SOLUTIONS TO PROBLEMS FROM CHAPTER 2

2.1. A K thermocouple produces a voltage which is measured by the potentiometer as 25mV. Determine the temperature T when the Reference Junction isothermal block is indicated by a thermistor as 0 ^{o}C. Use the Seebeck coefficient for 20^{o}C.

Solution

At 0 ^{o}C., the two junctions produce each voltages of 0 V. For the Seebeck coefficient at 20 ^{o}C of 40 μV/^{o} C, the temperature at junction J is

T = (25000 μV) /(40 μV/^{o} C) = 625 ^{o}C

This temperature is much higher than 20 ^{o}C and, consequently, the result using the Seebeck coefficient for 20^{o}C is approximate. A much more precise result can be obtained using NIST table or p(V) polynomial for K thermocouple.

2.2. A PC based data acquisition system is considered for a K type thermocouple. Two limit designs will have to be evaluated.

Design a) a cheap and less accurate design with:

-a 4-bit A/D converter;

-a standard electronic reference junction block with a temperature uncertainty of $\forall 0.57^0$ C at 0^0 C;

-a standard thermocouple with $\forall 2.3^0$ C error limit.

Design b) an expensive and more accurate design with:

-a 8-bit A/D converter

-a special electronic reference junction block with a temperature uncertainty of $\forall$ 0.15^0 C at 0^0 C;

-a special thermocouple with $\forall$1.15^0 C error limit.

Compare the two designs based on the following calculations:

1) Determine scale range for the analog input voltage in A/D converter;
2) Calculate the resolution;
3) Calculate the combined uncertainty (of A/D converter, Reference Junction and Thermocouple) regarding the temperature measurement.

Solution

1) K thermocouple can measure temperatures up to a maximum of 1372 ^{0} C. Using the Seebeck coefficient at 20 ^{o}C of 40 μV/^{o} C, this gives the approximate maximum voltage output of (1372 ^{0} C)(40 μV/^{o} C) = 54.88 mV. 0 to 0.1 V and 0 to 10 V). If 0-10V, 0-01 V and 0-0.1 V (0-100 mV) ranges are available, the range -100 mV should be chosen, such that the thermocouple output, with unipolar voltages in the domain 0 to 54.88 mV, covers most of the 0 to 100 mV range.

2) For the 4-bit converter of the design a, $2^4 = 16$ distinct digital readings are possible, i.e. a resolution of 100 [mV]/ 16 = 6.25 [mV] or 6.25 [mV]*100/100 [mV] = 6.25 % of the scale range.

For a 8-bit converter of the design b, $2^8 = 256$ distinct digital readings are possible, i.e. a resolution of 100 [mV]/ 256 = 0.39 [mV] or 0.39 [mV]*100/100 [mV] = 0.39% of the scale range

3) The combined uncertainty, u, of the uncertainties of the ADC, $u_{A/D,}$ of the Reference Junction, u_{RJ}, and of the thermocouple ,u_{TC}, is evaluated with a square root of sum of squares (RSS) method:

$$u=\forall/(u_{A/D}^2 + u_{RJ}^2 + u_{TC}^2)$$

For the case of no calibration error, the (resolution)/2 in mV of the A/D converter can be converted into temperature ($\forall$) uncertainty $u_{A/D}$ using the Seebeck coefficient for 20^0 C for K type thermocouple of 40 $\mu V/^0$ C or 0.04 $mV/^0$ C, such that

for design "a"

-for 4-bit converter

$$u_{A/D} = \forall\ (6.25\ [mV]/0.04\ [mV/^0\ C]\ )/2= \forall\ 78.1\ ^0C$$

and the combined uncertainty is

$$u \quad =\forall/\ (u_{A/D}^2 + u_{RJ}^2 + u_{TC}^2) = \forall/\ (78.1^2 + 0.57^2 + 2.3^2) = \forall 78.7\ ^0\ C$$

for design "b"

-for 8-bit converter

$$u_{A/D} = \forall(0.39\ [mV]/0.04\ [mV/^{0} C]\ )/2 = \forall\ 4.88\ ^{0}C$$

and the combined uncertainty is

$$u \quad = \forall/(u_{A/D}{}^{2} + u_{RJ}{}^{2} + u_{TC}{}^{2}) = \forall/(4.88^{2} + 0.15^{2} + 1.15^{2}) = \forall\ 5.01\ ^{0}C$$

The temperature errors of both the design "a" and design "b" are dominated $u_{A/D}$. If a reduction of overall uncertainty u is desired, it can be reduced by selecting an analog to digital converter with higher resolution than 8 bit.

2.3. A strain gauge with nominal resistance R = 120 Σ is installed in a branch of a Wheatstone bridge having for unstrained strain gauge $R_1 = R_2 = R_3 = R_4 = R$ and $V_i = 10$ V. As a result of bending the beam, on which it is cemented, the strain gauge is subject to a strain. A digital voltmeter with input resistance $R_m = 10$ MΣ gives a reading of $V_o = 5$ mV $= 5\ 10^{-3}$ V.

Calculate the:

a) change of the resistance ΔR

b) the strain ε for gauge factor G = 2 .

Solution

a) The change in resistance is given by

$$\Delta R = 4\, V_o\, R / V_i \quad = 4 * 5\ 10^{-3} * 120 / 10 = 0.24\ \Omega$$

b) the strain is given by

$$\varepsilon = (1/G)\ \delta = (1/G)\ \Delta R / R = (1/2)\ 1.2 / 120 = 0.005\ [m/m].$$

2.4. A resistive strain gauge, G = 2.2, is cemented on a rectangular steel bar with the elastic modulus E = 205 x 10^6 kN/m^2, width 3.5 cm and thickness 0.55 cm. An axial force of 12kN is applied.

Determine the change of the resistance of the strain gauge, ΔR, if the normal resistance of the gauge is R=100 Σ.

