2.7 Shear Strength of Unsaturated Soils

D. G. FREDLUND, *University of Saskatchewan, Saskatoon, Saskatchewan, Canada*

S. K. VANAPALLI, *Lakehead University, Thunder Bay, Ontario, Canada*

2.7.1 Introduction

Geotechnical, geoenvironmental, and agricultural engineers, along with soil scientists, are interested in understanding the shear strength behavior of soils. The shear strength of a soil is required for addressing numerous problems, such as the design of foundations, retaining walls, and pavements in civil engineering applications and the resistance to traction and tillage tools for agriculture engineering applications.

The shearing behavior of a saturated soil is related to one stress-state variable; namely, the effective stress, σ', defined as $(\sigma - u_w)$. The term, σ, is the total stress, and u_w is the pore-water pressure. The pore-water pressures in saturated soils are typically positive or zero. Shear strength tests for saturated soils can be performed in most geotechnical and agricultural laboratories. Detailed test procedures related to the determination of the shear strength of saturated soils are not discussed in this section. Standard testing procedures as per American Society for Testing and Materials (ASTM) methods for various shear strength tests for saturated soils are summarized in D2850-95e1, D3080-98, D2166-98a, and D4767-95 (ASTM, 1995a, b, 1998a, b). More information related to the test procedures is available in Lambe (1951), Holtz and Kovacs (1981), and Bishop and Henkel (1962).

Soils in an unsaturated state have negative pore-water pressures. The difference between the pore-air pressure, u_a, and pore-water pressure, u_w, is referred to as *matric suction* $(u_a - u_w)$. Unlike saturated soils, the mechanical behavior of unsaturated soils depends on two independent stress-state variables. These variables are the stress tensor, $(\sigma - u_a)$, which is referred to as *net normal stress*, and matric suction $(u_a - u_w)$ (Fredlund & Rahardjo, 1993). Soil behavior is independent of the individual valves of u_a, u_w, or the total stress, σ, so long as the stress-state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, are invariant. A special case of this principle, that the water content of unconfined soil specimens in a pressure membrane apparatus is uniquely dependent on the matric suction $(u_a - u_w)$, regardless of the individual values of u_a and u_w, is familiar to soil physicists.

The complete form of the stress state for an unsaturated soil in terms of two independent stress tensors can be represented in a matrix form as shown below:

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix} \qquad [2.7\text{–}1]$$

and

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix} \qquad [2.7\text{–}2]$$

Figure 2.7–1 illustrates two independent stress tensors in an unsaturated soil. The three normal stresses, σ_x, σ_y, and σ_z are mutually orthogonal with respect to x, y, and z directions. The six shear-stress components acting on the boundaries are symmetric, $\tau ij = \tau ji$. Because of this, it is possible to choose an orientation of the x-y-z axes, known as the *principal stress space*. The shear stresses vanish in the principal stress space, and the net applied stress tensor for this condition in a matrix form is shown below.

$$\begin{bmatrix} (\sigma_1 - u_a) & 0 & 0 \\ 0 & (\sigma_2 - u_a) & 0 \\ 0 & 0 & (\sigma_z - u_a) \end{bmatrix} \qquad [2.7\text{–}3]$$

The three principal stresses, σ_1 (major principal stress), σ_2 (intermediate principal stress), and σ_3 (minor principal stress), respectively, are applied to the boundaries, and the diagonal components of the principle stress tensor represent the eigenvalues of the general stress tensor from continuum mechanics principles.

The shear strength of an unsaturated soil is conventionally measured loading a cylindrical specimen in compression using triaxial shear-testing equipment. The pore pressures, u_a and u_w, and principal stresses, σ_1, σ_2, and σ_3, are independently controlled (similar to a pressure plate apparatus) in the triaxial cell of the

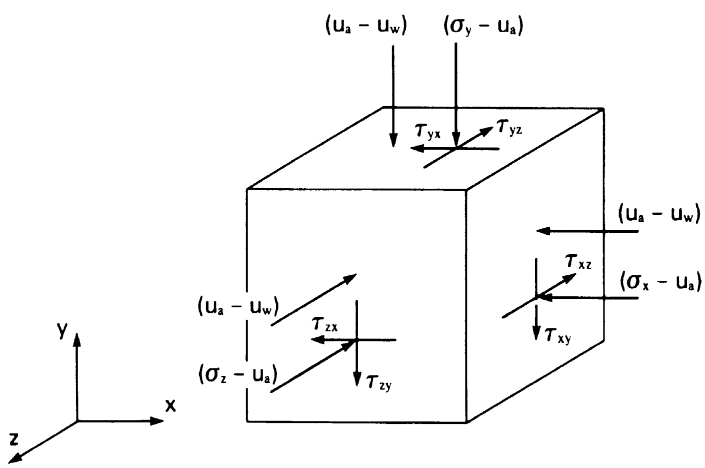


Fig. 2.7–1. The stress state variables for an unsaturated soil using the combination of $(\sigma - u_a)$ and $(u_a - u_w)$.

shear-testing equipment. Typical constraints in this equipment are that σ_2 is equal to σ_3, and σ_3 is less than σ_1. The resulting deviator stress $(\sigma_1 - \sigma_3)$ due to specimen loading is measured in compression. The shear stresses are always directly proportional to the deviator stress $(\sigma_1 - \sigma_3)$ for any given coordinate axis system.

The direct shear apparatus is another experimental apparatus to measure soil-shearing behavior. Unlike the triaxial shear apparatus, where all applied stresses are principal stresses, the direct shear apparatus has one applied normal stress component, σ_n, and the shear resistance, τ, is measured perpendicular to σ_n. More details about the triaxial and direct shear equipment and Mohr–Coulomb approach for interpreting the unsaturated shear strength behavior are described later.

This section discusses the basic principles and experimental procedures related to the measurement of the shear strength of unsaturated soils using triaxial and direct shear apparatus. The procedures for estimating the shear strength of an unsaturated soil from the soil water characteristic curve and the saturated shear strength parameters are also discussed.

2.7.2 Shear Strength Equation for Unsaturated Soils

The shear strength behavior of an unsaturated soil can be interpreted using triaxial or direct shear test results. The shear strength of an unsaturated soil is determined using "identical" specimens. The soil specimens need to be prepared at the same initial water content and dry density conditions to qualify as identical specimens (Fredlund & Rahardjo, 1993).

In a typical triaxial test series, the shear strength is determined over a range of matric suction values using the same net normal stress $(\sigma - u_a)$. If the major principal stress, σ_1, and minor principal stress, σ_3, acting on a specimen are known, the shear stress, σ, and the normal stress, σ_n, acting on a plane oriented at any given angle, σ, can be calculated relative to the principal axes. Also, Mohr's circles can be plotted with respect to failure conditions for combinations of σ_1 and σ_3 and different values of matric suction. Figure 2.7–2 shows the failure envelope drawn tangent to the Mohr's circles.

Fredlund et al. (1978) proposed the equation shown below for interpreting the shear strength of unsaturated soils in terms of two independent stress-state variables, $(\sigma - u_a)$ and $(u_a - u_w)$:

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \qquad [2.7\text{–}4]$$

where τ_{ff} is the shear stress on the failure plane at failure, c' is the intercept of the extended Mohr–Coulomb failure envelope on the shear stress axis where the net normal stress and the matric suction at failure are equal to zero—it is also referred to as *effective cohesion*, $(\sigma_f - u_a)_f$ is the net normal stress state on the failure plane at failure, u_{af} is the pore-air pressure on the failure plane at failure, ϕ' is the angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$, $(u_a - u_w)_f$ is the matric suction on the failure plane at failure, and ϕ^b is the angle indicating the rate of increase in shear strength relative to the matric suction, $(u_a - u_w)_f$.

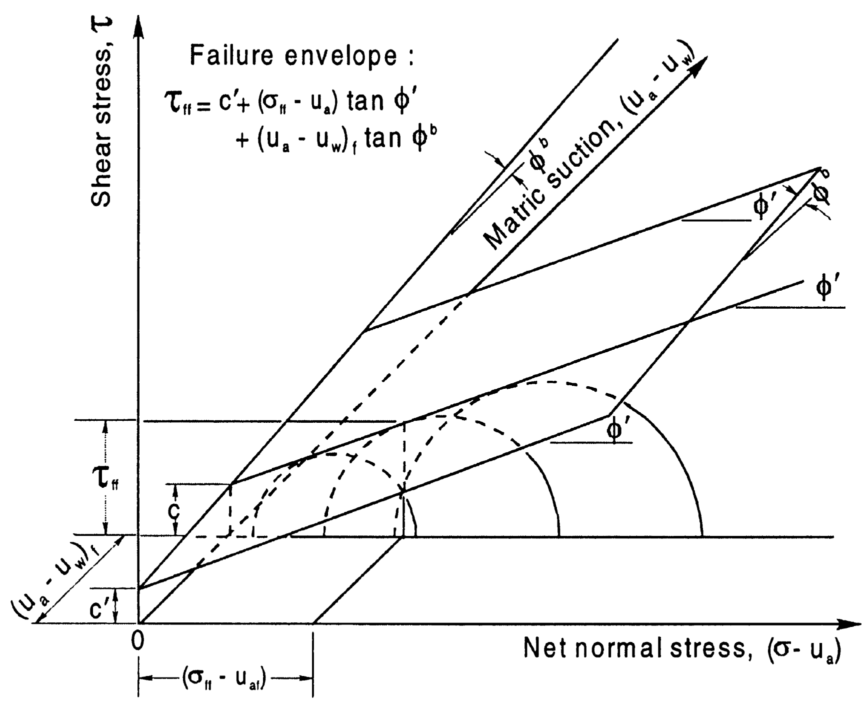


Fig. 2.7–2. Shear strength envelope for unsaturated soils (from Fredlund & Rahardjo, 1993).

The shear strength equation for an unsaturated soil (Eq. [2.7–4]) is also valid for direct shear tests. This equation is an extension of the Mohr–Coulomb shear strength equation for a saturated soil.

The shear strength envelope for unsaturated soils was originally proposed as a planar surface in nature based on a limited set of data available in the literature (Fredlund et al., 1978). Later experimental evidence by several investigators established that the shear strength for unsaturated soils is nonlinear when tested over a large range of suction (Gan et al., 1988; Escario & Juca, 1989). The Fredlund et al. (1978) equation is valid for interpreting data for both linear and nonlinear shear strength envelopes.

2.7.3 Triaxial Shear Tests for Unsaturated Soils

Conventional triaxial and direct shear equipment used to measure the shear strength of saturated soils requires modifications when testing the shear strength of unsaturated soils. These modifications must accommodate independent measurement (or control) of the pore-air pressure, u_a, and the pore-water pressure, u_w. The axis-translation technique forms the basis for these modifications. Laboratory testing of unsaturated soils with matric suction values >101.3 kPa (i.e., 1 atm) can then be undertaken without encountering cavitation problems (Hilf, 1956).

The axis-translation technique allows the pore-water pressure, u_w, in an unsaturated soil to be measured (or controlled) using a ceramic disk with fine pores

Table 2.7–1. Different triaxial tests for unsaturated soils.

Test methods	Consolidation prior to shearing process	Drainage: Pore air	Drainage: Pore water	Shearing process†: Pore air pressure, u_a	Shearing process†: Pore water pressure, u_w	Shearing process†: Soil volume change, V
Consolidated drained	Yes	Yes	Yes	C	C	M
Constant water content	Yes	Yes	No	C	M	M
Consolidated undrained	Yes	No	No	M	M	--
Undrained compression	No	No	No	--	--	--
Unconfined compression	No	No	No	--	--	--

† M = measurement, C = controlled.

(i.e., a high air-entry disk). These disks are used in unsaturated soil testing in place of conventional porous disks used in saturated soil testing. The ceramic disk must be epoxied around the edge to form a seal with the base pedestal. The high air-entry disk acts as a semipermeable membrane that separates the air and water phases. The separation of the water and air phases can be achieved only when the air-entry value of the disk is greater than the matric suction of the soil. The air-entry value refers to the maximum matric suction to which the high air-entry disk can be subjected before free air passes through the disk.

Various types of triaxial test methods are defined on the basis of drainage conditions during the application of confining pressure and during the application of the shear stress. The net total confining pressure $(\sigma_3 - u_a)$ generally remains constant during shear. The axial stress is continuously increased until a failure condition is reached. The axial stress generally acts as the total major principal stress, σ_1, in the axial direction, while the isotropic confining pressure acts as the total minor principal stress, σ_3 [i.e., $(\sigma_2 = \sigma_3)$].

