Landslides, Bell (ed.) © 1991 Balkema, Rotterdam. ISBN 90 5410 032 X

Use of linear and nonlinear shear strength versus matric suction relations in slope stability analyses

H. Rahardjo
School of Civil and Structural Engineering, Nanyang Technological University, Singapore

D.G. Fredlund & S.K. Vanapalli
Department of Civil Engineering, University of Saskatchewan, Saskatoon, Sask., Canada

ABSTRACT: An extended formulation for slope stability analysis, incorporating the effect of matric suction, is presented in this paper. The influence of nonlinearity in the shear strength versus matric suction relation on the computed factor of safety is demonstrated. An example involving a steep slope in residual soils is used to illustrate the above effects.

1 INTRODUCTION

Many steep slopes have a deep groundwater table. The soil above the groundwater table is unsaturated with pore-water pressures which are negative. Under these conditions, failure surfaces often occur at relatively shallow depths. The pore-water pressures along the failure surface may be negative but have increased in magnitude as a result of rainfalls (Rulon and Freeze, 1985; and Johnson and Sitar, 1990). The shear strength contribution from the negative pore-water pressures may be of significant importance to a stability analysis.

Two stress state variables control the shear strength of an unsaturated soil (Fredlund, Morgenstern and Widger, 1978). The two stress state variables commonly used for an unsaturated soil are the net normal stress, $(\sigma - u_a)$, and the matric suction , $(u_a - u_w)$; where: σ = total stress, u_a = pore-air pressure and u_w = pore-water pressure. The relationship between shear strength and matric suction can be nonlinear (Fredlund, Rahardjo and Gan, 1987). The effect of this nonlinearity on a slope stability analysis for a steep slope is illustrated in this paper. The results show how the computed factor of safety is related to the shear strength mobilized along the slip surface.

2 LINEAR AND NONLINEAR SHEAR STRENGTH VERSUS MATRIC SUCTION RELATIONS

The shear strength of an unsaturated soil can be expressed in the following form (Fredlund, Morgenstern and Widger, 1978):

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \quad [1]$$

where:

τ_{ff} = shear stress at failure

c' = effective cohesion or the shear stress when the net normal stress and the matric suction at failure are equal to zero.

$(\sigma_f - u_a)_f$ = net normal stress at failure

σ_{ff} = total normal stress at failure

ϕ' = angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$

$(u_a - u_w)_f$ = matric suction at failure

ϕ^b = angle indicating the rate of change in shear strength relative to changes in matric suction, $(u_a - u_w)_f$

Eq. [1] corresponds to a planar failure envelope for an unsaturated soil (i.e., extended

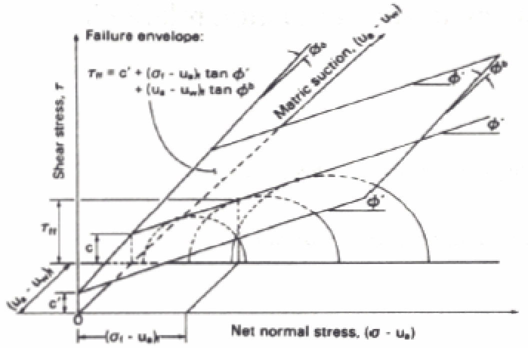

Fig. 1 Extended Mohr-Coulomb failure envelope for an unsaturated soil.

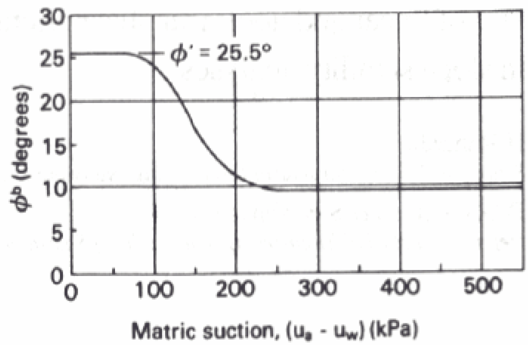

Fig. 2 Failure envelope obtained from unsaturated glacial till specimens.

Mohr-Coulomb failure envelope) as illustrated in Fig. 1. The shear strength of an unsaturated soil is considered to consist of an effective cohesion, c', and the independent contributions from net normal stress, $(\sigma - u_a)$, and the matric suction, $(u_a - u_w)$. The shear strength contributions from net normal stress and matric suction are characterized by ϕ' and ϕ^b angles, respectively.