Solution

The cross section area A is

$A = 3.5 \times 0.55[cm^2] = 1.925*10^{-4}\ [m^2]$

The stress σ [kN/ m^2] is

$\sigma = F/A = 12 / (1.025\ 10^{-4}) = 62.3\ \ 10^3$

The resulting strain is

$\varepsilon = \sigma / E = (62.3\ \ 10^3) / (205\ 10^6) = 0.304\ 10^{-3}[m/m]$

The change in resistance is:

$)R = R\gamma G = (100)\ (0.304*10^{-3})\ (2.2) = 0.067\ \Omega$

2.5. The strain gauge of the previous problem is connected to a resistance device having an accuracy ∀0.25 Σ. What is the uncertainty in determining the stress Φ?

Solution

The uncertainty of Φ, U_{Φ}, can be determined as function of the uncertainty of)R, $U_{)R,}$ as follows:

2.6. A steel bar with an elastic modulus E = 205 x 10^6 kN/m^2 and a cross section area A= 6.5 cm^2 is subject to an axial force F. For measuring this force, a strain gauge is cemented on the bar. The nominal resistance of the strain gauge is R= 100 Σ. The strain gauge is connected in a branch of Wheatstone bridge with all other branches with resistances equal to R = 100 Σ. Wheatstone bridge voltage output is conditioned, as shown in Fig. 2.10, using an inverting amplifier with a resistance R_o = 1 MΣ. The strain gauge factor is G = 2.1 and the voltage V_i = 8.5 V.

Calculate the force F given a measured voltage V_o= 6.5 V.

Solution

The force F is given by

$F = (2*100*6.5\ 10^{-4} * 205\ 10^{9})(6.5)/(8.5*\ 10^{6}*2.1) = 970\ N$

2.7. An accelerometer based on a strain gauge, shown in Fig. 2.12, consists in a cantilever beam of length L = 27 mm, width w = 2.5 mm and thickness t =1.5 mm, fitted with a (seismic) mass M = 0.017 kg. The modulus of elasticity of the beam is E = 205 x 10^9 N/m^2. The strain gauge cemented at l = 23 mm, from the free end of the beam and having G = 2.05 is connected to a bridge which is interfaced to an ADC, as shown in Fig. 2.10, through an operational amplifier with R_o = 1.15 MΣ. Assume nominal resistance of the strain gauge of R = 100 Σ and the supply voltage of the bridge V_i = 8.5 V.

Calculate the acceleration "a" which produces a voltage output V_o = 0.155 V.

Solution

The acceleration is given by:

$a = R\ E\ w\ t^2\ V_0 /(3G\ M\ l\ Vi\ R_0)$

or

$a=100*205\ 10^9 *0.0025*0.0015^2\ 0.155/ (3*2.05*0.017*0.023*8.5*1.15\ 10^6) = 0.76[m/s^2]$

2.8. In the previous problem, the resistance R_o of the operational amplifier was given as 1.15 MΣ and the voltage output from this amplifier to ADC was V_o = 0.155 V. In case the ADC has only the range 0 to 10 V available, calculate a value for R_o which will give the midscale 5 V for V_o, for an acceleration value and all other factors being kept the same.

Solution

Given

$a = R\ E\ w\ t^2\ V_0 / \ (3G\ M\ l\ Vi\ R_0)$

value for R_o is obtained as:

$R_0 = R\ E\ w\ t^2\ V_0 / \ (3G\ M\ l\ Vi\ a)$

or

$R_0 = 100*205\ 10^9\ *0.0025*0.0015^2\ *5/\ (3*2.05*0.017*0.023*8.5*0.76) = \ 37.1\ 10^6\ \Omega$

2.9. Assume that a positioning measuring potentiometer (with total resistance $R = 15.5\ k\Omega$, total length L=22 cm and input voltage $V_i = 10$ V) has the wiper at x= 12 cm. The voltage V_o of the potentiometer is measured with an analog voltmeter with input resistance $R_o = 35\ k\Omega$ connected as load of a potentiometer.

a) Calculate the voltage V_o measured by the analog voltmeter;

b) Calculate the error in analog voltmeter measurement of V_o with regard to the V_o measured by a digital voltmeter with $R_o = 12\ M\Omega$.

Solution

For y = 10 and for each voltmeter taking into account its R_v in kΩ, v_o is

$v_0\ (R_v) = (v_i/y_{max})[1+(R/R_v)\ (y/y_{max})\ (1- y/y_{max})]^{-1}y$

$\quad = (10/22)[1+(15.5/R_v)\ (10/22)\ (1- 10/22)]^{-1}10$

The linear approximation of the $y(R_v)$ is given by

$y\ (R_v) = (y_{max}/\ v_i)\ v_0\ (R_v) = (22/10)\ v_0\ (R_v)$

The measurement error $e(R_v)$ due to R_v is

$e(R_v) = 10 - y(R_v)1$

a)For $R_v = 35k\Omega$,

$v_0\ (35) = (10/22)[1+(15.5/35)(10/22)(1- 10/22)]^{-1}10 = 4.09$ V

$y\ (35) = (22/10)\ 4.09 = 9.01$ cm

$e(35) = 10 - 9.01 = 0.99$ cm

c) For $R_v = 12\ M\Omega = 12000\ k\Omega$,

$v_0\ (12000) = (10/22)[1+(15.5/12000)(10/22)(1- 10/22)]^{-1}10 = 4.544$ V

$y\ (35) = (22/10)\ 4.544 = 9.997$ cm

$e(12000) = 10 - 9.997 = 0.003$ cm

The digital voltmeter introduces an error of 0.003 cm, which is much smaller than the error of 0.99 cm introduced by an analog voltmeter. A digital voltmeter is chosen whenever precise measurements are required.

2.10. A tachometer has the tachometer constant K = 6 V/krpm (krpm= 1000 revolutions per minute) and is connected to a 8 bit ADC which has input voltage range from 0 to 10 V.

a) Calculate the maximum acceptable velocity which can be measured by the tachometer in this configuration.

b) Calculate the velocity measurement resolution of the tachometer with ADC.