The shear strength of unsaturated soils is interpreted in terms of the two stress-state variables, $(\sigma - u_a)$ and $(u_a - u_w)$, if the failure pore pressures are measured. Such an approach is referred to as a *stress-state-variable approach* for unsaturated soils. The shear strength has to be interpreted in terms of total stresses at failure if the pore pressures are not measured or controlled. The total stress approach for unsaturated soils should be applied in the field only for the case where it can be rightly assumed that the strength measured in the laboratory has relevance to the field drainage conditions. For example, rapid loading of a fine-grained soil can be assumed to be representative of undrained loading conditions.

The air, water, or total volume changes may or may not be measured during shear. A summary of various triaxial testing conditions, together with necessary measurements, is given in Table 2.7–1. The air, water, or total volume changes may or may not be measured during shear.

2.7.3.1 Test Procedures for Triaxial Tests

Figure 2.7–3 shows the assemblage of a triaxial cell used for testing the shear strength of an unsaturated soil. The installation of a high air-entry disk onto the triaxial cell base pedestal requires modifications to accommodate the pore-air pressure channel (i.e., valve C), the high air-entry disk, and the grooved water compartment below the disk (Fig. 2.7–4). The grooves inside the water compartment serve as water channels for flushing air bubbles that might be trapped or have accumulated as a result air diffusion. The high air-entry disk should be properly sealed onto the base pedestal using an epoxy resin seal around its circumference.

The high air-entry disk must be fully saturated before use in testing the shear strength of an unsaturated soil specimen, either in a triaxial shear test or a direct shear test. A head of distilled water can be applied to the high air-entry disk in the triaxial or direct shear equipment chamber. The water is then allowed to flow through the disk for a period of approximately 1 h, using an air pressure that is approximately equal to several times the air-entry value (i.e., five or six times). The air bubbles collected during the process from below the disk are flushed out. This procedure dissolves the air present in the disk and ensures full saturation of the high air-entry disk.

Typical diameters recommended for testing the shear strength of fine-grained and coarse-grained soil specimens are 38 and 76 mm, respectively. The ratio of length to diameter is recommended to be equal to 2. This value is similar to test-

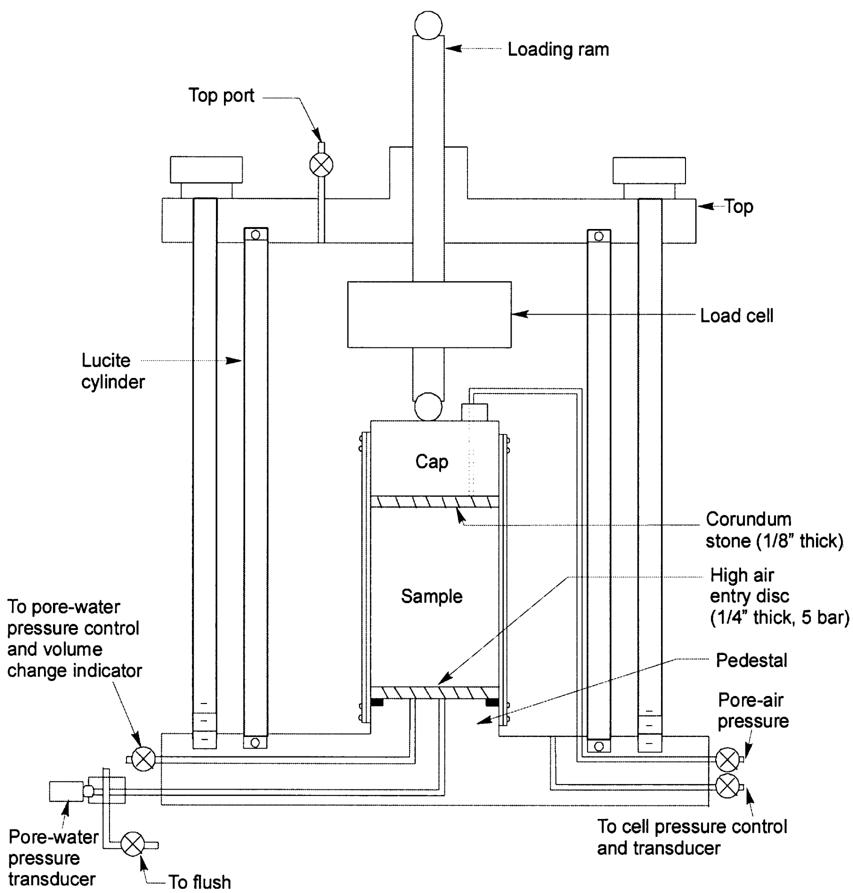


Fig. 2.7–3. Modified triaxial cell for testing the shear strength of unsaturated soils (from Fredlund & Rahardjo, 1993).

ing the shear strength of saturated soil specimens. The unsaturated soil specimens used for determining the shear strength, either undisturbed or compacted, are generally saturated before testing. Saturation is achieved in triaxial shear equipment by incrementally increasing the pore-water pressure, u_w. At the same time, the confining pressure, σ_3, is increased incrementally to maintain a constant effective stress $(\sigma_3 - u_w)$ in the specimen. As a result, the pore-air pressure increases and the pore-air volume decreases by compression and dissolution into the pore-water. The simultaneous pore-water and confining pressure increases are referred to as *back pressuring* the soil specimen.

Valve A in the base plate is used to control the pore-water pressure and to measure the water-volume change during a drained shear test (Fig. 2.7–4). Valve A can be connected to a twin-burette volume-change indicator (Bishop & Donald, 1961; Bishop & Henkel, 1962). Valve B on the base plate is used to measure the pore-water pressure during an undrained test. It can also be used to measure the prescribed pore-water pressure during a drained test. The pore-water pressure can be measured using a pressure transducer.

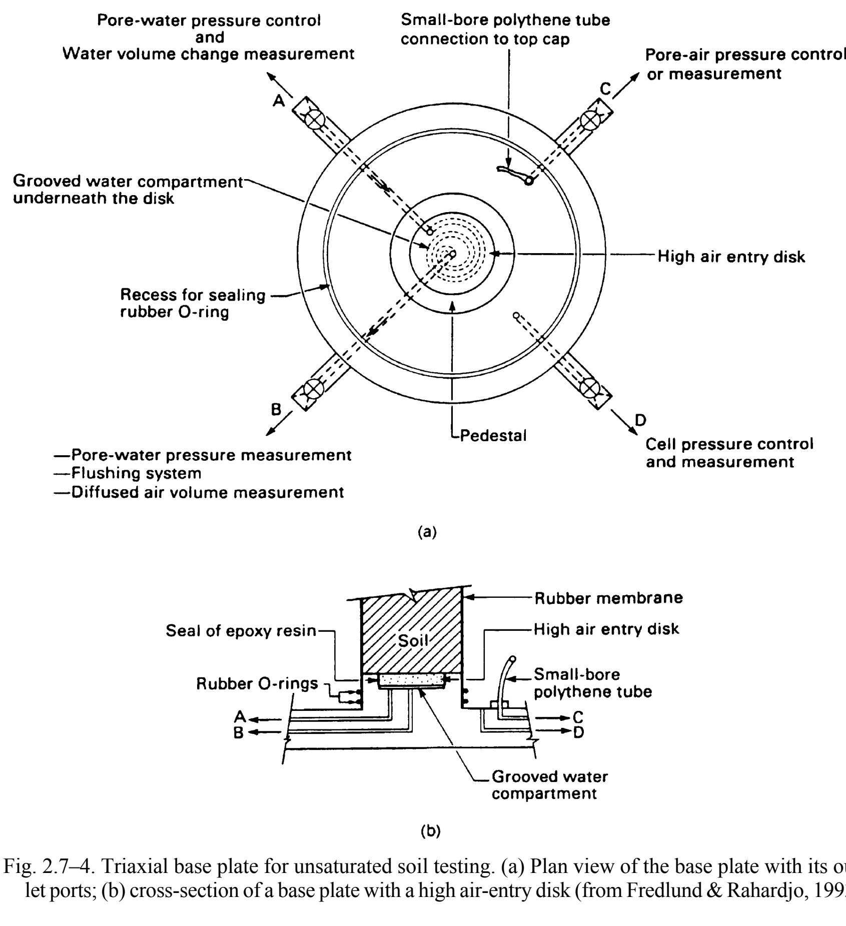


Fig. 2.7–4. Triaxial base plate for unsaturated soil testing. (a) Plan view of the base plate with its outlet ports; (b) cross-section of a base plate with a high air-entry disk (from Fredlund & Rahardjo, 1993).

An arrangement for pore-air pressure control using triaxial equipment is shown in Fig. 2.7–3. A 3.2-mm (1/8-in.) thick, coarse corundum disk is placed between the soil specimen and the loading cap. The disk is connected to the pore-air pressure control through a hole drilled in the loading cap and connected to a small-bore polyethylene tube. The pore-air pressure can be controlled at a desired pressure using a pressure regulator from an air supply. The measurement of pore-air pressure can be achieved using a small pressure transducer, preferably mounted on the loading cap. When measuring pore-air pressure, the air volume of the measuring system should be kept to a minimum to obtain accurate measurements. Pore-air pressure is difficult to measure because of the capability of air to diffuse through rubber membranes, water, polythene tubing, and other materials. The pore-air diffuses through water if the axis-translation technique is extended for a long time. A diffused air-volume indicator (DAVI) can be used in conjunction with the measuring system to flush diffused air from below the high air-entry disk. It is also possible to measure the volume of diffused air. Such a procedure enables a more accurate measurement of volume changes (Fredlund & Rahardjo, 1993).

The layout of the plumbing for the control board is shown in Fig. 2.7–5. The pore-air pressure line controls the pore-air pressure. In the case where the pore-air pressure is measured, a pressure transducer can be connected to the data-acquisition system through the base plate.

The initial confining air and water pressures to be applied to the soil specimen can be set on the pressure regulators before preparing the specimen. The con-

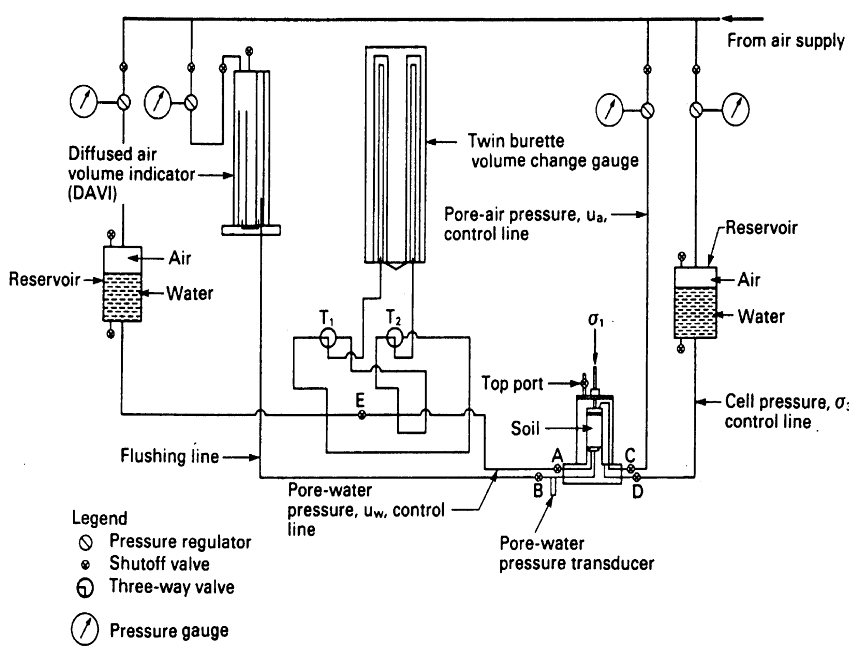


Fig. 2.7–5. Schematic diagram of the control board and plumbing layout for the modified triaxial apparatus (from Fredlund & Rahardjo, 1993).

fining air and water pressures are applied to form the required stress-state variables, $(\sigma_3 - u_a)$ and $(u_a - u_w)$, to the specimen through Valves D, C, and A, respectively (Fig. 2.7–5). An initial water pressure of 30 kPa or greater is desirable to provide sufficient pressure to flush air from the base plate.

