MCG 3307: Control Systems II

Quiz 2

April 1, 2010

Policy

The present test is closed book and closed notes. Illegible work and loose sheets will not be graded.

Problem 1

Consider a unity negative feedback control system with the following feed-forward transfer function

$$G(s) = \frac{8}{s(s+5)} \tag{1}$$

(a) (2 points): From the characteristic polynomial $1 + G(s)$ determine the damping ratio ζ and the undamped natural frequency ω_n.

(b) (2 points): Determine the coordinates of a pair of dominant closed-loop poles such that $\zeta = 0.5$ and $\omega_n = 4\text{rad/sec}$.

(c) (3 points): Explain which type of compensator is appropriate to design in order to modify the closed loop response accordingly to Question 1(b), and write the general expression for such a compensator.

(d) (4 points): Find the angle deficiency associated to the compensator in Question 1(c).

Problem 2

Consider a unity negative feedback control system with the following feed-forward transfer function

$$G(j\omega) = \frac{8}{(j\omega+1)(j\omega+5)} \tag{2}$$

(a) (2 points): Obtain the corner frequencies of the system.

(b) (2 points): Derive the expression for the log-magnitude as a function of ω.

(c) (3 points): Sketch the asymptotic approximation of the log-magnitude (Bode diagram of the magnitude).

(d) (3 points): Sketch the static position error constant on the log-magnitude asymptotic representation obtained from Question (c) (explain).

Quiz 2 - Solutions

Problem 1

1) Characteristic polynomial:

$$s + 5s + 8 = 0 \quad , \quad B = 5, \; K = 8, \; J = 1$$

We have

$$\xi = \frac{B}{2\sqrt{JK}} = 0.89$$

$$\omega_n = \sqrt{\frac{K}{J}} = 2.8$$

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2) For $\xi = 0.5$, $\omega_n = 4\,\text{rad/sec}$ we have

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$= -2 \pm j4\sqrt{1-\frac{1}{4}} =$$

$$= 2\left(-1 \pm j\sqrt{3}\right)$$

3) Since we want to modify the transient behaviour we need to design a lead compensator, with general expression

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha T s+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}, \quad 0<\alpha<1$$

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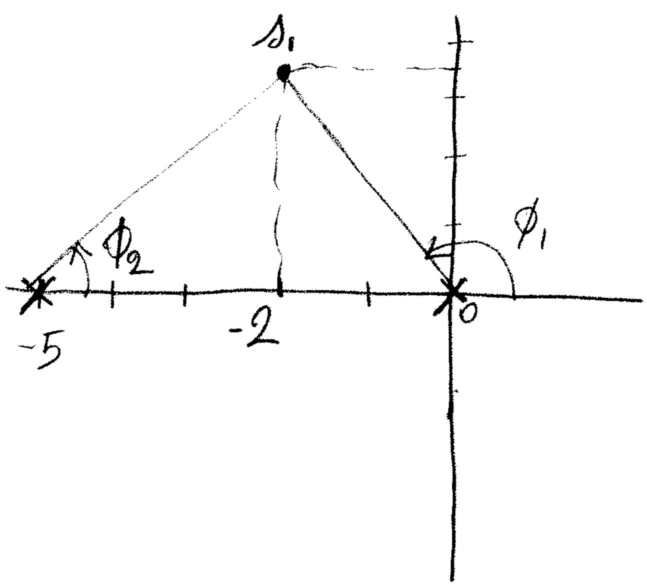
4) We consider the dominant closed-loop pole

$s_1 = 2(-1+j\sqrt{3})$



Open-loop poles:
$s=0$
$s=-5$

$$\phi_1 = 90^\circ + \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 90^\circ + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 90^\circ + 30^\circ = 120^\circ$$

$$\phi_2 = \tan^{-1}\left(\frac{2\sqrt{3}}{-2-(-5)}\right) = +\tan^{-1}\left(\frac{2\sqrt{3}}{3}\right) \cong 49^\circ$$

Angle deficiency:

$$\psi_K + \Sigma \phi_i = \pm 180^\circ(2K+1)$$

$$\psi_K = \pm 180^\circ(2K+1) - (120^\circ + 49^\circ)$$

$$\psi = 180^\circ - 169^\circ = 11^\circ$$

Problem 2

(a) Write

$$G(j\omega) = \frac{8}{(j\omega+1)\,5\left(\frac{j\omega}{5}+1\right)} = \frac{8/5}{(j\omega+1)\left(\frac{j\omega}{5}+1\right)}$$

Define

$$T_1 = 1 \quad , \quad T_2 = \frac{1}{5}$$

Corner frequencies:

$$\omega_1 = \frac{1}{T_1} = 1\,\text{rad/sec} \; ; \; \omega_2 = \frac{1}{T_2} = 5\,\text{rad/sec}$$

(b) Magnitude.

$$|G(j\omega)| = \frac{8}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+25}}$$

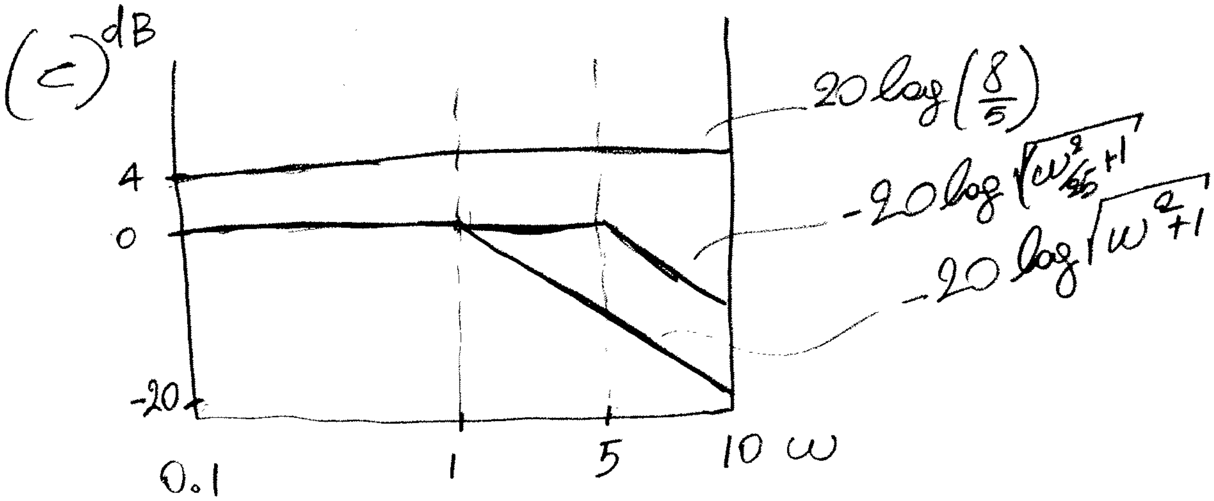
Log-magnitude

$$20\log|G(j\omega)| =$$

$$= 20\log 8 - 20\log\sqrt{\omega^2+1} - 20\log\sqrt{\omega^2+25}$$

$$= 20\log(8/5) - 20\log\sqrt{\omega^2+1} - 20\log\sqrt{\frac{\omega^2}{25}+1}$$

(c)



slope equals the summation of the indiv slopes

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(d) This is a type 0 system: therefore the position error constant is the asymptotic representation at $\omega = 0$:

$$20 \log \alpha_p = 20 \log \frac{8}{5}$$