Solution

a) For U_{max} = 10 V and K_n = 6 [V/krpm], angular velocity "n" in [krpm] is

$n_{max} = U_{max} / K_n$ = 10 V / (6 V/krpm) = 1.667 krpm

b) Voltage resolution at 8-bit is

r_d = 10 V / 2^8 = 10 / 256 = 0.039 V

Ripple voltage of 2% of 10V gives 0.2 V, an error about 5 times higher than the digital resolution. Reduction of ripple voltage effect is required for a more accurate angular velocity measurement.

2.11. A two-track incremental encoder installed on a rotating shaft has 1000 slots evenly distributed along the circumference and has the output sampled at 10 MHz. If the photocell for the outer track is counted “on” for 10000 sampling pulses, calculate the angular velocity of the rotating shaft.

Solution

Angular velocity n [revolutions per sec] or [rps] is given by:

n = f/(Nm)

where f =10 MHz, N=1000 slots and m=10000 sampling pulses, such that

n=10 000 000 /(1000*10000) =1 [rps]

2.12. An incremental encoder with 1^o slots evenly distributed along the circumference of 1 cm radius is installed on a shaft rotating at 1000 rpm.

Calculate the frequency of the pulse output (in pulses per second) of the encoder before sampling.

Solution

For 1^0 /slot, i.e. 2^0 between two adjacent slots, the number of slots is

$N= (360^0/2^0) – 1=179$

The frequency of the pulse output F[pulses/sec] is given by:

F= Nn/60

where

$n = 1000$ rpm

or

F= 170*1000/60= 2833 [pulses/sec]

SOLUTIONS TO PROBLEMS FROM CHAPTER 3

3.1. Consider a more precise model of the PM-DC motor in which the effect of rotor inductance L= 0.002 [H] is not negligible. Obtain the corresponding Simulink model with a DC supply and a pure viscous load. For the same data used for the simulation results shown in fig. 3.6, compare the results obtained for non-negligible rotor inductance.

Solution

Newton second law equation

$K\, i = J d\omega/dt + b\omega + T$

remains unchanged in this case and gives

$d\omega/dt = (K/ J)\, i - (b/J)\, \omega - (1/J)\, T$

$= B_i\, i - B_o\, \omega - B_T\, T$

This corresponds to the bottom part of the Simulink model shown in Fig. 3.4 for b=0.

Voltage drop equation for non-negligible rotor inductance L

$U = L di/dt + Ri + K_e\omega$

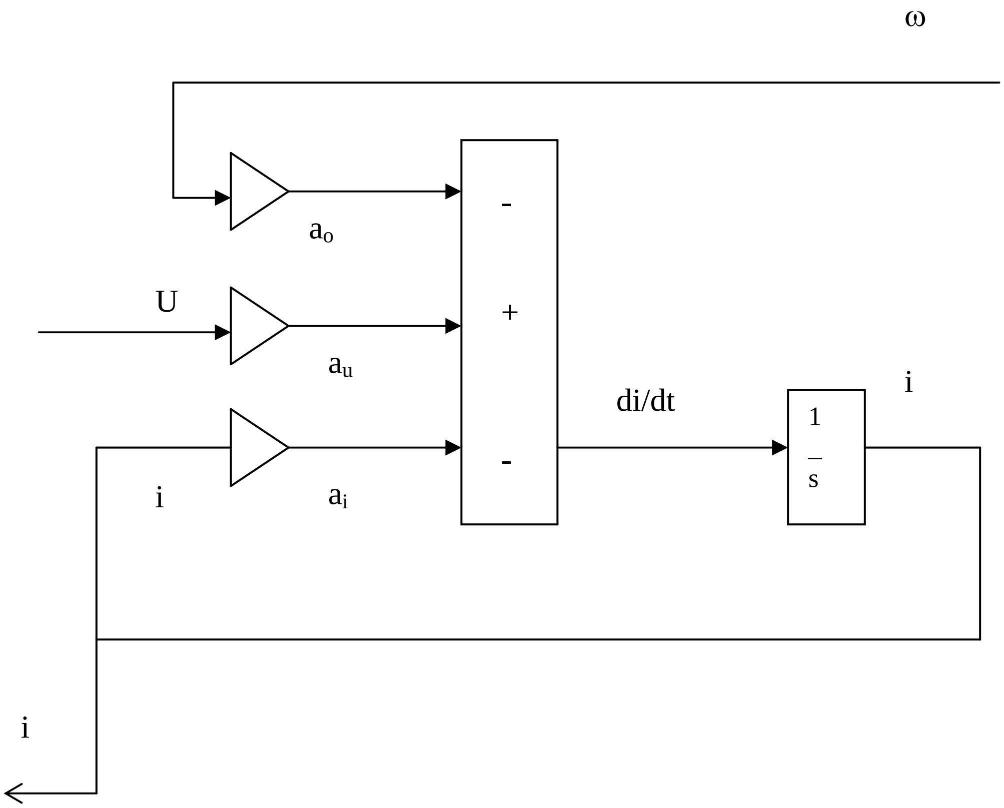
gives

$di/dt = (1/L)U - (R/L)i - (K_e/L)\omega$

$= a_u U - (a_i)i - a_o \omega$

$i = \int(di/dt)dt$

These two equations give the following Simulink model:



This Simulink model replaces the Current balance equation part of the Simulink model from Fig. 3.4 while retaining the same inputs U and ω and the same output i.