2.7.3.1.a Consolidated Drained Test

The consolidated drained (CD) test is a common test conducted in the laboratory to measure the shear strength of an unsaturated soil. Figure 2.7–6 shows the stress conditions during the consolidated drained triaxial compression test. In this type of test, the soil is consolidated to a stress state representative of the stress range likely to be encountered in the field or in design. The soil specimen in the triaxial cell is generally consolidated under an isotropic confining pressure of σ_3, while the pore-air and pore-water pressures are controlled at pressures of u_a and u_w, respectively, by opening Valves D, C, and A (Fig. 2.7–5). Valves B and E are always closed during the test, except during the flushing of diffused air from the base plate. The water pressure, applied to the base plate, is registered on a transducer.

The vertical deflection and the deformation are periodically monitored to measure the overall volume change of the specimen. The volume of water flowing into (or out from) the specimen is recorded on the twin-burette volume-change indicator. Therefore, the three-way valves, T_1 and T_2, are always open during the test, except during the process of flushing diffused air from the base plate (Fig. 2.7–5). The air-volume change is generally not measured. Consolidation is assumed to have reached an equilibrium condition when there is no longer a tendency for the overall volume change or the flow of water from the specimen. At the end of the consolidation process, the soil specimen has a net confining pressure of $(\sigma_3 - u_a)$ and matric suction of $(u_a - u_w)$. The specimen is then sheared by axial compression, at an appropriate strain rate. More details with respect to the strain rates of shear strength testing are discussed below.

Diffused air is generally flushed from the base plate once a day during both consolidation and shearing. The frequency of the diffused-air measurement depends primarily on the applied air pressure. For a low applied air pressure, the diffused-air volume can be measured less frequently. In any case, the diffused-air volume should be measured before changing applied pressures. The water-volume-change correction, due to the diffused air volume, becomes necessary whenever tests extend over a period of several days.

The soil specimen is compressed in the axial direction by applying a deviator stress [i.e., $(\sigma_1 - \sigma_3)$] during the shearing process. The drainage valves for both pore air and pore water remain open during the shearing stage (i.e., under drained conditions). The pore-air and pore-water pressures are usually controlled at constant pressures (i.e., their pressures at the end of consolidation). The deviator stress is applied slowly to prevent the development of excess pore-air pressure or pore-water pressure in the soil. The net confining pressure, $(\sigma_3 - u_a)$, and the matric suction, $(u_a - u_w)$, remain constant throughout the test until failure conditions are reached, as shown in Fig. 2.7–6, that is, $(\sigma_3 - u_a)_f = (\sigma_3 - u_a)$ and $(u_a - u_w)_f = (u_a - u_w)$. The deviator stress $(\sigma_1 - \sigma_3)$ keeps changing during shear until the net major principal stress reaches a value of $(\sigma_1 - u_a)_f$ at failure.

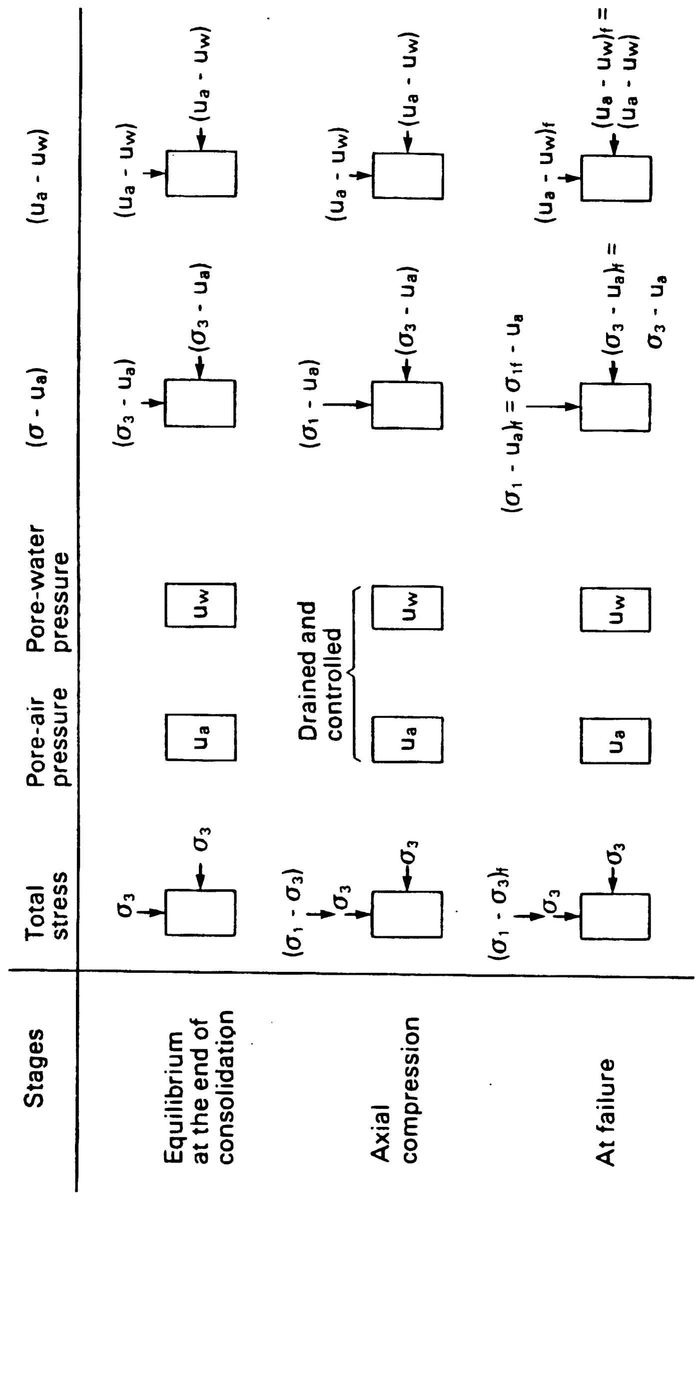


Fig. 2.7–6 Stress conditions during a consolidated drained triaxial compression test (from Fredlund & Rahardjo, 1993).

2.7.3.1.b Constant Water-Content Tests

For the constant water content (CW) test, the specimen is first consolidated and then sheared, with the pore-air phase allowed to drain while the pore-water phase is in an undrained mode. The consolidation procedure is similar to that of the consolidated drained test. When equilibrium is reached at the end of consolidation, the soil specimen has a net confining pressure of $(\sigma_3 - u_a)$ and a matric suction of $(u_a - u_w)$. The specimen is sheared by increasing the deviator stress until failure is reached. During shear, the drainage valve for the pore air remains open (i.e., under drained conditions). The pore-air pressure, u_a, is maintained at the pressure applied during consolidation; that is, Valve C is open during consolidation and shear (Fig. 2.7–5). The pore-water pressure is measured using a pressure transducer mounted on the base plate.

During shear, under undrained water-phase conditions, the diffused air volume should be measured. In this case, the water pressure in the base plate should be recorded before the flushing process and reset after flushing. The water in the pore-water pressure control line should first be subjected to the same pressure as recorded in the base plate. Valves A, T_1, and T_2 should remain closed, while Valve E is opened when adjusting the water-line pressure. The air back pressure in the diffused air-volume indicator should be adjusted to a pressure slightly lower than the recorded water pressure in the base plate, while Valve B remains closed. When Valve A is opened, the water in the base plate will quickly equalize to the pore-water pressure control line. The diffused air is then removed from the base plate by momentarily opening Valve B, which produces a pressure difference across the base plate. Valves A and B are closed at the end of the diffused air-volume measurement. The undrained pore-water pressure is then returned to the value existing before the flushing process. If the diffused-air removal is performed in a short period of time, disturbance to the undrained condition of the specimen should be minimal.

2.7.3.1.c Consolidated Undrained Tests with Pore-Pressure Measurements

For the consolidated undrained (CU) test, the soil specimen is first consolidated following the procedure described for the consolidated drained test. After equilibrium conditions have been established under the applied pressures (i.e., σ_3, u_a, and u_w), the soil specimen is sheared under undrained conditions with respect to the air and water phases. Undrained conditions during shear are achieved by closing Valves A, B, and C (Fig. 2.7–5).

The pressure transducer should be mounted on the loading cap, if possible, for measuring pore-air pressure changes. However, it is difficult to maintain an undrained condition for the pore air because of its capability to diffuse through the pore water, the rubber membrane, and the water in the high air-entry disk.

The diffused-air volume can be measured in a manner similar to that used during the CW test. These tests are generally not performed because of the problems associated with air diffusion.

2.7.3.1.d Undrained Tests

The procedure for performing an undrained test on an unsaturated soil specimen using the triaxial shear apparatus is similar to the procedure used for performing a typical undrained test on a saturated soil specimen. The unsaturated soil specimen is tested at its initial water content or matric suction. In other words, the initial matric suction in the specimen is not relaxed or changed before commencing the test. Both confined and unconfined compression triaxial tests can be conducted, and the shear strength contribution due to suction can be interpreted in terms of total stresses.

Matric suction changes in an unsaturated soil under undrained loading conditions are analogous to the changes in pore-water pressures in saturated soils under similar conditions of loading. Volume change in unsaturated soils under undrained loading is due mainly to the compression of air. Undrained pore pressures are assumed to be generated immediately after loading.

Figure 2.7–7 shows the development of pore-air and pore-water pressures during undrained compression under an applied isotropic stress. Skempton (1954) and Bishop (1954) introduced the concept of pore-pressure parameters. Tangent pore-pressure parameters for air and water phases for undrained loading conditions are:

$$B_a = du_a/d\sigma_3 \qquad [2.7\text{–}5]$$

$$B_w = du_w/d\sigma_3 \qquad [2.7\text{–}6]$$

where B_a is the tangent pore-air pressure parameter; du_a is the increase in pore-air pressure due to an infinitesimal increase in isotropic pressure, $d\sigma_3$; $d\sigma_3$ is the infinitesimal increase in isotropic pressure; B_w is the tangent pore-water pressure pa-

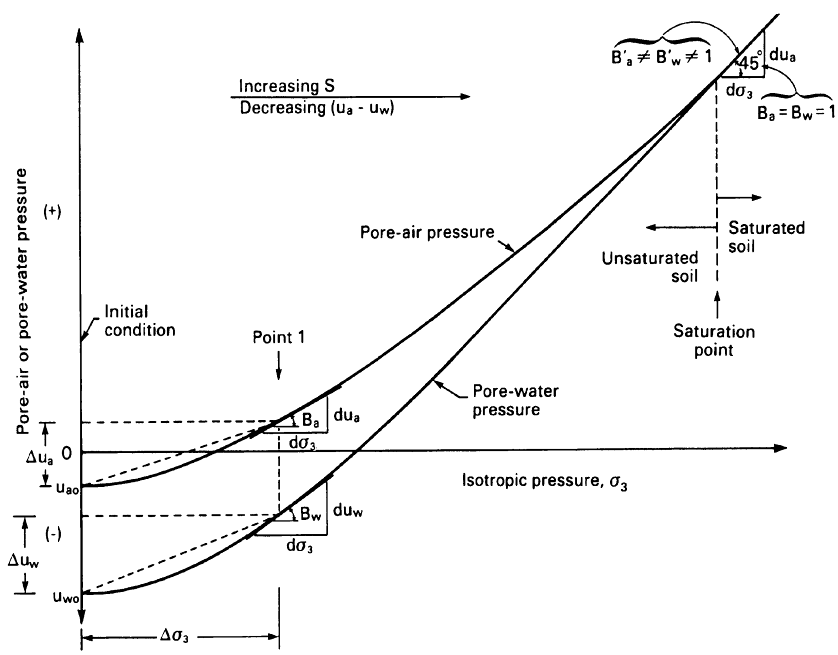


Fig. 2.7–7. Development of pore pressures due to undrained loading conditions (from Fredlund & Rahardjo, 1993).

rameter; and du_w is the increase in pore-water pressure due to an infinitesimal increase in isotropic pressure, $d\sigma_3$.

Pore-pressure parameters B_a and B_w take into account the changes in matric suction occurring under increasing total stress. The pore-pressure parameters are mainly a function of degree of saturation, compressibility, and loading and change at varying rates in response to the applied total stress. B_a and B_w are less than one at degrees of saturation less than 100%, and at complete saturation, B_a equals B_w and approach one. The matric suction $(u_a - u_w)$ in soil approaches zero once the soil approaches saturation under the applied loading condition.

