400				max bs1 = 352	400			0.000889	0.001262
					500	0.00086 @ max bs2		max bs2	0.001009
					600			= 413	max 586
Strut Angle $\gamma$ with vertical = 65°									
Vertical Av bs1(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88	Horizontal Av bs2(sq in)	2#3 0.22	2#4 0.40	2#5 0.62	2#6 0.88
50	0.00399	0.00725	0.01124	0.01595	50	0.00041	0.00338	0.00524	0.007438
100	max bs1	0.00363	0.00562	0.00798	100	0.00021	0.00169	0.00262	0.00372
150	= 88	0.00242	0.00375	0.00532	150	max bs2	0.00113	0.00175	0.00248
200		max bs1	0.00281	0.00399	200	= 147	0.00085	0.00131	0.00186
250		= 153	max bs1	0.00319	250		0.00068	0.00105	0.00149
300			= 248	0.00266	300		max bs2	0.00087	0.00124
350	0.00227 at max bs1			0.00228	350		= 267	0.00075	0.00106
400				max bs1 = 352	400			0.00066	0.00093
					500	0.00063 @ max bs2		max bs2	0.00074
					600			= 413	max 586