SHEAR STRENGTH BEHAVIOR OF A SILTY SOIL OVER THE SUCTION RANGE FROM 0 TO 1,000,000 kPa

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ABSTRACT Several semi-empirical models are available in the literature to predict the shear strength of an unsaturated soil. These models use the saturated shear strength parameters and the soil-water characteristic curve in the prediction of shear strength for a limited suction range (i.e., between 0 to 500 kPa). In this paper, the shear strength behavior of a silty soil is interpreted using conventional, unconfined compressive shear strength tests for the entire suction range of 0 to 1,000,000 kPa (i.e., from a fully saturated condition to a total dry condition). The unconfined compressive strength test results were compared using a prediction procedure developed at the University of Saskatchewan that uses the entire soil-water characteristic curve and the saturated shear strength parameters. There is a good comparison between the predicted and measured values of shear strength for a large suction range for the tested silty soil. The results of this study are encouraging as simple, unconfined compression strength tests can be used for interpreting the shear strength behavior of fine-grained, unsaturated soils.

1. INTRODUCTION

Several semi-empirical models were proposed in the last five years to predict the drained, shear strength behavior of unsaturated soils (Vanapalli et al. 1996, Fredlund et al. 1996, Oberg and Sallfors 1997, Khallili and Khembaz 1998, and Bao et al. 1998). The philosophy for predicting the shear strength of unsaturated soils used in each of these models is slightly different. All the models use the soil-water characteristic curve as a tool, either directly or indirectly, along with the saturated shear strength parameters to predict the shear strength. The soil-water characteristic curve defines the relationship between the soil suction and gravimetric water content, \( \theta \), or the degree of saturation, \( S \). Figure 1 shows the entire soil-water characteristic curve in which the soil suction varies from 0 to 1,000,000 kPa (i.e., from fully saturated conditions to completely dry conditions).

Various types of soils have been studied and comparisons have been made between the predicted and measured shear strength values by several investigators for a limited suction range (i.e., between 0 to 500 kPa) (Vanapalli et al. 1996, Fredlund et al. 1996, Khallili and Khembaz 1998, Bao et al. 1998, Russam and Williams, 1999). Typically, the soil suction range of 0 to 500 kPa forms the boundary effect stage and the transition stage of the soil-water characteristic curve for several fine-grained soils (Figure 1). Experimental studies by several investigators have shown that the variation of shear strength with respect to soil suction is linear in the boundary effect stage and non-linear in the transition stage irrespective of the type of soil tested. The movement of water in both the boundary effect stage and the transition stage is in the liquid phase. Soils are in a residual effect stage at low degrees of saturation and soil suction values are relatively high (Figure 1). The movement of water in this stage may be both in the liquid and as well as the vapor phase. Graphical construction procedures and computing techniques are available to identify the various stages from the entire soil-water characteristic curve data (Vanapalli et al. 1996, Vanapalli et al. 1999). There is limited information in the literature with respect to the shear strength behavior in the residual effect stage (Escario and Juca 1989, Nishimura and Fredlund 2000).

Shear strength behavior in the residual effect stage is of importance to geotechnical and geo-environmental engineers for addressing certain practical problems. For example, suction in a soil cover can be greater than 3,000 kPa. The suction range from 1,000 to 4,000 kPa generally
falls in the residual effect stage for many soils. The engineering properties such as the shear strength and the coefficient of permeability at such high values of suction are of practical interest in the design of soil covers. Shallow slope failures in coarse-grained and silty soils with low water contents that represent high values of suction are another example.