For a compacted soil, the initial pore-air pressure can be assumed to be zero. The negative pressure in the soil with reference to atmospheric pressure is the matric suction. Under the application of total stress, the degree of saturation of the soil increases because of a decrease in total volume. Changes in pore-air pressure, du_a, and pore-water pressure, du_w, due to a finite change in total isotropic pressure, σ_3, can be computed knowing the initial conditions of the soil (Hasan & Fredlund, 1980; Fredlund & Rahardjo, 1993). A marching-forward technique can be applied with finite increments of total stress to estimate the changes in pore pressures commencing from a known initial unsaturated condition of the soil.

Matric-suction changes can also occur in an unsaturated soil during the shearing stage of triaxial loading conditions. These changes are expressed in terms of D pore-pressure parameters. Tangent pore-pressure parameters are measured under triaxial undrained loading conditions. The pore-air pressure parameter, D_a, can be defined as:

$$D_a = du_a/[d(\sigma_1 - \sigma_3)] \qquad [2.7\text{–}7]$$

where D_a is the tangent pore-air pressure for uniaxial, undrained loading; $d\sigma_1$ is the finite increment in major principal stress; $d(\sigma_1 - \sigma_3)$ is the finite increment in deviator stress.

The pore-water pressure parameter, D_w, is defined as:

$$D_w = du_w/[d(\sigma_1 - \sigma_3)] \qquad [2.7\text{–}8]$$

In a triaxial test, the total stress increment $d\sigma_2$ equals $d\sigma_3$. The major principal stress increment of total stress, $d\sigma_1$, is applied axially. The development of pore pressures in the undrained triaxial test are influenced both by the total stress increment, $d\sigma_3$, and from the change in the deviator stress, $d(\sigma_1 - \sigma_3)$.

The changes in pore pressures are given by:

$$du_a = B_a d\sigma_3 + D_a d(\sigma_1 - \sigma_3) \qquad [2.7\text{–}9]$$

$$du_w = B_w d\sigma_3 + D_w d(\sigma_1 - \sigma_3) \qquad [2.7\text{–}10]$$

Equations [2.7–9] and [2.7–10] can be expressed in a different form as:

$$du_a = B_a[d\sigma_3 + A_a d(\sigma_1 - \sigma_3)] \qquad [2.7\text{–}11]$$

$$du_w = B_w[d\sigma_3 + A_w d(\sigma_1 - \sigma_3)] \qquad [2.7\text{–}12]$$

where:

$$B_aA_a = D_a \qquad [2.7\text{–}13]$$

$$B_wA_w = D_w \qquad [2.7\text{–}14]$$

Shear strength can be related to the total stresses when the pore pressures at failure are unknown.

Unconfined Compression Test. The unconfined compression test is a special case of the undrained test and is usually performed in a simple loading frame. The soil specimen is sheared by applying an axial load until failure is reached. The compressive load is applied quickly (i.e., a strain rate of 1.2 mm min^{-1}) to maintain undrained conditions. Theoretically, this should apply to both the pore-air and pore-water phases. The pore-air and pore-water pressures are not commonly measured during compression. The deviator stress ($\sigma_1 - \sigma_3$) is equal to the major principal stress, σ_1, since the confining pressure, σ_3, is equal to zero. The deviator stress at failure is referred to as the unconfined compressive strength, q_u.

Soil samples in the field can remain saturated in some situations (i.e., S is equal to 100%), although the pore-water pressure is negative (i.e., pore-water pressures less than the air-entry value). The pore-water pressure can be negative because the soil has been taken from some distance above the groundwater table, or it can be negative due to the release of overburden pressure, or both.

Confined Compression Test. The unconfined compressive strength is assumed to be equal to twice the undrained shear strength, c_u. However, as the confining pressure increases, the undrained shear strength for the unsaturated soil also increases. As a result, the compressive-strength value may not satisfactorily approximate the undrained shear strength, c_u, at a confining pressure greater than zero. In the next paragraph, explanations are provided such that the confined compression test can be interpreted in terms of stress-state variables.

Four “identical” soil specimens are subjected to different confining pressures during the undrained tests shown in Fig. 2.7–8. The pore-air pressure, B_a, and pore-water pressure, B_w, in these soil specimens increase with the increase in confining pressure. The change in matric suction that arises in the specimen due to the application of confining pressure, σ_3, can be either computed using the theory of undrained pore-pressure parameters (i.e., B_a and B_w parameters) or measured at the end of the test (Hasan & Fredlund, 1980; Fredlund & Rahardjo, 1993).

The shear strength increases with an increasing confining pressure. The matric suction in the soil decreases with an increase in the degree of saturation accompanied by a reduction in the volume. The four identical soil specimens are brought to different initial states of stress because of the changes in pore pressures under undrained loading conditions. In undrained loading conditions for unsaturated soils, the increase in shear strength caused by an increase in confining pressure is greater than the reduction of shear strength associated with the decrease in matric suction. In Fig. 2.7–8, the diameter of the Mohr circles increases with an increase in confining pressure. The envelope defines a curved relationship between the shear strength and total normal stress for unsaturated soils tested under undrained conditions. Once the soil becomes saturated under the application of confining pres-

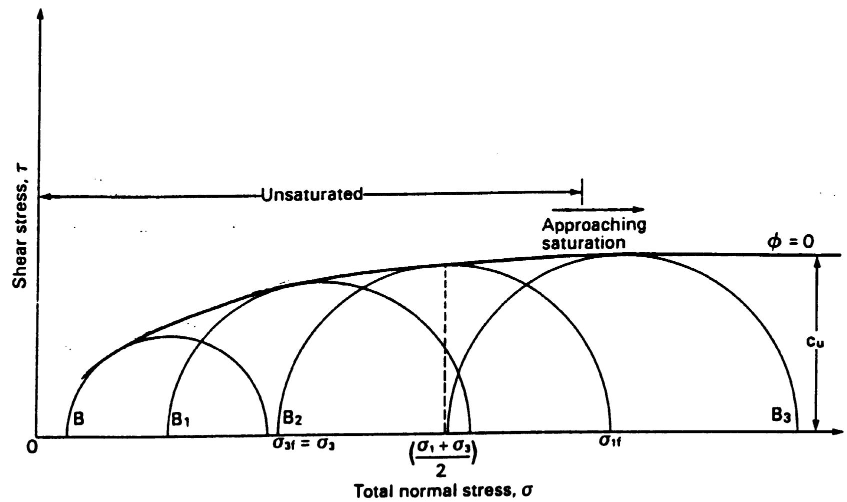


Fig. 2.7–8. Shear stress vs. total normal stress relationship for the undrained tests (from Fredlund & Rahardjo, 1993).

sure (i.e., B_a and B_w equals 1), a horizontal envelope develops with respect to the shear strength axis.

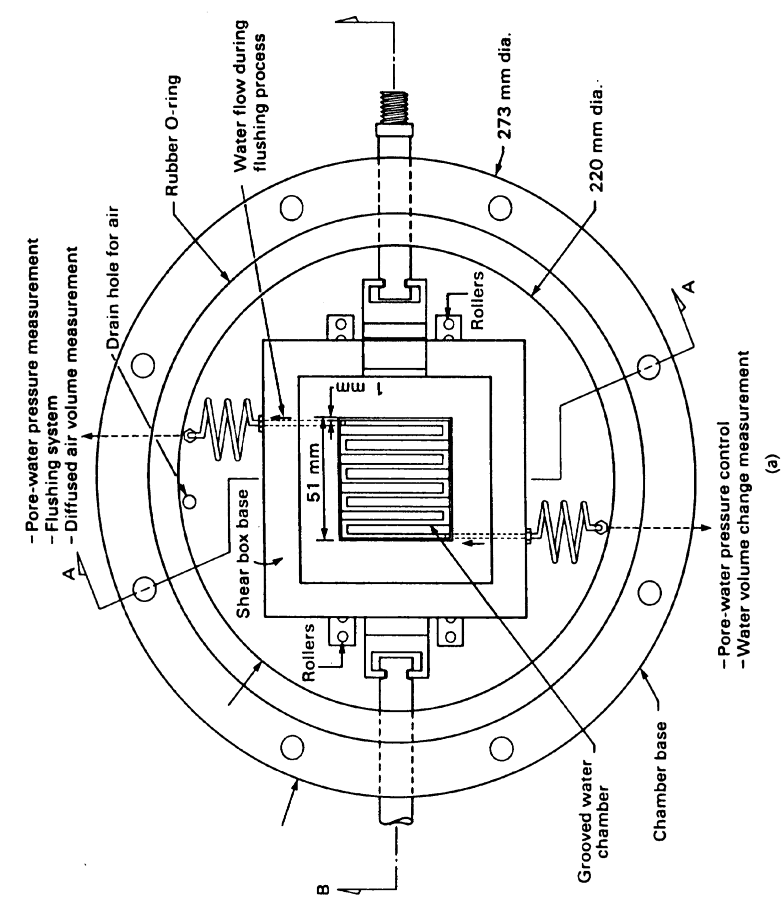
Under saturated conditions, where a single stress state variable $(\sigma - u_w)$ controls the strength, an increase in the confining pressure will be equally balanced by a pore-water pressure increase. The effective stress, $(\sigma_3 - u_w)$, remains constant regardless of the applied confining pressure, σ_3. Once the soil is saturated, the shear strength behavior is in accordance with the $\phi_u = 0$ concept (Skempton, 1948).

The pore-pressure changes due to the application of deviator stress (i.e., D_a and D_w parameters) are commonly neglected in undrained tests for unsaturated soils (Fredlund & Rahardjo, 1993). The shear strength contribution due to suction, ϕ^b, can be estimated assuming a planar-failure envelope both for confined and unconfined compression tests. These details are provided in a later section.

2.7.4 Direct Shear Tests for Unsaturated Soils

Direct shear tests on unsaturated soil specimens are commonly conducted in consolidated drained conditions. Typical dimensions recommended for testing the shear strength of fine-grained and coarse-grained soil specimens is 50 by 50 by 25 mm and 100 by 100 by 30 mm, respectively. Figure 2.7–9 provides the details related to the experimental setup, including the installation of a high air-entry disk in the base of the shear box. A cross-sectional view of the direct shear equipment is shown in Fig. 2.7–10.

A saturated soil specimen is placed in the direct shear box and consolidated under a vertical normal stress, σ. During the process of consolidation, pore-air and pore-water pressures must be controlled at selected pressures. The axis-translation technique can be used to impose a desired value of matric suction. The pore-water pressure can be controlled below the specimen using a high air-entry disk. At the


-Pore-water pressure measurement
-Flushing system
-Diffused air volume measurement
Drain hole for air
Rubber O-ring
Water flow during flushing process
273 mm dia.
220 mm dia.
Rollers
1 mm
51 mm
Shear box base
Rollers
A
A
B
Grooved water chamber
Chamber base
-Pore-water pressure control
-Water volume change measurement

(a)

