MCG 3307: Control Systems II

Quiz 1

March 11, 2010

Policy

The present test is closed book and closed notes. Illegible work and loose sheets will not be graded.

Problem

Consider a unity negative feedback control system with the following feed-forward transfer function

$$G(s) = \alpha \frac{s+5}{(s+1)(s^2+4s+13)} \tag{1}$$

where α is a real parameter in $[0, +\infty)$.

(1) (2 points): Find the intervals on the real axis where the root loci exist.

(2) (2 points): Find the number of asymptotes and, if they exist, the angles and the interception with the real axis.

(3) (2 points): For the pair of complex conjugates poles, find the angles of departure of the related branches.

(4) (2 points): The roots of the equation

$$\frac{\mathrm{d}\alpha}{\mathrm{d}s} = 0 \tag{2}$$

are

$$s = -7.23, \quad s = -1.38 \pm 1.75j \tag{3}$$

Determine (or explain) if there are break-in and/or breakaway points.

(5) (2 points): Based on the information obtained in previous points sketch the root-locus.

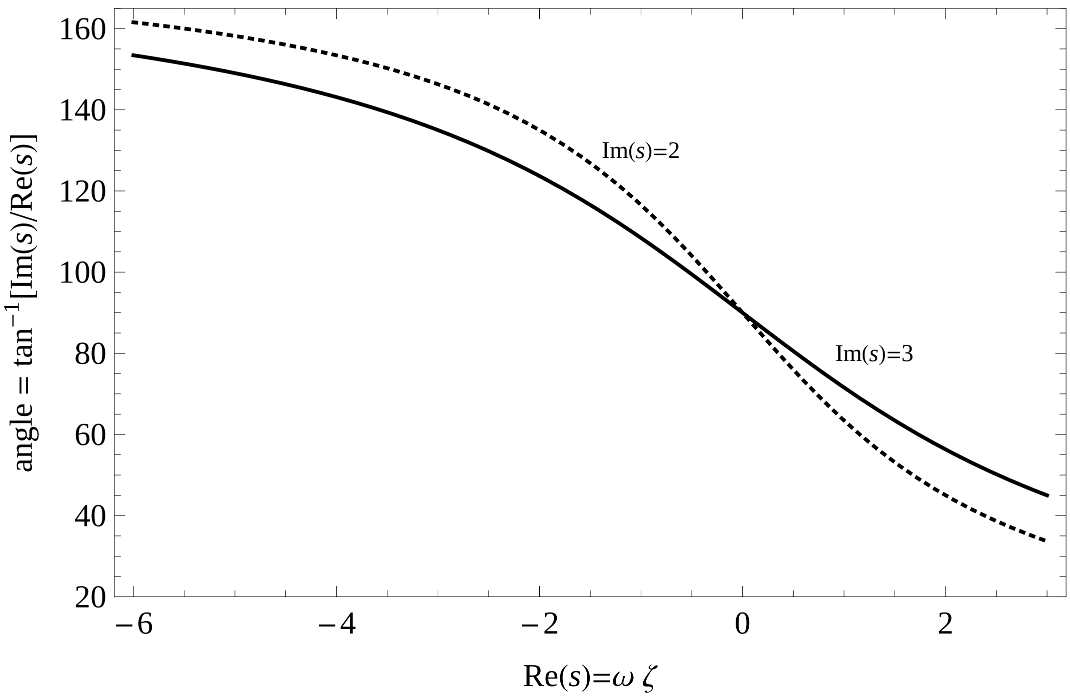


Figure 1: Plot of the function $\tan^{-1}(y/x)$=$\tan^{-1}(\mathrm{Im}(s)/\mathrm{Re}(s))$ for $\mathrm{Im}(s) = 2$ (dashed line) and $\mathrm{Im}(s) = 3$ (solid line), and $\mathrm{Re}(s)$ in the range $[-6, 3]$.