Recent experimental evidence indicates that there can be a significant nonlinearity in the shape of the failure envelope with respect to matric suction (Donald, 1956; Escario and Saez, 1986; and Fredlund, Rahardjo and Gan, 1987). As an example, the ϕ^b versus matric suction relationship for a compacted glacial till was found to be nonlinear (Gan and Fredlund, 1988) (Fig. 2). The slope of the failure envelopes with respect to the $(u_a - u_w)$ axis (i.e., ϕ^b angle) commence at an angle equal to ϕ' (i.e., 25.5^o) near saturation and significantly decrease at matric suctions in the range of 50 kPa to 100 kPa as indicated in Fig. 2. The ϕ^b angles reach a fairly constant value of 9.5^o when the matric suction exceeds 250 kPa.

Several procedures for handling the nonlinearity of the shear strength versus matric suction relationship have been proposed by Fredlund, Rahardjo, and Gan (1987) (see Fig. 3). A typical nonlinear failure envelope will normally have an ϕ^b angle equal to ϕ' at relatively low matric suctions when the soil remains essentially saturated (i.e., $\overline{AB}$ in Fig. 3). The ϕ^b angle reduces to a lower value than ϕ' when the soil became unsaturated (i.e., $\overline{BD}$ in Fig. 3). The matric suction at point B appears to be correlative with the air entry value of the soil, $(u_a - u_w)_b$.

Two procedures for handling the nonlinear shear strength versus matric suction relationship are suggested and are demonstrated using a slope stability analysis. The first procedure approximates the nonlinear envelope by a linear envelope (i.e., $\overline{AE}$ in Fig. 3) with the lowest measured ϕ^b angle for the entire range of matric suction. This simple procedure gives a lower shear strength than the actual shear strength. The second procedure approximates the nonlinear envelope with a bilinear envelope having two ϕ^b angles (i.e., $\overline{ABD}$ in Fig. 3). The two ϕ^b angles correspond to matric suctions below and above the air entry value of the soil.

3 SLOPE STABILITY INCORPORATING MATRIC SUCTION

The two methods for incorporating matric suction into the slope stability analysis are referred to as the "Total Cohesion" method and the "Extended Shear Strength" method (Rahardjo and Fredlund, 1991). In this paper, the "Extended Shear Strength" method is used to study the effect of the nonlinearity in the shear strength versus matric suction relation on the computed factor of safety.

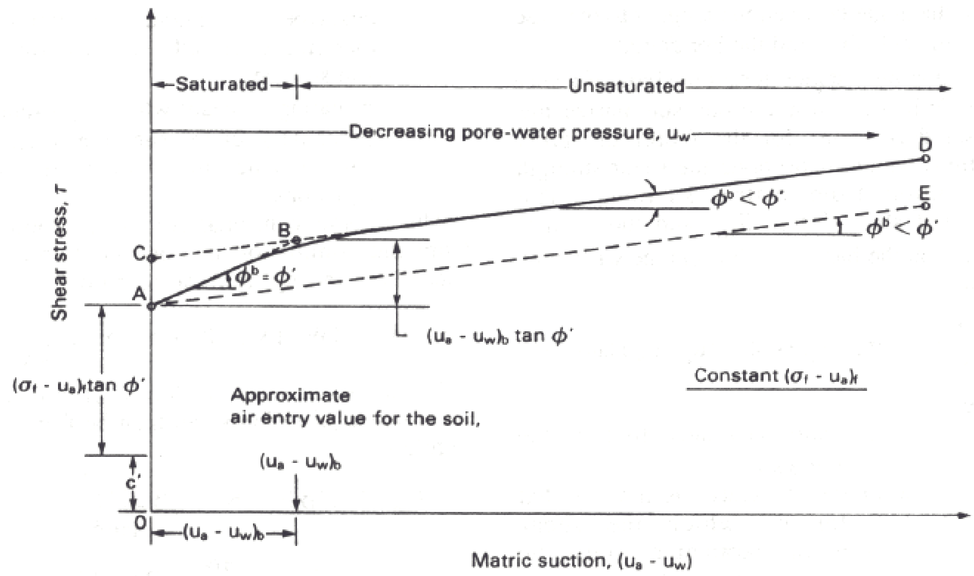

Fig. 3 Several procedures for handling the nonlinearity of the failure envelope.