In this paper, the shear strength behavior of a silty soil is interpreted using conventional, unconfined compressive shear strength tests for the entire suction range from 0 to 1,000,000 kPa (i.e., from a fully saturated condition to a total dry condition). The Fredlund et al. (1978) equation is extended for the interpretation of the test results. The

unconfined compressive strength test results were compared with a prediction procedure developed at the University of Saskatchewan. The entire soil-water characteristic curve and the saturated shear strength parameters are used in this procedure for predicting the shear strength (Vanapalli et al. 1996 and Fredlund et al. 1996). There is a good comparison between the predicted and measured values of shear strength for a large suction range for the tested silty soil. The results of this study are encouraging to practicing engineers as simple, unconfined compression tests can be used for interpreting the shear strength behavior of fine-grained, unsaturated soils for a large range of suction.

![Typical soil-water characteristic curve showing various stages.](image)

**FIGURE 1.** Typical soil-water characteristic curve showing various stages.

2. BACKGROUND

Fredlund et al. (1978) have proposed a relationship to explain the shear strength of unsaturated soils in terms two independent stress state variables as shown below:

\[
\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b
\]

where

- \(\tau\) = shear strength of unsaturated soil;
- \(c'\) = effective cohesion;
- \(\phi'\) = effective angle of frictional resistance for a saturated soil;
- \(\phi^b\) = angle of frictional resistance with respect to soil suction;
- \((\sigma_n - u_a)\) = net normal stress;
- \(u_a\) = pore-air pressure;
- \(u_w\) = pore-water pressure; and
- \((u_a - u_w)\) = matric suction.

Experimental studies using modified direct shear tests and modified triaxial shear tests can be undertaken to interpret the shear strength of unsaturated soils using Equation [1] (Fredlund and Rahardjo, 1993). This equation was found to be valid for explaining the shear strength of unsaturated soils that include compacted soils, residual soils and expansive soils (Gan et al. 1988, Abramento and Carvalho 1989, Drumright 1989, Vanapalli 1994, Iyer and Williams, 1995, Rahardjo et al. 1995, Bao et al. 1998, Khalili and Khabbaz 1998 and Russam and Williams 1999).

Vanapalli et al. (1996) and Fredlund et al. (1996) have proposed a function for predicting the shear strength of an unsaturated soil using the entire soil-water characteristic curve (i.e., 0 to 1,000,000 kPa) and the saturated shear strength parameters as shown below:
\[ \tau = [\kappa + (\sigma_n - u_a)\tan \phi^b] + \left[(u_a - u_w)\Theta^2\right][\tan \phi^b] \] \tag{2}

where:

- \( \kappa \) = fitting parameter used for obtaining a best-fit between the measured and predicted values; and
- \( \Theta \) = normalized volumetric water content, \( \theta_w/\theta_s \).

Equation [2] can also be written in terms of degree of saturation, \( S \), or gravimetric water content, \( w \), to predict the shear strength yielding similar results. The normalized volumetric water, \( \Theta \), is also equal to degree of saturation, \( S \).

The philosophy behind the interpretation of the shear strength of unsaturated soils using Equation [2] is similar to Equation [1]. The following relationship can be obtained by comparing Equations [1] and [2], with the realization that \( \phi^b \) is not a single value.

\[ \tan \phi^b = \Theta^2 \left[\tan \phi^b\right] \] \tag{3}

A best-fit soil-water characteristic curve for the entire range of suction can be obtained in terms of \( a, n, \) and \( m \) parameters using the equation proposed by Fredlund and Xing (1994) which is shown below:

\[ \theta_w(\psi) = \theta_s - \left[\ln \left(1 + \frac{\psi}{h_r}\right) - \frac{1}{\ln \left(1 + 10^6\right)} \ln \left(\exp(1) + \left(\frac{\psi}{a}\right)^n\right)^m\right] \] \tag{4}

where:

- \( \psi \) = soil suction;
- \( \theta_w \) = volumetric water content;
- \( \theta_s \) = saturated volumetric water content;
- \( a \) = suction related to the inflection point on the curve;
- \( n \) = soil parameter related to slope at the inflection point;
- \( m \) = soil parameter related to the residual water content; and
- \( h_r \) = suction related to the volumetric residual water content, \( \theta_r \).