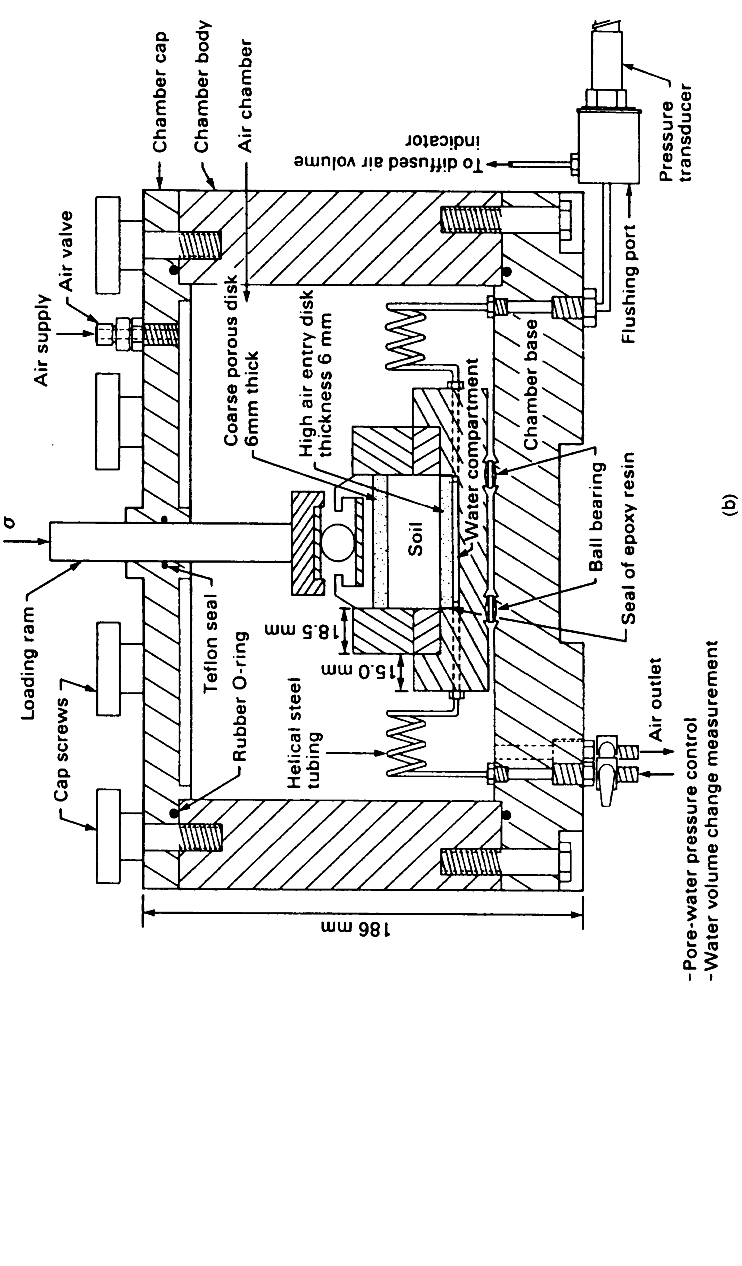


Fig. 2.7–9. Modified direct shear equipment for testing an unsaturated soil specimen. (a) Plan view of the pressure chamber for a direct shear box: (b) cross-sectional view of a direct shear box and pressure chamber (from Gan & Fredlund, 1988).

end of the consolidation process, the soil specimen has a net vertical stress of ($\sigma - u_a$) and a matric suction of ($u_a - u_w$).

During shear, the pore-air and pore-water pressures are controlled at constant values. Shearing is achieved by horizontally displacing the top half of the direct shear box relative to the bottom half. The soil specimen is sheared along a horizontal plane between the top and bottom halves of the direct shear box. Shear stress is increased until the soil specimen fails. This is the same procedure as used in the operation of a conventional direct shear apparatus. A motor that provides a constant horizontal shear-displacement rate is connected to the shear-box base.

The shear box is seated on a pair of rollers that can move along a pair of grooved tracks on a chamber base (Fig. 2.7–9a). The top box is connected to a load cell that measures the shear-load resistance (Fig. 2.7–10). The horizontal load required to shear the specimen, divided by the nominal area of the specimen, gives the shear stress on the shear plane. The water compartment below the high air-entry disk is connected to measuring systems through the stainless steel tubing (Fig. 2.7–9b). The plumbing layout and the procedure for conducting the consolidated drained shear test is similar to the consolidated drained, triaxial shear test procedure detailed earlier.

The direct shear test is particularly useful for testing unsaturated soils due to the short drainage path in the specimen. The low coefficient of permeability of unsaturated soils results in "times of failure" in triaxial tests that can be excessive. Other problems associated with testing unsaturated soils in a direct-shear apparatus are similar to those common to saturated soils (e.g., stress concentrations, deformations of the failure place, and the rotation of principal stresses). The failure envelope can be obtained for the results of direct shear tests without constructing the Mohr's circles.

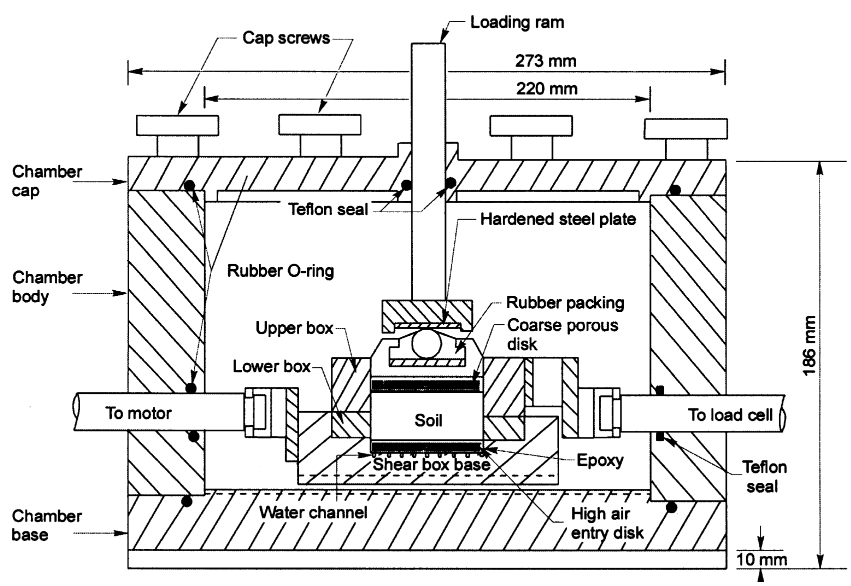


Fig. 2.7–10. Modified direct shear apparatus for testing unsaturated soils (from Gan & Fredlund, 1988).

2.7.5 Failure Criteria for Unsaturated Soils

The shear strength test is performed by loading a soil specimen with increasing applied loads such that a failure load is determined from the stress vs. strain relationships. *Stress* is defined as the load divided by area of the specimen. *Normal strain* for a triaxial specimen is defined as the ratio of the change in length to the original length. Shear displacement rather than strain is used for direct shear testing. Similar definitions are used in the interpretation of shear strength results for saturated soils. The load on the specimen is generally measured using a loading cell, and the strain is measured using a linear vertical displacement transducer (LVDT). The stress vs. strain data are collected using a personal computer with a data-logging system.

Various types of failure criteria for unsaturated soils were used in the literature (Bishop et al., 1960; Satija & Gulhati 1979). Typical stress–strain curves for consolidated drained triaxial tests are shown in Fig. 2.7–11. The curves show an increase in the maximum deviator stress as the net confining pressure $(\sigma_3 - u_a)$ is increased, while the matric suction $(u_a - u_w)$ remains constant. The volume change of the soil specimen during shear is usually measured with respect to the initial soil volume, that is, $-V/V_o$, and plotted vs. strain (Fig. 2.7–11c). Compression is conventionally given a negative sign, while expansion has a positive sign. Figure 2.7–11b presents a plot of gravimetric water-content change vs. strain where an increase in water content is given a positive sign.

In certain cases, the failure load is not well defined, as shown in Fig. 2.7–12. In this case, an arbitrary strain (e.g., 12%) is selected to represent failure conditions. The limiting strain-failure criterion is used when large deformations are required to mobilize the maximum shear stress. This criterion can also be used in direct shear testing. From a practical perspective, the different failure criteria produce similar shear strength parameters for unsaturated soils.

2.7.5.1 Strain Rates for Triaxial and Direct Shear Tests

The shear strength testing of unsaturated soils is generally performed at a constant rate of strain. An appropriate strain must be selected before commencing a test. In undrained shear, the selected strain must ensure equalization of pore pressures throughout the specimen. In drained shear, the selected strain must ensure complete dissipation of the pore pressures. The estimation of strain rate for testing soils in triaxial and direct shear can be made partly on the basis of the experimental evidence and partly on the basis of theory.

The *strain rate* for triaxial testing can be defined as the rate at which a soil specimen is axially compressed:

$$\varepsilon = \varepsilon_f / t_f \qquad [2.7–15]$$

where is the strain rate for shearing a specimen in the triaxial test, $_f$ is the strain of the soil specimen at failure, and t_f is the time required to fail the soil specimen or time to failure.

The rate of horizontal shear displacement in a direct-shear displacement test is analogous to the strain rate in a triaxial shear test. The horizontal shear-dis-

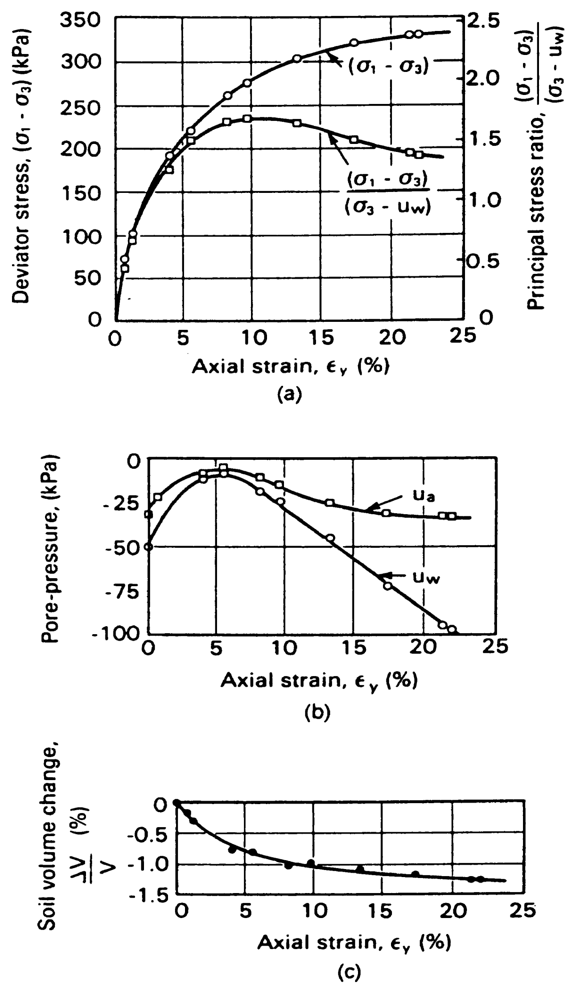


Fig. 2.7–11. Consolidated drained triaxial test results on Dhanauri clay. (a) Stress vs. strain curve; (b) water content change vs. strain curve; (c) soil volume change vs. strain curve (modified after Satija & Gulhati, 1979).

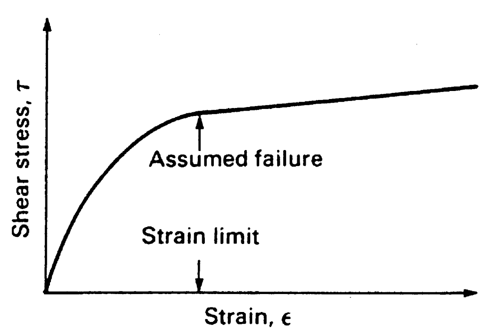


Fig. 2.7–12. Strain limit as a failure criterion.

placement rate can be defined as the relative rate at which the top and the bottom halves of the direct shear box are displaced.

$$\delta = d_h / t_f \qquad [2.7\text{–}16]$$

where δ is the horizontal shear-displacement rate for a direct shear test, and d_h is the horizontal displacement of the soil specimen at failure.

The strain at failure depends on the soil type and the stress history of the soil. Tables 2.7–2 and 2.7–3 present typical values of strain at failure, obtained from numerous triaxial and direct shear testing programs on unsaturated soils. This information can be of value as a guide when attempting to establish a suitable strain rate. The direct shear test has the advantage that a thin specimen can assist in expediting testing.

The strain rate for testing can also be estimated from a theoretical standpoint. The computed strain is approximate because of the assumptions involved in the theory and the difficulties in accurately assessing relevant soil properties. More details of the theory associated with the selection of strain rates are available in Bishop and Gibson (1963), Gibson and Henkel (1954), Ho and Fredlund (1982a), and Fredlund and Rahardjo (1993).

2.7.6 Interpretation of Drained Test Results Using Multistage Testing Procedures

Multistage testing can be performed using triaxial or direct shear testing equipment for consolidated drained tests. In this method, the stress path is altered to maximize the amount of shear strength information that can be obtained from one specimen. This method has the advantage of eliminating the effect of soil variability on the test results. For unsaturated soils, multistage testing has been performed in conjunction with a consolidated drained type of test using triaxial shear equipment. In these types of tests, the net confining pressure $(\sigma_3 - u_a)$ is usually maintained constant, while the matric suction is varied from one stage to another. However, the multistage testing is also applicable when using the constant water content or the consolidated undrained test procedure.