Conventional limit equilibrium methods of slices for slope stability can be extended to incorporate matric suctions by using the shear strength equation for an unsaturated soil (i.e., Eq. [1]). Consider a typical, steep slope with a deep groundwater table as shown in Fig. 4. The stability calculations for the slope are performed by dividing the soil mass above the slip surface into vertical slices. The forces acting on a slice within the sliding soil mass are shown in Fig. 4. The variables are defined as follows:

W = the total weight of the slice of width "b" and height "h"

N = the total normal force on the base of the slice

S_m = the shear force mobilized on the base of each slice

E = the horizontal interslice normal forces (the "L" and "R" subscripts designate the left and right sides of the slice, respectively)

X = the vertical interslice shear forces (the "L" and "R" subscripts designate the left and right sides of the slice, respectively)

R = the radius for a circular slip surface or the moment arm associated with the mobilized shear force, S_m for any shape of slip surface

f = the perpendicular offset of the normal force from the center of moments

x = the horizontal distance from the center line of each slice to the center of moments

a = the perpendicular distance from a resultant external water force to the center of moments.

A = the resultant external water forces. The "L" and "R" subscripts designate the left and right sides of the slope, respectively

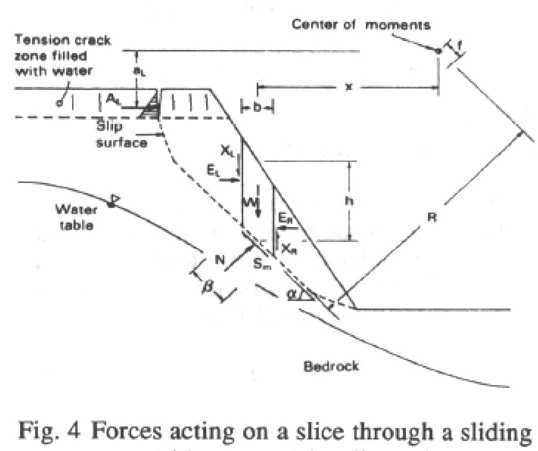

Fig. 4 Forces acting on a slice through a sliding mass with a composite slip surface.

α = the angle between the tangent to the base of each slice and the horizontal.

β = sloping distance across the base of a slice

The following formulations summarize the revised derivations for the factor of safety equations which incorporate the shear strength contribution from matric suction. The mobilized shear force, S_m, and the normal force, N, at the base of a slice can be written as follows:

$$S_m = \frac{\beta}{F}\{c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b\} \quad [2]$$

where:

S_m = the mobilized shear force at the base of a slice

F = factor of safety, defined as that factor by which the shear strength parameters must be reduced to bring the soil mass into a state of limiting equilibrium along the assumed slip surface. The factor of safety for all parameters are assumed to be equal for all soils involved and for all slices.

σ_n = total stress normal to the base of a slice

$(\sigma_n - u_a)$ = net stress normal on the base of a slice

$(u_a - u_w)$ = matric suction on the base of a slice

$$N = \{W - (X_R - X_L) - \frac{c'\beta\sin\alpha}{F} + u_a\frac{\beta\sin\alpha}{F}(\tan\phi' - \tan\phi^b) + u_w\frac{\beta\sin\alpha}{F}\tan\phi^b\}/m_\alpha \quad [3]$$

where:

$m_\alpha = \cos\alpha + (\sin\alpha\tan\phi')/F$

The interslice shear forces, X, in Eq. [3], are computed by assuming that the interslice shear force, X, is related to the interslice normal force, E, by a mathematical function (Morgenstern and Price, 1965):

$$X = \lambda f(x)E \quad [4]$$

where:

f(x) = a functional relationship which describes the manner in which the magnitude of X/E varies across the slip surface

λ = a scaling constant which represents the percentage of the function, f(x), used for solving the factor of safety equations

The interslice normal forces are computed from the summation of horizontal forces on each slice starting from left to right.

$$E_R = E_L + \{W - (X_R - X_L)\}\tan\alpha - \frac{S_m}{\cos\alpha} \quad [5]$$

The factors of safety with respect to moment and force equilibriums can then be formulated as follows:

$$F_m = \Sigma[c'\beta R + \{N - u_w\beta\frac{\tan\phi^b}{\tan\phi'} - u_a\beta(1 - \frac{\tan\phi^b}{\tan\phi'})\}R\tan\phi']/(A_L a_L + \Sigma Wx - \Sigma Nf) \quad [6]$$

where:

F_m = factor of safety with respect to moment equilibrium

$$F_f = \Sigma[c'\beta\cos\alpha + \{N - u_w\beta\frac{\tan\phi^b}{\tan\phi'} - u_a\beta(1 - \frac{\tan\phi^b}{\tan\phi'})\}\tan\phi'\cos\alpha]/(A_L + \Sigma N\sin\alpha) \quad [7]$$

where:

F_f = factor of safety with respect to force equilibrium

In most cases, the pore-air pressure, u_a, is atmospheric (i.e., $u_a = 0$) and as a result, the pore-air pressure terms vanish from the above equations. For a saturated soil condition, the ϕ^b value becomes equal to ϕ' and the above equations revert to the conventional slope stability formulations. Therefore, the above extended equations can be considered as a general formulation that is applicable to both saturated and unsaturated soil conditions.

4 EXAMPLE PROBLEMS

A steep slope of granite colluvium, as shown in Fig. 5, is used as an example problem. The

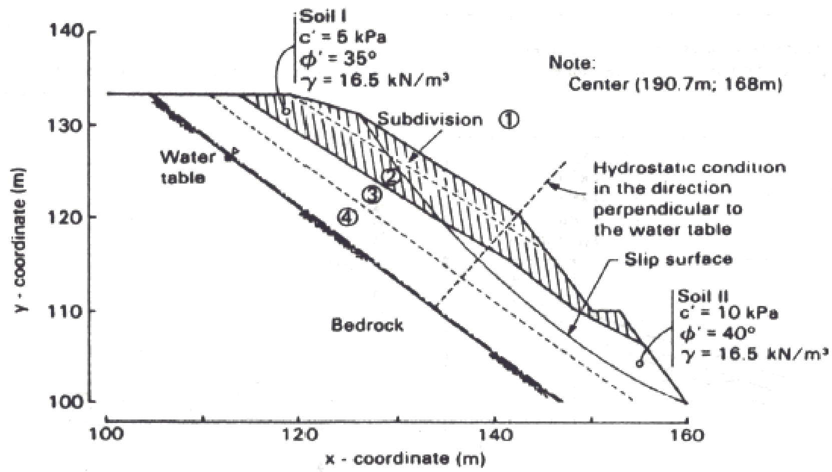

Fig. 5 Example of a typically steep slope (from Sweeney and Robertson, 1979)

slope consists of two soil layers of different properties which are underlain by a deep bedrock. The water table is assumed to be located at the interface between the bedrock and the soil layer. Numerous slope stability analyses have been performed in accordance with the "Extended Shear Strength" method. The analyses were performed using both linear and nonlinear shear strength versus matric suction relations (see Fig. 3). In addition, the soil layers were divided into four subdivisions or strata and a specified slip surface was used for all analyses. All analyses were performed using the PC-SLOPE software* which is programmed to accommodate unsaturated soil conditions with independent ϕ^b angles for each soil strata.

The slope was represented as an infinite slope with each stratum being parallel to the water table. Therefore, a hydrostatic condition of the pore-water pressures can be assumed along a direction perpendicular to the water table. The negative pore-water pressure distribution above the groundwater table was established as being hydrostatic perpendicular to the water table. Pore-water pressure conditions corresponding to 1/2 and 2 times the hydrostatic conditions were also investigated.

----

* PC-SLOPE is a proprietory product of GEO-SLOPE International Ltd., Calgary, Alberta, Canada.

The matric suction in each subdivision was computed from the pore-water pressure at the center of the subdivision and by assuming atmospheric pore-air pressure conditions.

The stability analyses were conducted using a constant interslice force function (i.e., f(x) = 1.0 in Eq. [4]) and λ value was computed such that the moment and force equilibrium factors of safety were equal (i.e., $F = F_m = F_f$).

Fig. 6 presents the results of analyses using a hydrostatic pore-water pressure distribution perpendicular to the water table. The factors of safety are computed using linear and nonlinear shear strength relations versus matric suction and various ϕ^b angles. In all cases, the computed factors of safety increase with increasing ϕ^b values.