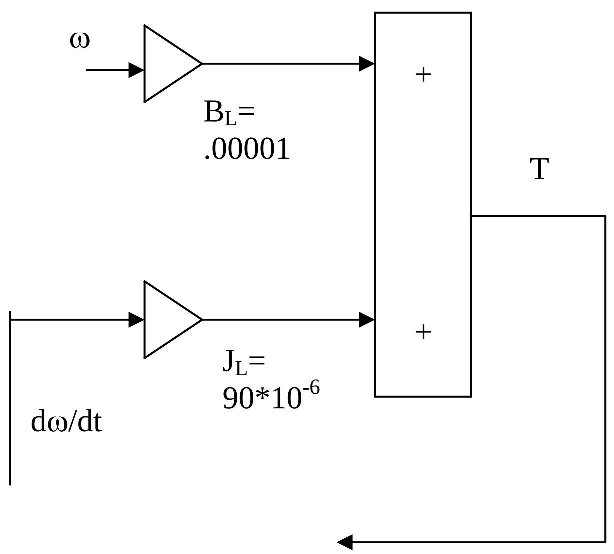
3.2. Consider the PM-DC motor model in which the load is not a pure viscous load, but an inertial-viscous load with a load moment of inertia of $90*10^{-6}$ kg m^2. Obtain the corresponding Simulink model and, for the same data used for the simulation results shown in fig. 3.6, compare the results obtained for an inertial-viscous load

Solution

Load torque equation for an inertial-viscous load is

$T = J_L \, d\omega/dt + B_L \, \omega$

This equation gives the following Simulink model:



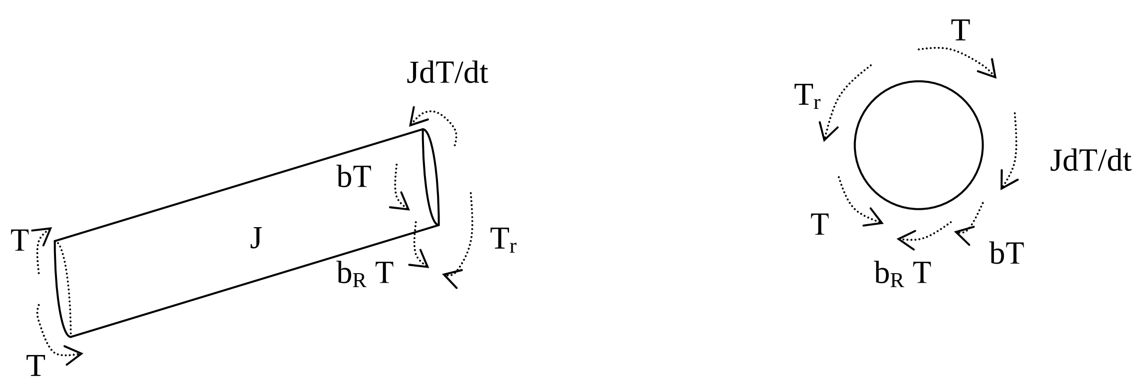
This Simulink model replaces the T=(BL)omega part of the Simulink model from Fig. 3.5 while retaining the same input ω the same output T and adding an extra input dω/dt from the output of the Newton second law block.

3.3 Obtain the block diagram representation of a PM-DC motor which has non-negligible viscous friction in the rotor bearings b_R.

Compare with the block diagram shown in Fig. 3.8.

Solution

The block diagram from Fig. 3.8 contains the coefficient b, which accounts for rotor air drag. Given that the rotor is modeled as a rigid body with moment of inertia J, the effect of the non-negligible viscous friction in the rotor bearings can be included in the free body diagram from Fig. 3.2 as shown:



Torque balance equation gives:

$J\, d\omega/dt = T_r - b\,\omega - b_R\,\omega - T = T_r - (b + b_R)\omega - T$

This indicates that in this case the block diagram remains the same as in Fig. 3.8 with the exception of replacing the viscous coefficient b by $b + b_R$.

3.4 Modify the Simulink model of a PM-DC motor from Fig. 3.5 to account for non-negligible viscous friction in the rotor bearings b_R = 0.00003 Nms/rad. For the same data used for the simulation results shown in fig. 3.6, compare the results obtained for non-negligible viscous friction in the rotor bearings.

Solution

The model derived in problem 3.3 gives

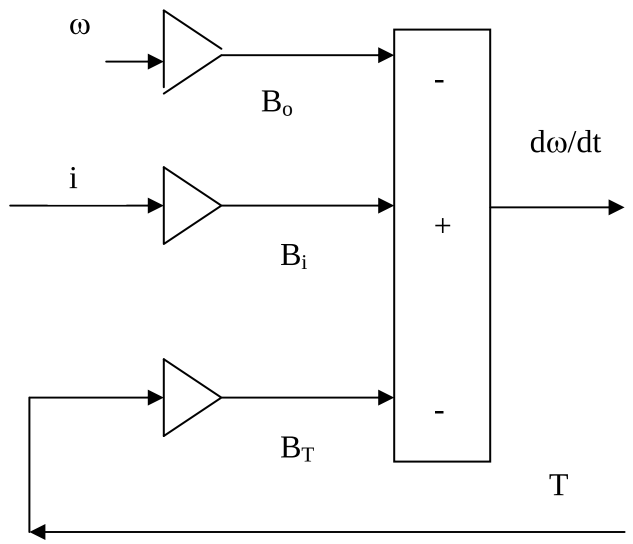
$J\, d\omega/dt = T_r - (b + b_R)\omega - T$

or

$d\omega/dt = (K/J)\, i - ((b + b_R)/J)\, \omega - (1/J)\, T$

$\quad = B_i\, i - B_o\, \omega - B_T\, T$

Simulink model from Fig. 3.5 considers a negligible $B_o = 0$. A non-negligible B_o requires to modify the block referring to the Newton second law as shown:



This Simulink model replaces the Newton second law part of the Simulink model from Fig. 3.5 retaining the same inputs i and T, the same output dω/dt while adding an extra input ω from the output of the Integrator of d(omega)/dt block.