Vanapalli and Fredlund (2000) in a recent study have shown comparisons between the measured and predicted values of shear strength for a large suction range that includes residual effect stage for several soils using Equation [2]. The analyses of the results have shown that the shear strength of an unsaturated soil can be predicted with a reasonable degree of accuracy for a large suction range using Equation [2]. A relationship between the fitting parameter, \( \kappa \), and the plasticity index, \( I_p \), was also proposed based on this study (Figure 2). From this relationship, the fitting parameter \( \kappa \) can be estimated and used in Equation [2] to predict the shear strength of an unsaturated soil. While these studies are useful, it is important to have more experimental data to better understand the shear strength behavior of various types of soils from a fully saturated condition to a total dry condition that covers all the stages (Figure 1).


![FIGURE 2. The relationship between the fitting parameter, \( \kappa \), and the plasticity index, \( I_p \). (from Vanapalli and Fredlund, 2000).](image)
study of the shear strength behavior for a large suction range (Escario and Juca, 1989 and Nishimura and Fredlund 2000). It is time consuming and complex to measure the shear strength data that includes the residual effect stage. The coefficient of permeability of unsaturated soils are extremely low at high values of suction and hence a long period of time is required to come to equilibrium conditions before the specimens can be sheared to measure the strength of an unsaturated soil. This is true irrespective of the type of soil (i.e., coarse grained and fine-grained soils). Laboratories may not have the necessary equipment and facilities to undertake experimental studies at such high values of suction. It would be both practical and useful to propose simple experimental procedures to understand the shear strength behavior for the entire range of suction (i.e., 0 to 1,000,000 kPa).

3. THEORY

The Fredlund et al (1978) shear strength equation (i.e., Equation 1) can be extended for interpreting the triaxial shear strength test results using the equation given below.

\[
\frac{\sigma_1 - \sigma_3}{2} = c' \cos \phi' + \left( \frac{\sigma_1 + \sigma_3}{2} - u_a \right) \sin \phi' + (u_a - u_u) \tan \phi' \cos \phi' \]


\[
\frac{\sigma_1 - \sigma_3}{2} = c' \cos \phi' + \left( \frac{\sigma_1 + \sigma_3}{2} - u_a \right) \sin \phi' + (u_a - u_u) \left( \Theta^* \right) (\tan \phi' \cos \phi') \]

Equation [7] can be used for interpreting unconfined compression strength test results. This equation is obtained by setting \( \sigma_3 \) equal zero and \( u_3 \) equal zero to simulate unconfined compression strength test results in Equation [6].

\[
c_u = \frac{\sigma_1}{2} = c' \cos \phi' + (u_a - u_u) \left( \Theta^* \right) (\tan \phi' \cos \phi') \left( 1 - \sin \phi' \right) \]

where:

- \( c_u \) = unconfined compressive shear strength.

Comparisons between the predicted and measured unconfined compression shear strength tests can be provided using Equation [7]. Conventionally, modified triaxial shear testing equipment or modified direct shear testing equipment is used to measure the shear strength of unsaturated soils (Fredlund and Rahardjo 1993). The theory presented in this section facilitates to determine the shear strength of unsaturated soils using conventional unconfined compression test equipment. These tests can be conducted safely and quickly, and can be undertaken in all soil mechanics laboratories.

The required data for undertaking the analysis would include the entire soil-water characteristic curve, saturated shear strength parameters, \( c' \) and \( \phi' \), initial suction values in the unsaturated soil specimens used for testing unconfined compression tests, and the unconfined compression strength test results. The procedure for estimating the suction in the unsaturated soil specimens is detailed in the experimental program section. The theory presented in this section is based on the assumption that the initial soil suction in the unsaturated soil specimens does not appreciably change during the shearing stage. In other words, the interpretation is based on initial suction values. This assumption can be said to be valid good with a reasonable degree of accuracy for fine-grained soils.