In the nonlinear analyses, an air entry value, $(u_a - u_w)_b$, was selected for all soil strata. The linear analyses are equivalent to conditions with a zero air entry value. The results in Fig. 6 indicate that an increase in the air entry value for the soil resulted in a higher shear strength which in turn causes a higher factor of safety. However, the effect of the nonlinearity of the shear strength envelope was most significant at low ϕ^b angles as demonstrated in Fig. 7. As the ϕ^b angle approaches the ϕ' angle, both linear and nonlinear analyses produced an insignificant difference in the computed factors of safety (Figs. 6 and 7).

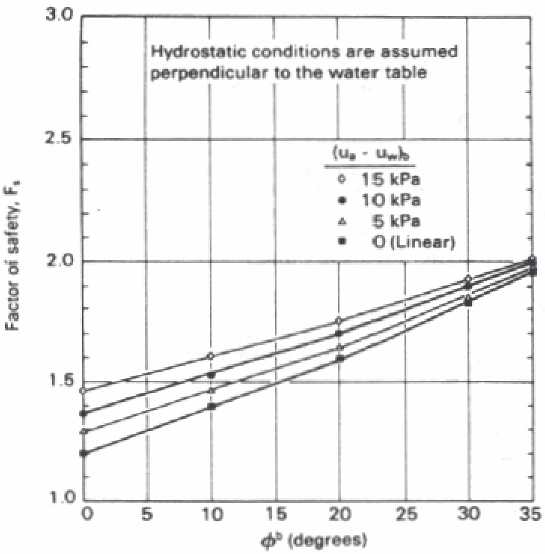

Fig. 6 Factors of safety computed using linear and nonlinear shear strength versus matric suction and hydrostatic pore-water pressure conditions.

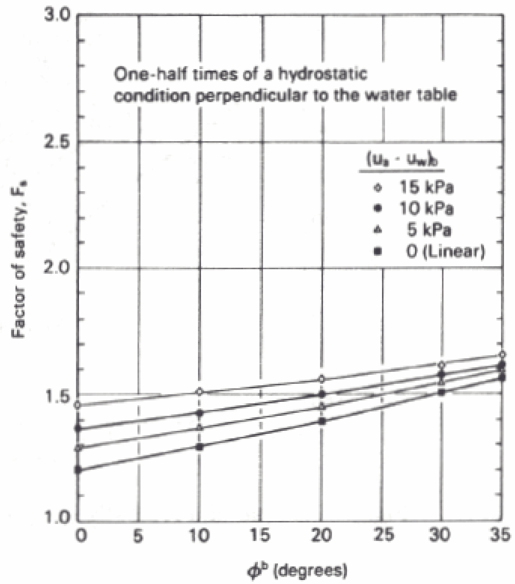

Fig. 8 Factors of safety computed using 1/2 times the hydrostatic conditions.

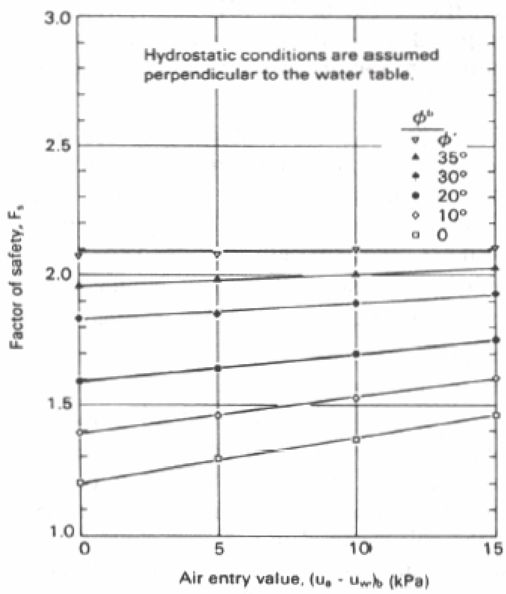

Fig. 7 Effects of various air entry values on the computed factor of safety.

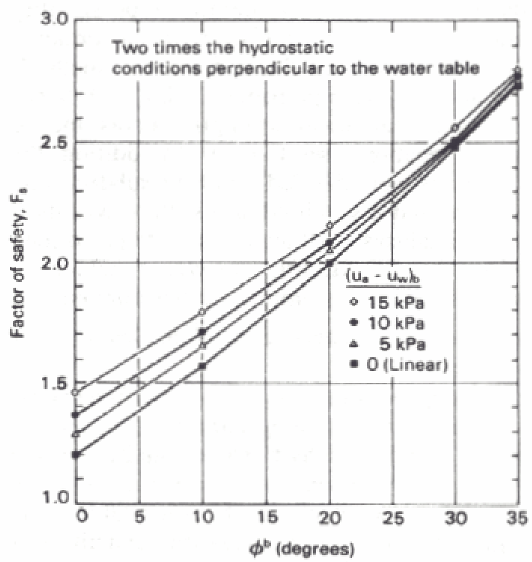

Fig. 9 Factors of safety computed using 2 times the hydrostatic conditions.