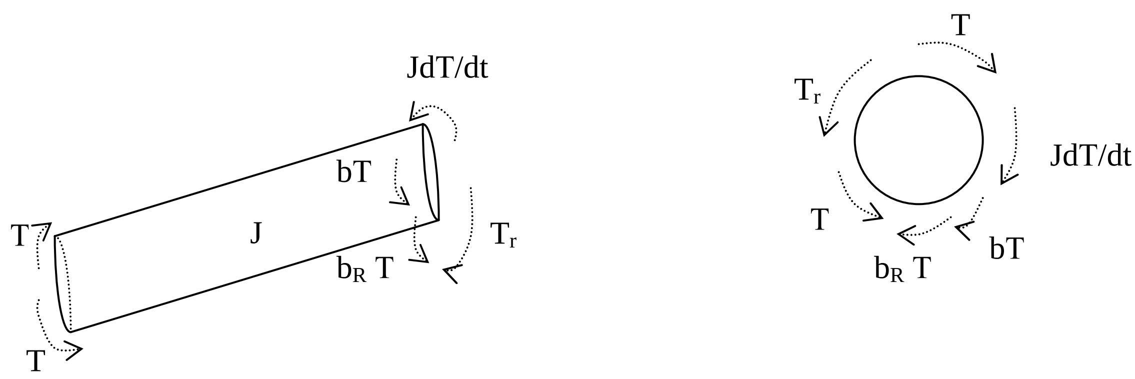
3.5 Modify the Simulink model of a shunt DC motor from Fig. 3.13 to account for non-negligible viscous friction in the rotor bearings b_R = 0.00005 Nms/rad. For the same data used for the simulation results shown in fig. 3.14, compare the results obtained for non-negligible viscous friction in the rotor bearings.

Solution

The model of the electrical part remains, in this case, unchanged:

$$i = (1/R)\,(1 + R/R_f)\,U - (K_e/R)\,\omega = A_u\,U - A_o\,\omega$$

Given that the rotor is modeled as a rigid body with moment of inertia J, the effect of the non-negligible viscous friction in the rotor bearings can be included in the free body diagram from Fig. 3.10 as shown:



Torque balance equation gives:

$J\, d\omega/dt = T_r - b\,\omega - b_R\,\omega - T = T_r - (b + b_R)\omega - T$

This indicates that in this case the effect of non-negligible viscous friction in the rotor bearings b_R can be taken into account by replacing in torque balance equation the viscous coefficient b by $b + b_R$, such that the nonlinear model of the shunt DC motor becomes:

$d\omega/dt = ((K_a K_f)/(J R_f))\, Ui - ((K_a K_f)/)J R_f^{\,2}))\, U^2 - ((b+ b_R)/J)\, \omega - n(1/J)\, T = B_{ui}\, Ui - B_{uu} U^2 - B_o\omega - B_T T$

where

$A_u = (1/R)\,(1 + R/R_f)$

$A_o = K_e/R$

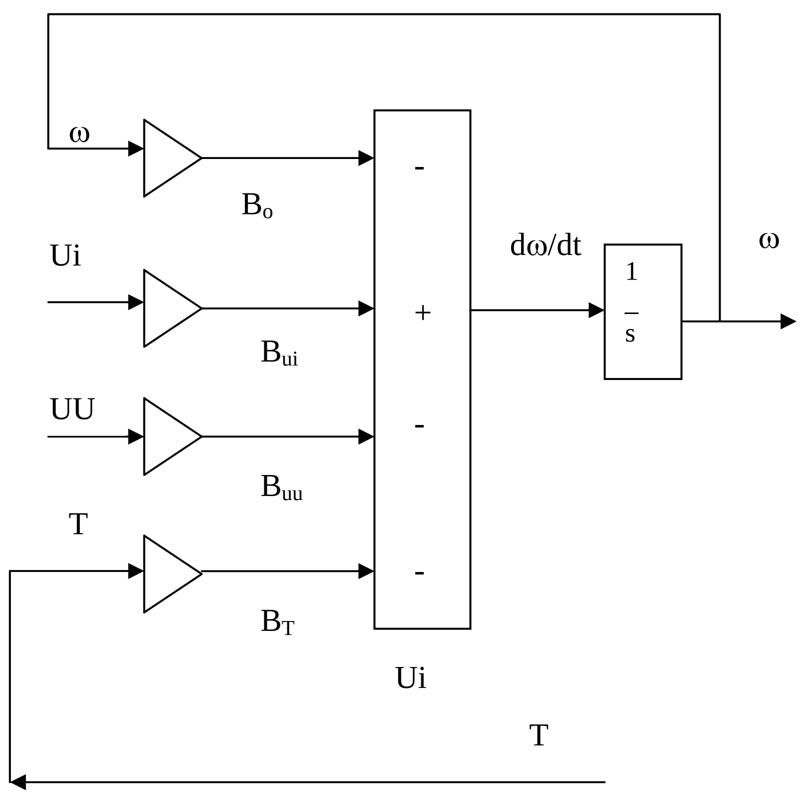
$B_{ui} = (K_a K_f)/(J R_f)$

B_{uu} = $(K_a K_f)/(J R_f^2)$

$B_o = ((b + b_R)/J$

$B_T = 1/J$

Simulink model from Fig. 3.13 considers a negligible $B_o = 0$. A non-negligible B_o requires to modify the block referring to the Newton second law as shown:



This Simulink model replaces the Newton second law part of the Simulink model from Fig. 3.13 retaining the same inputs Ui, UU and T, the same output dω/dt while adding an extra input to B_o ω from the output of the Integrator of d(omega)/dt block

3.6 Choose DC Gearmotor Series GM 8000 LO-COG from the Pittman Catalogue on the Internet [138]. Use the catalogue data for the model developed in problem 3.2 and compare the simulation results with those displayed in Fig. 3.6.

Solution

For a small PM-DC motor from Pittman Catalogue on the Internet, http://www.pittmannet.com/pdf/lcg_bulletin.pdf , parameters for GM 8x13, for reference voltage of 12 V, are:

K= 0.0131 Nm/A,

K_e = 0.0131 Vs/rad,

R= 3.2 Σ, b=0

J= 1.3 10^{-6} kg m^2

For this motor, the coefficients for the Simulink model shown in fig. 3.5 are:

A_u = 1/R = 0.313

A_o = K_e/R = 0.0041

B_i = K/ J = 10077

B_o = b/J = 0

B_T =1/J = 769 231

3.7 A two phase hybrid stepper motor is considered for driving a load. The application requires an angular step of 0.9 [^{0}/step]. Determine the number of rotor teeth needed in this case.