Ho and Fredlund (1982b) proposed a cycling loading procedure for multistage testing. Figure 2.7–13a illustrates this procedure for a decomposed rhyolite. The rhyolite specimen was tested under a net confining pressure $(\sigma_3 - u_a)$, which was equal to 138 kPa. The deviator stress $(\sigma_1 - \sigma_3)$ is released to zero once the estimated, maximum value of shear stress is reached. A test with three different stages and differing matric suction values is illustrated. Two-dimensional projections of the failure envelope on to the shear stress, τ, vs. net normal stress $(\sigma - u_a)$ plane is presented in Fig. 2.7–13b. A tangent line is drawn to the Mohr circles, and the intercept ordinates are measured. The measured intercepts, which represent the failure shear stress for various matric-suction values for which the specimen was tested, are plotted as shown in Fig. 2.7–13c to obtain the shear strength envelope. The shear strength at failure increased with increasing values of matric suction for a constant net confining pressure $(\sigma_3 - u_a)$. The results suggest that the envelope is linear in character, and the shear strength variation with respect to matric suction, ϕ^b, is a constant value.

Table 2.7–2. Strain rate and strain at failure for triaxial tests on unsaturated soils (from Fredlund & Rahardjo, 1993).

	Triaxial shear test†	Strain rate ε	Approximate strain at failure, ϕ	References
		% s^{-1}	%	
Boulder clay; $w = 11.6$ and clay = 18%	CW	3.5×10^{-5}	15	Bishop et al. (1960)
Braehead silt	CW CD	4.7×10^{-5} 8.3×10^{-6}	11 12	Bishop & Donald (1961)
Talybont boulder clay; $w = 9.75$% and clay = 6%	Undrained with pore-pressure measurements	4.7×10^{-7}	$\sigma_3 = 83$ kPa: 8.5 $\sigma_3 = 207$ kPa: 11	Donald (1963)
Dhanauri clay; $w = 22.2$% and clay = 25%	CW CD	6.7×10^{-4} 1.3×10^{-4}	20 20	Satija & Gulhati (1979)
Undisturbed decomposed granite and rhyolite	CD Multistage	6.7×10^{-5}	Stage I: 3–5 Stage II: 1–3 Stage III: 1–3	Ho & Fredlund (1982a)
Clayey sand: $w = 14$–17% and clay = 30%	Undrained and unconfined	1.7×10^{-3}	15–20	Chantawarangul (1983)

†CW, constant water content; CD, consolidated drained.

Table 2.7–3. Horizontal displacement rate and horizontal displacements at failure for several direct shear tests (from Fredlund & Rahardjo, 1993).

	Direct shear test	Displacement rate d_h	Displacement at failure, d_h†	References
		mm s^{-1}	mm	
Madrid gray clay	CD	1.4×10^{-4}	3.5–5	Escario (1980)
Madrid gray clay	CD	2.8×10^{-5}	6.0–7.2	Escario & Sáez (1986)
Red clay of Guadalix de la Sierra	CD	2.8×10^{-5}	4.8–7.2	Escario & Sáez (1986)
Madrid clayey sand	CD	2.8×10^{-5}	2.4–4.8	Escario & Sáez (1986)
Glacial till	CD (multistage)	1.7×10^{-5}	1.2	Gan (1986)

† Square specimen of 50 by 50 mm.

Multistage testing should be terminated when failure is imminent. Generally, this can be determined by observing when the deviator stress tends a maximum value. The soil specimen should not be subjected to excessive deformation, particularly during the early stages of loading. The specimen will tend to develop distinct shear planes, and the strength may be reduced from its peak strength. The shear strength measured at successive stages may tend towards an ultimate or residual strength condition. The ultimate or residual shear strength condition is obtained when the deviator stress has leveled off after reaching its peak or maximum value. Excessive strain accumulation can be a problem in multistage testing, particularly for soils whose structure is sensitive to disturbance.

2.7.7 Nonlinearity of Failure Envelope

Several investigators have observed nonlinearity in the shear strength vs. matric suction relationship (Gan et al., 1988; Escario & Juca, 1989; Vanapalli et al., 1996). Figure 2.7–14 illustrates a typical nonlinear matric-suction failure envelope. Experimental results have shown that the ϕ^b angle equals σ' value as long as the soil is in a saturated condition, even when a tension is applied to the water phase.

Studies of various investigators have shown that the ϕ^b angle is generally equal to ϕ' at low matric suction and decreases to a lower value at high matric suctions. In other words, there is a relationship between the ϕ^b angle and the matric suction. As the specimen desaturates, the rate at which the shear strength increases gradually decreases with an increase in the matric suction. This relationship can be visualized by examining the soil-water characteristic curve of a soil. Several investigators have used a soil-water characteristic curve and the saturated-shear strength parameters to predict the shear strength of unsaturated soils (Vanapalli et al., 1996; Fredlund et al., 1996; Oberg & Sallfours, 1997; Bao et al., 1998). The use of a soil water characteristic curve in predicting the shear strength of unsaturated soils is discussed briefly in a later section.

2.7.8 Interpretation of Undrained Test Results

Undrained shear strength can be interpreted by extending the Fredlund et al. (1978) shear strength equation (i.e., Eq. [2.7–4]). Effective shear strength parameters, c' and ϕ', along with the initial matric suction and the results from unconfined and confined compression tests, are required for the analysis. Changes in matric suction due to applied total isotropic or confining pressure, σ_3, can be computed by knowing the initial conditions of the soil using a marching-forward technique. This procedure is detailed in Fredlund and Rahardjo (1993) and is not repeated here. This technique is not necessary if the matric suction at failure condition is measured.

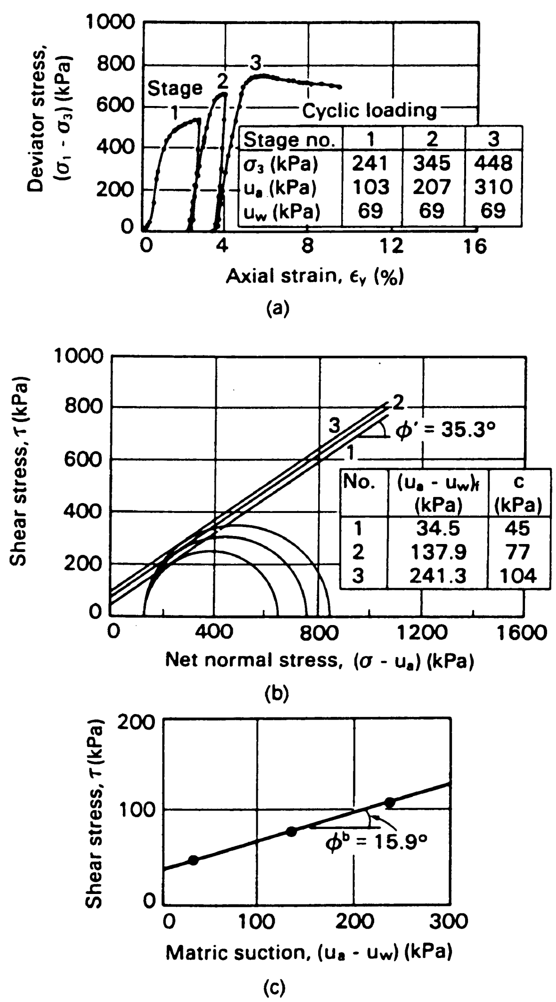


Fig. 2.7–13. Stress vs. strain curves and two-dimensional presentations of failure envelope for decomposed rhyolite specimen no. 11C using cyclic loading. (a) Deviator stress vs. strain curve (b) failure envelope projected onto the τ vs. $(\sigma - u_a)$ plane; (c) intersection line between failure envelope and the τ vs. $(u_a - u_w)$ plane at zero net normal stress, i.e., $(\sigma_f - u_a)_f = 0$ (from Ho & Fredlund, 1982).

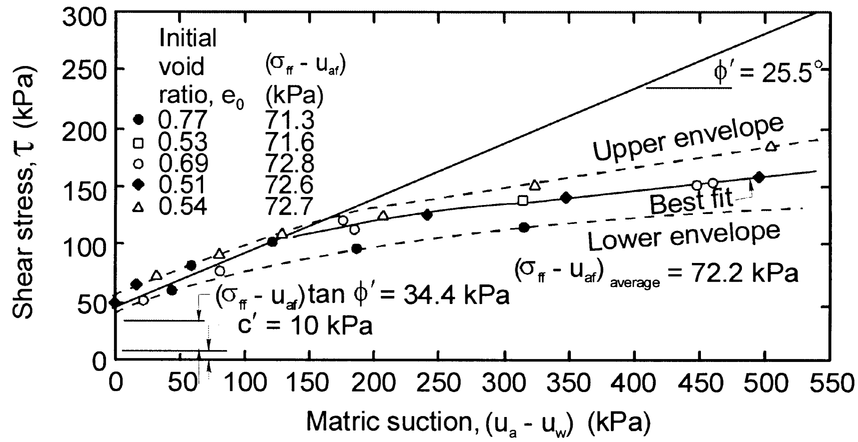


Fig. 2.7–14. Failure envelopes obtained from unsaturated glacial till specimens (from Gan et al., 1988).

2.7.8.1 Confined Compression Tests

The failure conditions for a confined compression test are shown in Fig. 2.7–15. The shear strength contribution parameter, $\tan\phi^b$, for unsaturated conditions assuming planar conditions will be equal to:

$$\tan\phi^b = \frac{[c_u(\cos\phi' + \sin\phi' \tan\phi') - (c_u + \sigma_3)\tan\phi' - c']}{(u_a - u_w)} \qquad [2.7\text{–}17]$$

where $c_u = (\sigma_1 - \sigma_3)/2$ and is the failure deviator stress from the undrained triaxial test, $(u_a - u_w)$ is the matric suction in the specimen at failure condition, and σ_3 is the confining pressure.

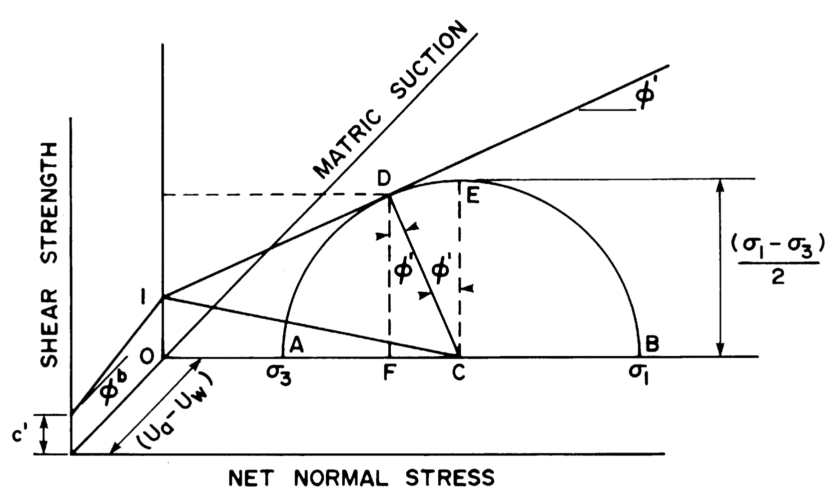


Fig. 2.7–15. Three-dimensional representation of confined compression test expressed in terms of stress state variables (from Vanapalli et al., 1999a).

The above equation has been derived assuming that the pore air dissolves in the water of the specimen and the pore-air pressure, u_a, is equal to zero. More details of the derivation of this expression are available in Vanapalli et al. (1999a).

2.7.8.2 Unconfined Compression Tests

Equation [2.7–17] can be applied for unconfined compression tests also by setting σ_3 equal to zero.

$$\tan\phi^b = \frac{[(\sigma_1/2)(\cos\phi' + \sin\phi' \tan\phi')] - [(\sigma_1/2)\tan\phi' - c']}{(u_a - u_w)} \qquad [2.7–18]$$

For unconfined compression tests, the pore-air pressure can be assumed to be atmospheric, and the results can be interpreted by assuming constant matric suction. In other words, the changes in matric suction during shear are assumed to be small compared to the initial value of matric suction.

Figure 2.7–16 shows the results of three unconfined compression tests. The initial matric suction in these specimens was 152 kPa. The specimens failed after reaching peak strength at about 3 to 5% axial strain. Using Eq. [2.7–18], the shear strength contribution due to matric suction (i.e., ϕ^b) from the unconfined compression test results is 23°. The average value of ϕ^b is the same as the effective angle of friction, ϕ', due to the high degree of saturation in the soil (i.e., 95.4%). The value of ϕ^b for specimens tested under drained conditions at the same value of suction is approximately the same (Vanapalli et al., 1999a).