Similar patterns of results were obtained from the analyses using 1/2 and 2 times the hydrostatic pore-water pressure distribution along a line perpendicular to the water table as shown in Figs. 8 and 9, respectively. The

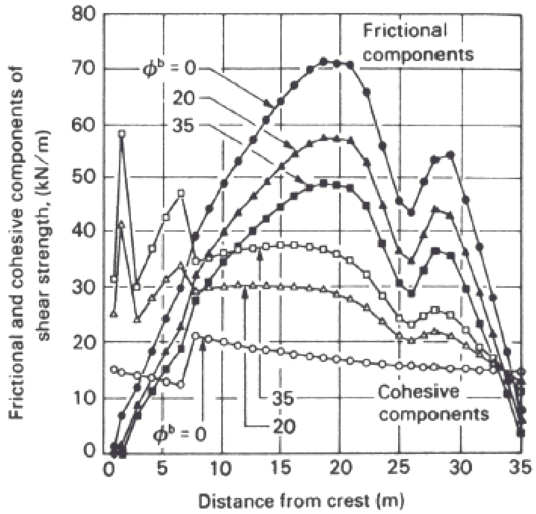

Fig. 10 Strength components under hydrostatic pressure conditions with an air-entry value of 10 kPa.

results show that the effect of the ϕ^b angle on the factor of safety is more pronounced at high values of matric suction (Fig. 9) as compared to low matric suction conditions (Fig. 8). However, the effect of the air entry value (or the nonlinear failure envelope) on the factor of safety is essentially the same for various distributions of matric suction.

Fig. 10 illustrates the contribution of matric suction to the mobilized shear strength. An increase in the ϕ^b angle results in an increase in the cohesive component of shear strength. The frictional component decreases due to the influence of ϕ^b on the normal force at the base of each slice.

5 CONCLUSIONS

Slope stability analyses using linear and nonlinear shear strength versus matric suction relations can be performed in accordance with the "Extended Shear Strength" method. The linear analyses under-estimate the soil shear strength and therefore, result in the lowest factor of safety. The effect of the nonlinearity of the shear strength relationship on the computed factor of safety is most significant when there is a large change in the ϕ^b angle.

REFERENCES

Donald, I.B. 1956. Shear strength measurements in unsaturated non-cohesive soils with negative pore pressures. *Proc. of the 2nd Australian - New Zealand Conf. on Soil Mechanics and Found. Eng., The New Zealand Inst. of Engineers,* Christchurch, N.Z.

Escario, V. and J. Saez 1986. The shear strength of partly saturated soils. *Geotechnique* 36(3).

Fredlund, D.G., N.R. Morgenstern, and R.A. Widger 1978. The shear strength of unsaturated soils. *Can. Geotech. J.* 15: 313-321.

Fredlund, D.G., H. Rahardjo, and J.K.M. Gan 1987. Nonlinearity of strength envelope for unsaturated soils. *Proc. 6th Int. Conf. Expansive Soils*: 49-54. New Delhi, India.

Gan, J.K.M., and D.G. Fredlund 1988. Multistage direct shear testing of unsaturated soils. *Geotech. Testing J.*, 11(2), 132-138.

Johnson, K.A. and N. Sitar 1990. Hydraulic conditions leading to debris-flow initiation. *Can. Geotech. J. 27:789-801.*

Morgenstern, N.R. and V.E. Price 1965. The analysis of the stability of general slip surfaces. *Geotechnique* 15: 79-93.

Rahardjo, H. and D.G. Fredlund 1991. Calculation procedures for slope stability analyses involving negative pore-water pressures. *Int. Conf. on Slope Stability Eng. Developments and Application,* April 15-19, U.K.

Rulon, J.J. and R.A. Freeze 1985. Multiple seepage faces on layered slopes and their implications for slope stability analysis. *Can. Geotech.* 22: 347-356.

Sweeney, D. and P. Robertson 1979. A fundamental approach to slope stability in Hong Kong. *The Hong Kong Engineer*, Hong Kong, 35-44.