Solution

For a hybrid stepper motor the step angle, or the angular resolution r is:

r [^{0} /step] = 360 [^{0} /revolution] / N [steps/revolution]

For r = 0.9[^{0} /step]

N = 360 [^{0} /revolution] /r [^{0} /step] = 400 [steps/revolution]

For the motor with n phases on the stator and m teeth on the rotor, the total number of steps per revolution N is

N = n*m

For n= 2 phases

m= N/n= 400/2 = 200 [teeth]

3.8 Choose hybrid stepper motor from the PDS or PDX Series in Parker Hannifin Catalogue on the Internet [139]. For a typical step angle, determine the number of rotor teeth required for that motor.

Solution

A PDX hybrid stepper motor from:

http://www.compumotor.com/wwwroot/manuals/pd/PDX1.pdf

has typical step angles of r=1.8 [^{0}/step], i.e

N = 360 [^{0} /revolution] /r [^{0} /step] = 200 [steps/revolution]

For n= 2 or the number of phases on the stator

that the number of rotor teeth is

m= N/n = 200/2 = 100 [teeth]

3.9 The PDS or PDX Series, in Parker Hannifin Catalogue on the Internet [139], offer various optional ministepping resolutions in Steps/Rev. A standard two phase hybrid stepper motor, listed in the catalogue, has 50 teeth for the N section and 50 teeth for the S section of the rotor. Calculate the resolution with full stepping. How many times the ministepping driver improves the resolution of the motor using full stepping?

Solution

For a typical step angles of r=1.8 [^{0}/step], full stepping resolution is

N = 360 [^{0} /revolution] /r [^{0} /step] = 200 [steps/revolution]

PD-E Series Ministepping systems from

http://www.compumotor.com/catalog/c_57.pdf

offer selectable resolutions of 400 to 4000 [steps/ revolution], i.e 2 to 20 times higher than the full stepping resolution.

3.10 Assume that a hybrid stepper motor using full stepping, has N= 400 [steps/revolution]. The DDA method is employed for a PC with M=64 [bit] to generate a pulse train to achieve a constant velocity of 0.5 [revolutions/sec] for 2 [sec] and then, 0.55 [revolutions/sec] for 1 [sec]. Ignoring the duration of acceleration and deceleration, calculate number and the frequency of the steps to be generated.

Solution

A full step excitation of a N =400 [steps/revolution] stepper motor, this corresponds to a desired velocity V_d [steps/sec] given by

$V_d = v_d * N$

for a rotation of T_d [steps].

$T_d = V_d * t_d$

For v_d =0.5 [revolutions/sec] and t_d =2 [sec]

$V_d = 0.5* 400 = 200$ [steps/sec]

$T_d = 200*2 = 400$[steps].

For v_d =0.55 [revolutions/sec] and t_d =1 [sec]

$V_d = 0.55* 400 = 220$ [steps/sec]

$T_d = 220*1 = 220$[steps].

Assuming a PC with M=64 [bit] and a clock frequency of F = 1,000,000 [cycles/sec] and 1 [addition/cucle, i.e. addition frequency og f= 1,000,000 [addition/sec], the DDA algorithm calculates for V_d = 200 [steps/sec]

VV= V_d /f =200/1,000,000= 0.0002 [steps/addition

And for V_d = 220 [steps/sec]

VV= V_d /f =220/1,000,000= 0.00022 [steps/addition]

3.11 For the case given in the previous problem, calculate the error due do binary representation in case the digital representation is reduced to M= 8 [bit].

Solution

The error due to a M-1 representation is $1\text{-}1/2^{M-1}$

For M= 64, the error is $1\text{-}1/2^{64-1} = 1 - 1.08 * 10^{-19}$

For M=8, the error is $1\text{-}1/2^{8-1} = 1 - 0.008 = 0.992$

3.12 For the block diagram of a hydraulic actuator with mass-spring-damper load shown in Fig. 3.23, the following values are given: m_L = 2 [Kg], m_p = 1]Kg], b_L = 0.1 [Ns /m] , b_p = 0, k_L = 10 [N/m], V /2β =0.01 [m^4s/Kg] ,A = 0.005 [m^2], k_u = 0. 2 [kg/ms] k_p = 0.01 [m^4/Kg].

Obtain the Simulink model and simulate system dynamics for a step input with magnitude of 0.01 [m]. Is the response x(t) overdamped or underdamped? Change the value of the damping coefficient b_L from 0 to 3 and observe the change in the response x(t).

Solution

The coefficients of the Block diagram not given in the text of the problem are:are:

$m= m_L + m_p = 1 + 2 =3$ [Kg]

$=b_L + b_p = 0.1 + 0= 0.1$ [Ns /m]

$k=k_L = 10$ [N/m]

$k/m =10/3 = 3.3$

$b/m = 0.1/3 = 0.033$

$A/m = 0.005/3 = 0.0017$

3.13 Modify the diagram in fig. 3.22 considering that between the piston and the load, the shaft is flexible and has a spring coefficient k_s .

Modify the load model to include the flexible shaft and obtain the new block diagram.

Solution

A flexible shaft imposes to modify the load model by letting x be the position variable for the puston and X the position variable of the Mass-Spring-Damper load.

Newton second law equations in this case are:

For the piston:

$m_p\, d^2x/dt^2 + b_p\, dx/dt + k_L\, (x-X) = Ap$

For the Mass-Spring-Damper load:

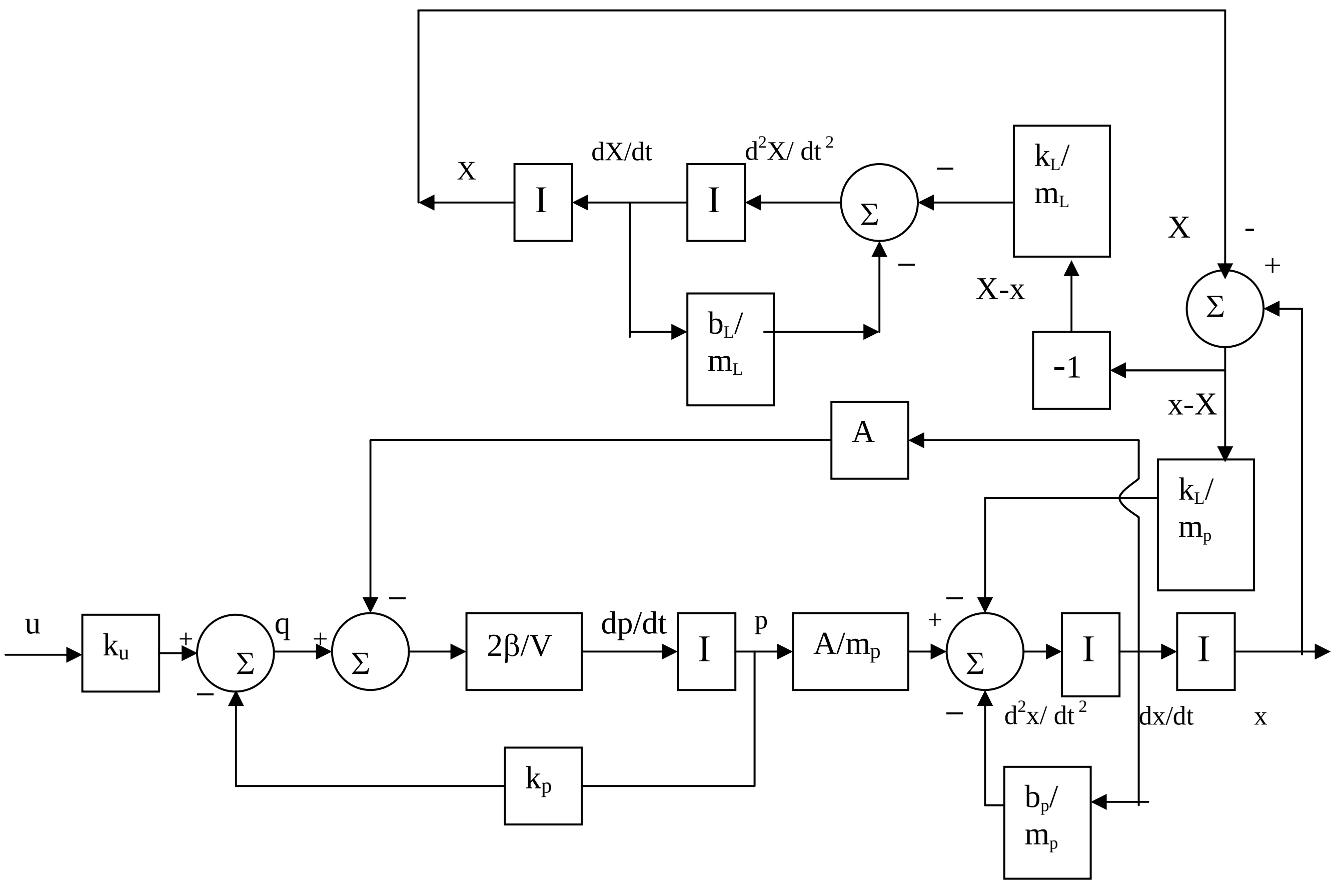
$$m_L\, d^2X/dt^2 + b_L\, dX/dt + k_L\,(X-x) = 0$$

The model for the block diagram is given by

$$dp/dt = (2\beta/V)\,(q - A\, dx/dt\,)$$

$$d^2x/dt^2 = (A/m_p)p - (\,b_p/m_p)\, dx/dt - (k_L/m_p)\,(x-X)$$

$$d^2X/dt^2 = -(\,b_L/m_L)\, dX/dt - (k_L/m_L)\,(X-x)$$



3.14 For a piezoceramic element BM500 Type I [22] of thickness 0.04", width 1.0" and length 3.0", the catalog parameters are:

$g_{31} = -11.5 * 10^{-3}$ [Vm/N]

compliance = $15.5 * 10^{-12}$ [m^2 /N]

relative dielectric constant = 1750

Calculate the capacitance and the axial strain $\Delta L/L$ for the value V = 5 [V] of the applied voltage.

Solution

The capacitance C [F] of the piezoelement of dimensions 3.0" * 1.0" * 0.040" (76.2 mm * 25.4mm * 1.016mm).

can be calculated as follows

$C = c\,WL/T = (11.95\ 10^{-9})\ (25.4\ 10^{-3})\ (76.2\ 10^{-3})/(1.016\ 10^{-3}) = 22.76\ 10^{-9}$ [F]

The axial strain is given by

$\Delta L/L = (g_{31}\ c / T)\ V = ((-11.5 * 10^{-3})\ (11.95\ 10^{-9})/(1.016\ 10^{-3}))\ 5 = 0.676\ 10^{-6}$ [m/m]

SOLUTIONS TO PROBLEMS FROM CHAPTER 4

4.1 A Weighted Resistor Digital-to-Analog Converter has n = 8 bit, the reference voltage V_R = 10 V, the Most Significant Bit resistance R = 12 k Ω, and the feedback resistance of the operational amplifier R_F = 6 k Ω.

Calculate:

a) The output voltage V_0 corresponding to the Least Significant Bit;

b) The output voltage V_0 corresponding to the Most Significant Bit;

c) The maximum value of the output voltage V_0 ;

d) The nominal full-scale output voltage V_0 ;

e) The resolution;

f) The output voltage V_0 corresponding to the binary input 10101100.

Solution

Given that for n=8

$V_o = -(2R_F\, V_R /R)\,(b_1 /2+ b_2/2^2+\ldots b_8 /2^8) = -(2*6*10/12)\,(b_1 /2+ b_2/2^2+\ldots b_8 /2^8) =$

$-10(b_1 /2+ b_2/2^2+\ldots b_8 /2^8)$

a) for LSB the binary input is 00000001 such that

$V_{o\ LSB} = -10/2^8 =-0.039$ V

The resolution is also 0.039 V.

b) for MSB the binary input is 10000000 such that

$V_{o\ MSB} = -10/2^1 = -5$ V

c) the maximum value of the digital input is 11111111 such that

$V_o = -10\,(1/2^1 + 1/2^2 +\ldots.+1/2^8) = -\ 0.961$ V

d) for the binary input 10101100

$V_o= -10\,(1(1/2^1) + 0(1/2^2)+ 1(1/2^3)+ 0(1/2^4)+ 1(1/2^5)+ 1(1/2^6)+ 0(1/2^7)+$

$0(1/2^8)) = 6.41$ V

e) $V_{fsr} = 2R_F V_R /R = 10$ V

It can be observed that

V_{fsr} - $V_{o\ MSB}$ = 10 - 0.961 = 0.039 V

i.e. the same value as the resolution.