2.7.9 Relationship Between the Soil Water Characteristic Curve and the Shear Strength of Unsaturated Soils

The soil water characteristic curve defines the relationship between the amount of water in the soil and soil suction. The amount of water can be a gravi-

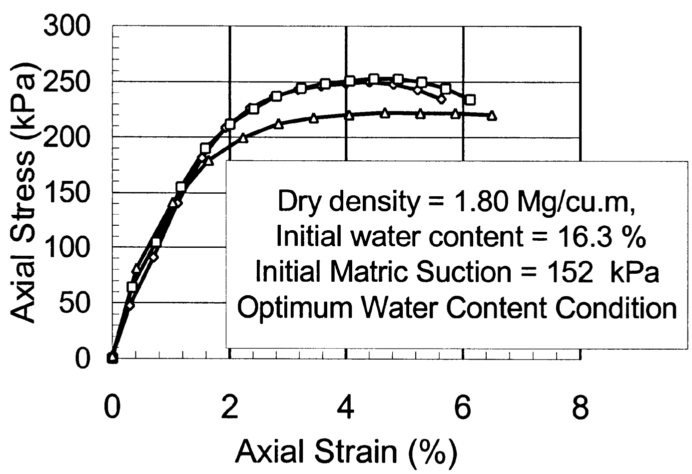


Fig. 2.7–16. Stress vs. strain relationships from unconfined compression tests (from Vanapalli et al., 1999a).

metric water content, *w*, or a volumetric water content, θ. The soil water characteristic curve can also plotted as a relationship between the degree of saturation, *S*, and soil suction. Experimental procedures for measuring the soil water characteristic curve in the laboratory are available in Fredlund and Rahardjo (1993). Standard testing procedures as per ASTM methods for determining the soil water characteristic curves for coarse and fine-textured soils are summarized in Methods D2325-68 (1981)e1 and D3152-72 (1994)e1, respectively (ASTM, 1981, 1994).

Typical soil water characteristic curves with various zones of desaturation are shown in Fig. 2.7–17. This figure also provides information with respect to the determination of the air-entry value and the residual degree of saturation. More details of the graphical construction procedure for determining the air-entry value and the residual degree of saturation are available in Vanapalli et al. (1999b). From Fig. 2.7–17, it can be seen that as the soil moves from a saturated state to drier conditions, the distribution of soil, water, and air phases changes as the stress state changes. The wetted area of contact between the soil particles decreases with an increase in the soil suction. There is a relationship between the rate at which the shear strength changes in unsaturated conditions to the wetted area of water contact between the soil particles or aggregates. In other words, a relationship exists between the soil water characteristic curve and the shear strength of unsaturated soils.

The typical relationship between the shear strength and the soil water characteristic curve can be seen by comparing Fig. 2.7–18a and 2.7–18b. There is a linear increase in shear strength with respect to suction up to the air-entry value. This value of suction identifies the point at which air enters the largest pores of the soil. The soil is essentially saturated at this stage, and there is no reduction in the area

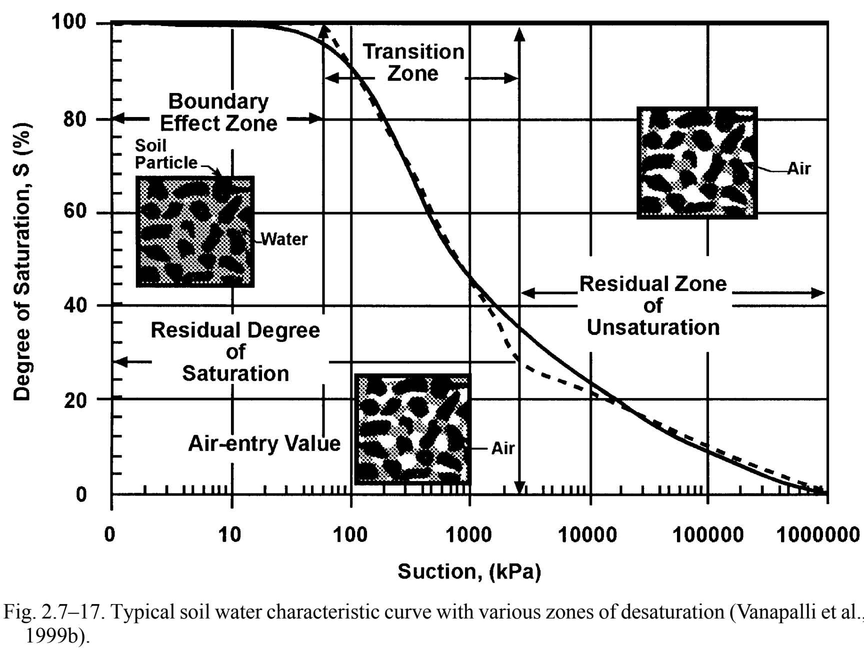


Fig. 2.7–17. Typical soil water characteristic curve with various zones of desaturation (Vanapalli et al., 1999b).

of water in this stage. The rate of desaturation with respect to an increase in matric suction, that is, $dS/(u_a - u_w)$, is greatest between the air-entry value and the suction corresponding to residual water-content conditions. The amount of water at the soil particle (or aggregate) contacts reduces as desaturation continues (i.e., the water-menisci area in contact with the soil particles or aggregates is not continuous and starts reducing) in this stage. There is a nonlinear increase in shear strength in this region.

Several equations are available in the literature to fit the soil water characteristic curve data for a limited range of suction (Brooks & Corey 1964; van Genuchten, 1980). Fredlund and Xing (1994) provided an analytical basis for mathematically defining the entire soil water characteristic curve. The equation ap-

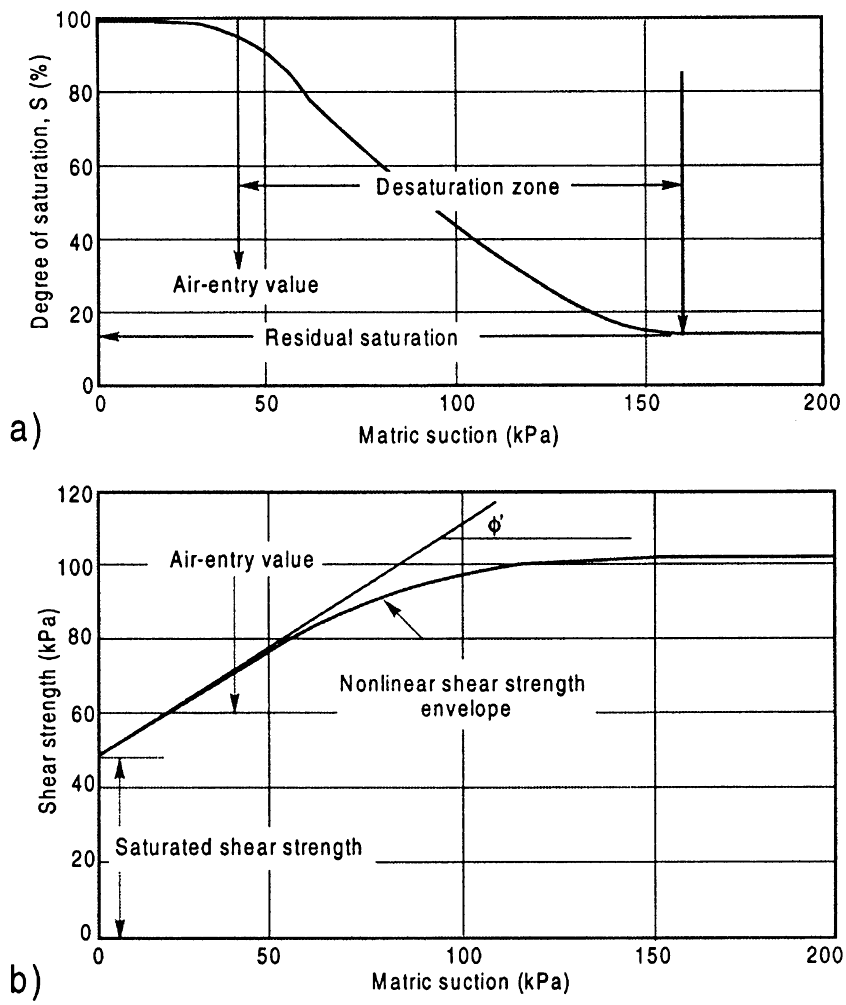


Fig. 2.7–18. (a) A typical soil water characteristic curve; (b) shear strength behavior of soil as it relates to the soil water characteristic curve (from Vanapalli et al., 1996).

plies to the entire range of suctions from 0 to 1 000 000 kPa. This relationship is empirical, but is derived based on the pore-size distribution, assuming that the soil consists of a set of interconnected pores that are randomly distributed. The equation is most commonly written in terms of volumetric water content, θ.

$$\theta = C(\psi)\left[\frac{\theta_s}{\ln\{[e + (\psi/a)^n]\}^m}\right] \qquad [2.7\text{–}19]$$

where θ is the volumetric water content, θ_s is the saturated volumetric water content, a is a suction related to the air-entry value of the soil, n is a soil parameter related to the slope at the inflection point on the soil water characteristic curve, ψ is soil suction, m is a soil parameter related to the residual water content, θ_r is the volumetric water content at residual conditions, e is a natural number (2.71828...), and $C(\psi)$ is a correction function that forces the soil water characteristic curve through a suction of 1 000 000 kPa and zero water content.

The correction factor forces the soil water characteristic curve through 1 000 000 kPa and is defined as:

$$C(\psi) = \left[1 - \frac{\ln[1 + \psi/C_r)]}{\ln[1 + (1000000)/C_r)]}\right] \qquad [2.7\text{–}20]$$

where C_r is the suction value corresponding to residual water content, θ_r.

Equation [2.7–19] can be written in a normalized form by dividing both sides of the equation by the volumetric water content at saturation:

$$\Theta = C(\psi)\left[\frac{1}{\ln[e + (\psi/a)^n]}\right] \qquad [2.7\text{–}21]$$

The dimensionless volumetric water content, Θ, is defined as:

$$\Theta = \theta/\theta_s \qquad [2.7\text{–}22]$$

where θ is the volumetric water content, and θ_s is the volumetric water content at a saturation of 100%.

The degree of saturation, S, however, is also equal to the dimensionless volumetric water content (i.e., Θ = S).

Equation [2.7–19] or [2.7–21] can be used to best-fit soil water characteristic curve data of any soil for the entire range of suctions. The fitting parameters (i.e., a, n, and m values) must be determined using a nonlinear regression procedure (Fredlund & Xing, 1994).

2.7.10 Procedure for Predicting the Shear Strength of Unsaturated Soils

Vanapalli et al. (1996) and Fredlund et al. (1996) proposed that the shear strength of an unsaturated soil at any given value of suction be written as follows:

$$\tau = [c' + (\sigma_n - u_a)\tan\phi'] + (u_a - u_w)(\Theta^{\kappa}\tan\phi') \qquad [2.7\text{–}23]$$

The first part of the equation is the saturated shear strength when the pore-air pressure, u_a, is equal to the pore-water pressure, u_w. This part of the equation is a function of normal stress since the shear strength parameters c' and ϕ' are constant for a saturated soil. For a particular net normal stress, this value is a constant. The second part of the equation is the shear strength contribution due to suction, which can be predicted using the soil water characteristic curve.

To obtain a better correlation between predictions and experimental shear strength data, a fitting parameter such as κ is useful in Eq. [2.7–23]. Vanapalli and Fredlund (2000) have shown comparisons between the measured and predicted values of shear strength for several statically compacted soils using Eq. [2.7–23]. The analyses of the results have shown that the shear strength for unsaturated soils can be predicted with a reasonable degree of accuracy for a large suction range (i.e., 0–10 000 kPa). A relationship between the fitting parameter, κ, and the plasticity index, I_p, was also proposed based on this study (Fig. 2.7–19).

Extending the same philosophical concepts, Vanapalli et al. (1996) proposed another equation for predicting the shear strength without using the fitting parameter, κ. The equation is given below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\{[\tan\phi'][(\theta - \theta_r)/(\theta_s - \theta_r)]\} \qquad [2.7\text{–}24]$$

where θ_r is the residual volumetric water content.

The forms of Eq. [2.7–23] and [2.7–24] are consistent with the Fredlund et al. equation (Eq. [2.7–4]) for determining the shear strength of an unsaturated soil.