4. EXPERIMENTAL PROGRAM

The primary objective of the experimental program was to understand the effect of suction on the shear strength behavior of a silty soil using conventional, unconfined compression shear strength tests over the entire suction range. The secondary objective was to examine the validity of the proposed theory presented in this paper and provide comparisons between the measured and predicted shear strength values of the unconfined compression test results.

![Grain size analysis for Botkin soil.](image)
The plasticity index, \( I_p \), of the soil tested in the study; called Botkin silt was equal to 8%. The soil had sand, silt and clay equal to 27.5, 48.5 and 24 percentages respectively. The grain size analysis of the soil is shown in Figure 3. The experimental program includes the determination of the saturated shear strength parameters, determination of the entire soil-water characteristic curve using a Tempe cell apparatus and desiccators, and the unconfined compressive shear strength of silt specimens over the entire suction range.

### 4.1 Unconfined compressive shear strength

The unconfined compressive strength of statically compacted silty soil specimens with a dry density of 1.85 Mg/m\(^3\) were proposed to be tested for the research study. The water content required to prepare these specimens with the pre-decided dry density (i.e., 1.85 Mg/m\(^3\)) was calculated from volume-mass relationships and added to the silty soil. The water content was chosen such that the specimens are initially in a state of fully saturated condition. The pre-calculated water content was added to the soil mass and wrapped in a plastic bag. The prepared soil sample was placed in a humidity-controlled room for a period of 24 hours to achieve uniform water content conditions. The specimens for testing unconfined compression strength values were statically compacted in six equal layers using specially designed molds with spacers (Figure 4). The compacted specimens were 50 mm in diameter and 104 mm height. The statically compacted saturated specimens were then subjected to different levels of air drying such that the suction in the specimens increases (i.e., the degree of saturation in the compacted specimens will vary). The compacted specimens were wrapped in plastic sheets and placed in a humidity-controlled room. The water content and suction attained equilibrium conditions throughout the specimens over a period of four days. The water content and the degree of saturation in the compacted specimens were measured from volume-mass relationships.

The initial suction value in the compacted specimens was estimated from soil-water characteristic curve data. The soil-water characteristic curve was measured along the drying path for the entire suction range using an “identical” soil specimen. More details of the procedure used for the determination of the soil-water characteristic curve are available in a separate section. Figure 5 demonstrates the procedure for estimating the soil suction in the specimens by determining the degree of saturation in the specimens from volume-mass properties.

The specimens were sheared in a conventional unconfined compression testing loading frame at a rate of 1.2 mm/min. All the specimens after testing were cut into two halves at approximately mid-height of the specimen and the water content was determined. There was negligible or no variation in the water content of both halves of the same specimen. Hence, it can be assumed that the suction value in the specimen is uniform throughout the specimen. Some variation in the water content was observed in specimens that were air-dried for a longer period of time to achieve low water content conditions in the specimens to simulate high suction values.

![FIGURE 4. Specially designed molds for preparing specimens for unconfined compression testing.](image)

![FIGURE 5. Indirect procedure for estimating the soil suction in the specimens knowing the degree of saturation.](image)

### 4.2 Soil-water characteristic curve

A saturated soil specimen was prepared under “identical” conditions with respect to dry density and water content as discussed in the earlier section to determine the soil-water characteristic curve. Two sets of equipment were used to determine the entire soil-water characteristic curve for the suction range of 0 to 1,000,000 kPa. The modified pressure plate apparatus (i.e., Tempe cell) was used to measure the soil-water characteristic curve for a suction range of 0 to 300 kPa (Figure 6). The prepared saturated soil specimen was placed on the saturated high air-entry ceramic disk in a sealed air-pressure chamber of the Tempe cell apparatus. The chamber was pressurized to a desired value of suction. Equilibrium condition was assumed in the specimen under the applied suction when no water discharged from the Tempe cell. The amount of water discharged from the soil specimen was determined from the mass determinations of the Tempe cell. The soil specimen was removed from the Tempe cell after attaining equilibrium conditions under a suction value of 300 kPa. Small blocks (pieces) of soil specimen were cut from the tested specimen that was removed from the Tempe cell and were placed in glass desiccators under controlled environment conditions (Figure 7). A series of glass desiccators with different salt solutions...
of known concentrations were used. The degree of saturation or water content was determined in the specimen after it had reached to equilibrium conditions with the atmosphere in the desiccator. More details of the experimental procedure for measuring the entire soil-water characteristic curve are available in Vanapalli et al. 1999. The measured soil-water characteristic curve is shown in Figure 8.