Quiz 1 – Solutions

1

Feedforward transfer function

$$G(s) = \alpha \frac{s+5}{(s+1)(s^2+4s+13)}$$

(1) Root loci exist on the left of an odd number of poles and zeros. For this system we have

$$a(s) = (s+1)(s^2+4s+13) =$$
$$=(s+1)(s+2-3j)(s+2+3j)$$

Therefore $n=3$,

$$p_1 = 2-3j$$
$$p_2 = 2+3j$$
$$p_3 = 1$$

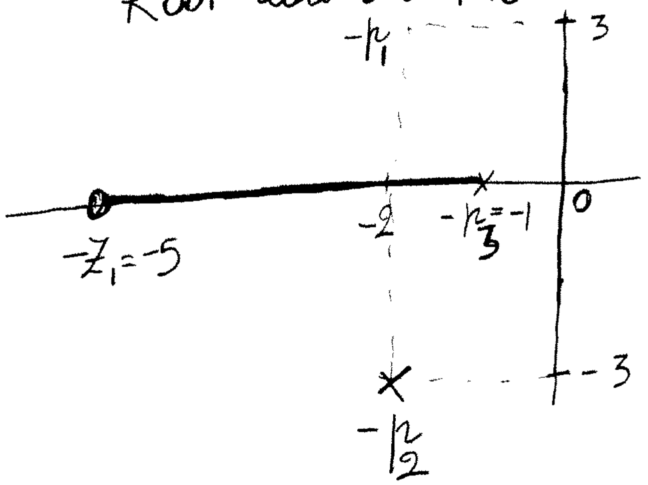
Moreover

$$b(s) = s+5$$

$$m=1$$

$$z_1 = 5$$

Root loci on the real axis:



Root loci exist on the real axis on the interval

$[-5,-1]$

(2) Since $n=3$ and $m=1$ we have $n-m=2$ asymptotes. One branch will end at the zero $-z_1$.

Angles:

$$\psi_\ell = \frac{\pm 180^\circ(2\ell+1)}{n-m}, \quad \ell = 0,1,\dots,n-m$$
$$= 0,1$$

Therefore

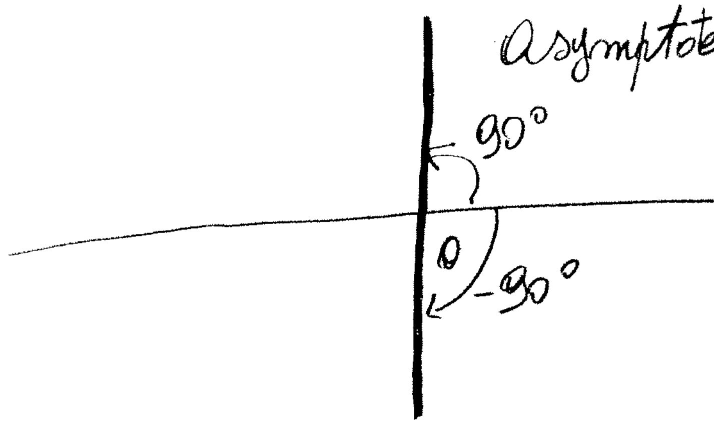
$$\psi_1 = \frac{180^\circ}{2} = 90^\circ$$

$$\psi_2 = \frac{3\times180^\circ}{2} = -90^\circ = 270^\circ$$

Interception:

$$\beta = -\frac{\sum p_i - \sum z_i}{n-m} = -\frac{1}{2}[(2+2+1)-5]$$

$$= -\frac{1}{2}[0] = 0$$



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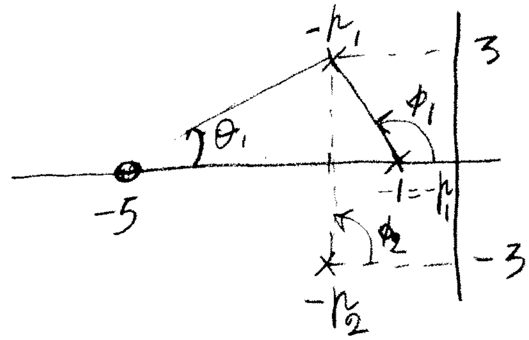
(3) Consider the pole $s=-p_1=-2+3j$. The phase angles with respect to the other poles are



$\phi_2 = 90°$

$$\tan\phi_1 = \frac{3-0}{-2-(-1)} = \frac{3}{-1} = -3; \quad \phi_1 \simeq 108° \text{ (plot)}$$

$$\tan\theta_1 = \frac{3-0}{-2-(-5)} = \frac{3}{3} = 1; \quad \theta_1 = 45° \text{ (plot)}$$

Angle of departure (pole of multiplicity 1)

$$\gamma = 180° - \Sigma \phi_i + \Sigma \theta_i$$

$$= 180° - (90° + 108°) + 45°$$

$$= 27°$$

for the pole at $-p_3$ is $-27°$

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(4) No, there are neither break-in nor break-away points since two roots are complex conjugates, and the real one does not belong to the root-locus as explained in point 1.

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(5)