Figure 2.7–20 shows the soil water characteristic curves of statically compacted clay-till specimens at three different initial water-content conditions representing dry-of-optimum, optimum, and wet-of-optimum conditions for the entire suction range of 1 000 000 kPa. The specimens compacted at dry-of-optimum water-content conditions desaturated at a faster rate in comparison with specimens compacted at higher water contents (i.e., optimum and wet-of-optimum water-content conditions). The effective saturated shear strength parameters for specimens com-

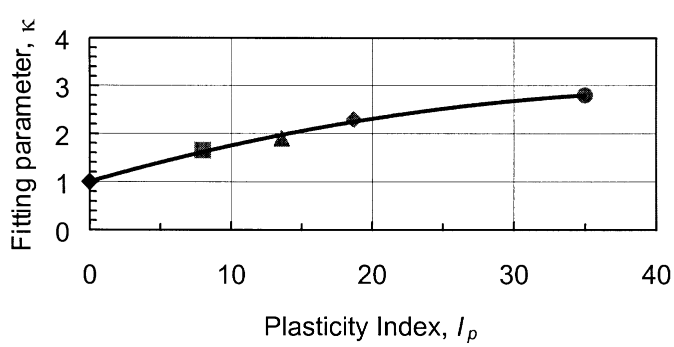


Fig. 2.7–19. The relationship between the fitting parameter κ, and plasticity index, I_p (Vanapalli & Fredlund, 2000).

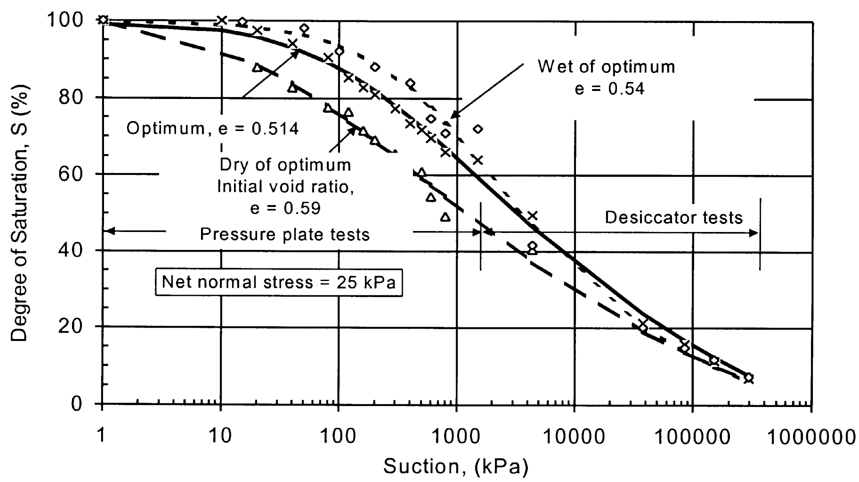


Fig. 2.7–20. Soil water characteristic curves for specimens at three different initial water content conditions (from Vanapalli et al., 1996).

pacted at these three water-content conditions were also determined. The shear strength parameters for the specimens compacted at three different water contents fall in a narrow range. The shear strength parameters, effective cohesion, and angle of internal friction were equal to zero and 23°, respectively.

The variation of shear strength for a suction range of 0 to 500 kPa was predicted using Eq. [2.7–23] and [2.7–24]. Experimental values of shear strength for the same suction range were also measured under consolidated drained conditions using direct shear test equipment described in this section. The experimental data are shown as symbols, and the predicted values are shown as continuous lines (Fig. 2.7–21). There is a good correlation between the measured and predicted values of shear strength. More details of the soil properties and testing procedures are available in Vanapalli et al. (1996).

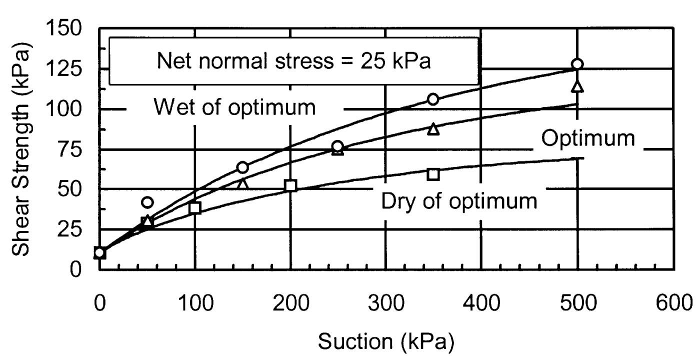


Fig. 2.7–21. Variation of shear strength with respect to matric suction at three different initial water content conditions (Vanapalli et al., 1996).

2.7.11 Summary

Geotechnical, agricultural, geological, hydrogeological, and geoenvironmental engineers and soil scientists are commonly required to deal with soils that are both in saturated and unsaturated conditions. This section provides details with respect to theory, experimental procedures, and estimation of the shear strength of soils in unsaturated conditions.

2.7.12 References

American Society for Testing and Materials. 1981. Standard test method for capillary moisture relationships for coarse-and medium-textured soils by pressure–plate apparatus. ASTM D2325-68 (1981)e1. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

American Society for Testing and Materials. 1994. Standard test method for capillary-moisture relationships for fine-textured soils by pressure-membrane apparatus. ASTM D3152-72 (1994)e1. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

American Society for Testing and Materials. 1995a. Standard test method for consolidated undrained triaxial compression test for cohesive soils. ASTM D4767-95. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

American Society for Testing and Materials. 1995b. Standard test method for unconsolidated, undrained compressive strength of cohesive soils in triaxial compression. ASTM D2850-95e1. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

American Society for Testing and Materials. 1998a. Standard test method for direct shear test of soils under consolidated drained conditions. ASTM D3080-98. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

American Society for Testing and Materials. 1998b. Standard test method for unconfined compressive strength of cohesive soil. ASTM D2166-98a. Annual book of ASTM Standards. ASTM, West Conshohocken, PA.

Bao, C., G. Bewei, and Z. Liangtong. 1998. Properties of unsaturated soils and slope stability of expansive soils. Keynote Lecture, UNSAT 98. 2nd International Conference on Unsaturated Soils. Beijing, China. 27–30 Aug. 1998. China Academy of Railway Science, Beijing, China.

Bishop, A.W. 1954. The use of pore pressure coefficients in practice. Geotechnique 4:148–152.

Bishop, A.W., I. Alpan, G.E. Blight, and I.B. Donald. 1960. Factors controlling the shear strength of partly saturated cohesive soils. p. 503–532. *In* ASCE Research Conference on Shear Strength of Cohesive Soils. University of Colorado, Boulder, CO.

Bishop, A.W., and I.B. Donald. 1961. The experimental study of partly saturated soil in the triaxial apparatus. p. 13–21. *In* Proceedings 5th International Conference on Soil Mech. and Found. Eng. Paris, France. Vol. 1.

Bishop, A.W., and D.J. Henkel. 1962. The measurement of soil properties and the triaxial test, 2nd ed. Edward Arnold, London, England.

Bishop, A.W., and R.E. Gibson. 1963. The influence of the provisions for boundary drainage on strength and consolidation characteristics of soils measured in the triaxial apparatus. Laboratory Shear Testing of Soils. American Society for Testing and Materials (ASTM), STP No. 361:435–458. ASTM, Philadelphia, PA.

Brooks, R.H., and A.T. Corey. 1964. Hydraulic properties of porous media. Hydrology Paper, no. 3. Colorado State University, Fort Collins, CO.

Chantawarangul, K. 1983. Comparative study of different procedures to evaluate effective stress strength parameters for partially saturated soils. M.Sc. thesis. Asia Inst. of Tech., Bangkok, Thailand.

Donald, I.B. 1963. Effective stress parameters in unsaturated soils. p. 41–46. *In* Proceedings 4th Australia–New Zealand Conf. Soil Mech. Found. Eng. Adelaide, South Australia.

Escario, V. 1980. Suction controlled penetration and shear tests. p. 781–787. *In* Proceedings 4th International Conference Expansive Soils. Denver, CO. Vol. 2. ASCE, Reston, VA.

Escario, V., and J. Sàez. 1986. The shear strength of partly saturated soils. Géotechnique 36:453–456.

Escario, V., and J.F.T. Juca. 1989. Strength and deformation partly saturated soils. p. 43–46. *In* Proceedings of the 12th International Conf. on Soil Mech. and Found. Eng. Rio de Janeiro, Brazil. Vol. 3.

Fredlund, D.G., N.R., Morgenstern, and R.A. Widger. 1978. Shear strength of unsaturated soils. Can. Geotech. J. 15:313–321.

Fredlund, D.G., and A. Xing. 1994. Equations for the soil-water characteristic curve. Can. Geotech. J. 31:521–532.

Fredlund, D.G., and H. Rahardjo. 1993. Soil mechanics for unsaturated soils. John Wiley and Sons, New York, NY.

Fredlund, D.G., A. Xing, M.D. Fredlund, and S.L. Barbour. 1996. The relationship of the unsaturated soil shear strength to the soil-water characteristic curve. Can. Geotech. J. 33:440–448.

Gan, J.K.M. 1986. Direct shear strength of unsaturated soils. M.Sc. thesis. Department of Civil Engineering, University of Saskatchewan, Saskatoon, SK, Canada.

Gan, J.K.M., D.G. Fredlund, and H. Rahardjo. 1988. Determination of the shear strength parameters of an unsaturated soil using the direct shear test. Can. Geotech. J. 25:500–510.

Gibson, R.E., and D.J. Henkel. 1954. Influence of duration of tests at constant rate of strain on measured 'drain' strength. Géotechnique 4:6–15.

Green, R.E., and J.C Corey. 1971. Calculation of hydraulic conductivity: A further evaluation of some predictive methods. Soil Sci. Soc. Am. Proc. 35:3–8.

Hasan, J.V., and D.G. Fredlund. 1980. Pore pressure parameters for unsaturated soils. Can. Geotech. J. 17:395–404.

Hilf, J.W. 1956. An investigation of pore-water pressure in compacted cohesive soils. Ph.D. diss. Tech. Memo. no. 654. U.S. Dep. of the Interior Bureau of Reclamation, Washington, DC.

Ho, D.Y.F., and D.G. Fredlund. 1982a. Strain rates for unsaturated soil shear strength testing. *In* Proceedings 7th South-east Asian Geotechnical Conference. Hong Kong.

Ho, D.Y.F., and D.G. Fredlund. 1982b. A multi-stage triaxial test for unsaturated soils. American Society for Testing and Materials (ASTM) Geotech. Test. J. 5(1/2):18–25.

Holtz, W.G., and W.G Kovacs. 1981. An introduction to geotechnical engineering. Prentice-Hall, Engelwood Cliffs, NJ.

Lambe, T.W. 1951. Soil testing for engineers. Wiley Publications, New York, NY.

Oberg, A., and G. Sallfours. 1997. Determination of shear strength parameters of unsaturated silts and sands based on the water retention curve. American Society for Testing and Materials (ASTM) Geotech. Test. J. 20:40–48.

Satija, B.S., and S.K. Gulhati. 1979. Strain rate for shearing testing of unsaturated soil. p. 83–86. *In* Proceedings 6th Asian Regional Conference Soil Mech. and Found. Eng. Singapore.

Skempton, A.W. 1948. The $\phi_u = 0$ analysis for stability and its theoretical basis. p. 72–77. *In* Second International Conference of Soil Mechanics and Foundation Engineering. Vol. 1.

Skempton, A.W. 1954. The pore pressure coefficients A and B. Geotechnique 4:143–147.

Vanapalli, S.K., and Fredlund, D.G. 2000. Comparison of empirical procedures to predict the shear strength of unsaturated soils uses the soil-water characteristic curve, Geo-Denver 2000. American Society of Civil Engineers, Special Publication. No. 99, pp. 195–209.

Vanapalli, S.K., D.G Fredlund, D.E. Pufahl, and A.W. Clifton. 1996. Model for the prediction of shear strength with respect to soil suction. Can. Geotech. J. 33:379–392.

Vanapalli, S.K., D.E. Pufahl, and D.G. Fredlund. 1999a. Interpretation of the shear strength of unsaturated soils in undrained loading conditions. p. 643–650. 52nd Canadian Geotechnical Conference. Regina. 25–27 Oct. 1999.

Vanapalli, S.K., D.G., Fredlund, and D.E. Pufahl. 1999b. Influence of soil structure and stress history on the soil-water characteristics of a compacted till. Géotechnique 49:143–159.

van Genuchten, M.Th. 1980. A closed-form equation predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. 44:892–898.