4.2. A frequency sensor is connected to a computer through a Digital-to-Analog converter which has a 2% settling time of 130 μsec.

Calculate the maximum frequency which can be measured .

Solution

A DAC characterized by its 2% settling time of 130 μsec limits the frequency of binary input signals to less than 1/ 130 (10^{-6}) = 7693 Hz.

4.3. The Gain – Bandwidth Product (GBP) of an operational amplifier has the value12 Mhz. The operational amplifier is an amplifier with a gain of 120 dB.

a) Calculate the bandwidth of the amplifier;

b) Can this amplifier be used for amplifying signals of 120 Hz. If not, calculate the gain of the amplifier having the same GB= 12 MHz.

Solution

4.4. Assume a weighted resistors Digital to Analog Converter with n= 4 bit with R = 1 kΩ and V_R = 5 V. Calculate R_F such that analog output voltage V_0 will take values from 0 to 10 V.

Solution

The full scale range voltage V_{fsr} is defined as:

$V_{fsr} = 2R_F V_R /R$

or

$R_F = R\ V_{fsr} /(2V_R) = 1*10/(2*5) = 1\ k\Omega$

4.5. For the Weighted Resistors Digital to Analog Converter described in problem 4.4, build a Simulink model and simulate the following cases:

a) The output voltage V_0 corresponding to the Least Significant Bit, 0001;

b) The output voltage V_0 corresponding to the Most Significant Bit ,1000;

c) The maximum value of the output voltage V_0 , 1111 ;

d) The nominal full-scale output voltage V_0 ;

e) The output voltage V_0 corresponding to the binary input 1010.

Solution

In this case

$V_o=-(2R_F V_R /R)\ (b_1 /2+ b_2/2^2+ b_3 /2^3+ b_4 /2^4)=10\ (b_1 /2+ b_2/2^2+ b_3 /2^3+ b_4 /2^4)$

$= (10/2) *b_1 + (10/2^2)*b_2+(10/2^3)b_3 +(10 /2^4)b_4$

A suggested solution is to introduce the values for b_1, b_2, b_3 and b_4 from MATLAB workspace as a custom input signal and create a workspace block. The output of this block can then be applied as input to another block which calculates the weighted sum

$V_o = (5) * b_1 + (2.5) * b_2 + (1.25) * b_3 + (0.6125) * b_4$

or convert it in a matrix multiplication.

4.6. Assume an Analog to Digital Converter with n = 3 bit, R =1 kΩ, R_F = 2 kΩ, and V_R = 10 V. Calculate the full scale range and nominal scale range as well as the resolution.

Solution

$V_{fsr} = 2R_F V_R / R = 40V = V_N$

$r = V_N / 2^n = 10/ 8 = 1.25$ V

4.7. For the Analog to Digital Converter described in problem 4.6, build a Simulink model for Up Count method and simulate the following cases:

a) The output for full scale upper limit;

b) The output for 1 V analog input;

c) The outputs for inputs from 0 V, in steps equal to the resolution, up to the upper limit of the full scale.

Solution

The model of a DAC from problem 4.6 has to be augmented with a From Workspace block which generates a Matrix table with 2 columns and $2^n = 16$ rows of all distinct binary values of b_1, b_2, b_3 and b_4 in increasing order, which models the output of the Registers using Up count method . If-else-end construction from MATLAB provides the model for the comparator of Vin and Vo.

4.7. For the Analog to Digital Converter described in problem 4.6, build a Simulink model for Successive Approximations method and simulate the computation of the digital output for 1 V and 3 V analog inputs.

Solution

If-else-end construction from MATLAB provides the model for the construction of the Most Significant Bit sequence based on comparing each time Vin and Vo.

4.8. Use the Simulink model developed in Problem 3.4 and replace the battery model by a power operational amplifier model. For the same data as in Problem 3.4 from Chapter 3 and $R_1 = 1$ kΣ, $R_F = 10$ kΣ and $U^{(c)} = -1V$, compare the simulation results for the case of battery supply and power operational amplifier supply.

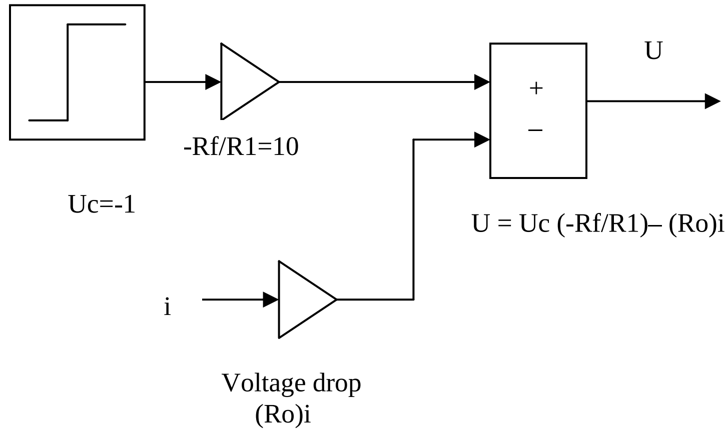
Solution

The block

U = Us – (Rs)i

from the Simulink model developed in Problem 3.4, has to be replaced by a block which models the power operational amplifier

U = Uc (-Rf/R1)– (Ro)i



This Simulink submodel will replace the block U = Us – (Rs)i, from the Simulink model developed in Problem 3.4, maintaining the same input i and the same output U.