4.3. Saturated shear strength parameters

The saturated shear strength parameters for the silty soil were determined under consolidated drained tests using the direct shear equipment. The soil specimens used for testing in the direct shear equipment were prepared using similar procedures detailed in earlier section to prepare specimens for determining the soil-water characteristic curve. The saturated shear strength parameters, \( c' \) and \( \phi' \) for the tested soil were equal 14.2 kPa and 36.5° (Figure 9).
5. PRESENTATION AND DISCUSSION OF RESULTS

Figure 10 provides a comparison between the measured and predicted unconfined compression strength test results of the tested silty soil. Symbols represent the measured shear strength values from the unconfined compression tests and the continuous line represents the predicted unconfined compressive strength. The predicted unconfined compression test values were obtained using Equation [7]. A fitting parameter value, \( \kappa \) equal 1.8 is used in the study. This value was obtained from Figure 2. There is a reasonably good comparison between the measured and predicted shear strength values in the boundary effect stage and the transition stage (i.e., suction range between 0 to 10,000 kPa). There was no defined trend in the strength data measured in the specimens with very high suction values (i.e., greater than 100,000 kPa). These results are not reported in the paper. This behavior could be attributed to the variation in water content values observed in two halves each of specimen tested with high suction values. The time period of four days may not have been sufficient to attain equilibrium conditions in the specimens with lower water contents (i.e., high suction values).

The results suggest that the Fredlund et al. (1978) et al. equation (i.e., Equation 1) can be used for the interpretation of the shear strength of soils from fully saturated conditions to total dry conditions (i.e., for the entire suction range of 0 to 1,000,000 kPa). Detailed theoretical explanations with respect to using the shear strength theory for the entire range of suction is available in Vanapalli et al. 1998. The unconfined compression test results in this study were interpreted assuming that there will be insignificant changes in the initial suction values in the tested specimen due to shearing. Such an assumption can be said to be valid for fine-grained soil specimens.
6. SUMMARY AND CONCLUSIONS

A simple procedure is presented in this paper to interpret the shear strength behavior of an unsaturated soil for a large suction range using conventional, unconfined compressive strength tests. The information required for undertaking the analysis requires the determination of the saturated shear strength parameters, c’ and φ’, the soil-water characteristic curve for the entire suction range and the unconfined compressive strength results. The presented theory is verified by undertaking an experimental program on a statically compacted silty soil. Comparisons between the measured and predicted values of unconfined compression strength data for the tested soil were undertaken using Equation [7]. The analysis of the results based on the assumption that the suction changes in the specimen are not significant during the shearing stage. There is a reasonably good comparison between the predicted and measured results of unconfined compression strength values. The relationship between the κ versus \( I_p \) was found to be valid for the tested silty soil (Figure 2). It is expected that better comparisons would be possible between the measured and predicted values of unconfined compressive shear strength results for fine-grained soils with higher plasticity index, \( I_p \) values. This is due to the reason that the changes in suction values of the specimen before and after failure conditions would not be significant for fine-grained soils with higher plasticity values. The results of the study presented in this paper are encouraging to practicing geotechnical engineers and geo-environmental engineers to put the theories related to the shear strength of unsaturated soils into practice. More studies are necessary on different types of soils to further validate the theory presented in this paper and better understand the shear strength behavior of soils for the entire suction range of 0 to 1,000,000 kPa.

7. REFERENCES


