A SIMPLE EXPERIMENTAL PROCEDURE FOR DETERMINING THE FITTING PARAMETER, κ FOR PREDICTING THE SHEAR STRENGTH OF AN UNSATURATED SOIL

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ABSTRACT

The shear strength of an unsaturated soil can be predicted with a semi-empirical shear strength function developed at the University of Saskatchewan both for low and as well as large suction ranges. A fitting parameter, κ is necessary in this function to provide comparisons between the predicted and measured shear strength values. A relationship is recently proposed between the fitting parameter, κ and the plasticity index, I_p. This relationship is useful to determine the required fitting parameter κ value and use in the function for predicting the shear strength. The I_p *vs* κ relationship is proposed using the available experimental results in the literature on statically compacted specimens. The validity of using the κ value from this relationship for different types of natural soils and compacted conditions is not well known. In this paper, a simple experimental procedure is proposed for determining the fitting parameter, κ value. This procedure requires the results of unconfined compressive shear strength along with the matric suction value of the unsaturated soil specimen. Using the value of fitting parameter, κ determined from this simple experimental procedure, comparisons were provided between the predicted and measured shear strength values for clay till specimens that were compacted at three different initial water content conditions representing dry of optimum, optimum, and wet of optimum conditions. There is a reasonably good comparison (i.e., with in a percentage of accuracy of 10 to 15%) between the predicted and measured shear strength values of clay till specimens.

RÉSUMÉ

La résistance au cisaillement d'un sol non saturé peut être prédite à l'aide d'une fonction semi-empirique de résistance au cisaillement développée à l'Université du Saskatchewan pour des succions faibles et élevées. Cette fonction nécessite un paramètre d'ajustement κ pour comparer les valeurs prédites et mesurées de la résistance au cisaillement. Une relation a été récemment proposée entre le paramètre κ et l'index de plasticité I_p. Cette relation est utile pour déterminer la valeur du paramètre d'ajustement κ qui est requise pour prédire la résistance au cisaillement. La relation proposée entre I_p et κ est basée sur des résultats expérimentaux disponibles dans la littérature pour des spécimens compactés de façon statique. La validité des valeurs de κ obtenues par cette relation pour différents types de sols naturels et conditions de compaction n'est pas bien connue. Dans cet article, on propose une simple procédure expérimentale pour déterminer la valeur du paramètre d'ajustement κ. Cette procédure requiert les résultats d'essais de résistance à la compression et de succion pour un spécimen de sol non saturé. On a comparé les valeurs mesurées de la résistance au cisaillement et celles prédites en utilisant le paramètre κ déterminé au moyen de cette simple procédure expérimentale pour des spécimens de till argileux. Les spécimens ont été compactés de façon à obtenir trois valeurs initiales différentes de teneur en eau : plus sec qu'optimum, optimum, et plus humide qu'optimum. L'accord entre les valeurs prédites et mesurées de la résistance au cisaillement est raisonnablement bon (La marge d'erreur varie entre 10 et 15%).

1. INTRODUCTION

Shear strength forms an important engineering property in the design of numerous geotechnical and geo-environmental structures such as earth dams, retaining walls, pavements, soil liners and soil covers. These structures are typically in a state of unsaturated condition and often do not become saturated during their design life period. Some soil structures such as soil covers are constructed to remain in an unsaturated state during their entire design life period. Thus, the engineering properties of unsaturated soils such as the shear strength and the coefficient of permeability are relevant in the design of several geotechnical and geo-environmental structures.

Several semi-empirical shear strength functions were proposed to predict the shear strength of unsaturated soils in recent years (Vanapalli et al. 1996a, Fredlund et al. 1996, Oberg and Sallfors 1997 and Khallili and Khabbaz 1998, Vanapalli and Fredlund, 1999). The soil-water characteristic curve is used as a tool either directly or indirectly along with the effective shear strength parameters in the proposed functions for predicting the shear strength. The soil-water characteristic curve defines the relationship between the soil suction and gravimetric water content, w or the volumetric water content, θ, or the degree of saturation, S. Typically, comparisons between predicted and measured values are provided using the shear strength functions for a suction range between 0 to 500 kPa (Vanapalli et al. 1996a, Oberg and Sallfors 1997, Khallili and Khabbaz 1998, Bao et al. 1998). Experimental studies are commonly undertaken in a suction range of 0 to 500 kPa as experimental procedures for testing the shear strength of unsaturated soils are elaborate and time consuming. Also, in many practical situations engineers are interested in the engineering of behavior of unsaturated soils in the low suction range (i.e., 0 to 500 kPa).

Vanapalli and Fredlund (2000) in a recent study have provided comparisons between the measured and predicted values of unsaturated shear strength using the shear strength functions published in the literature. Comparisons were provided both for low suction range (i.e., 0 to 1,500 kPa) as well as large suction range (0 to 10,000 kPa or higher) using the results of Escario and Juca (1989). The three soils used in the study have different gradation properties, percentage of clay and plasticity index, I_p values. These results were published in the literature prior to the development of the shear strength functions. The studies show that the shear strength function developed at the University of Saskatchewan provide good comparison between the predicted and measured shear strength values, both for low and large suction ranges. A fitting parameter, κ is however necessary in the University of Saskatchewan shear strength function to provide comparisons between the predicted and measured shear strength values.

Vanapalli and Fredlund (2000) proposed a relationship between the fitting parameter, κ and the plasticity index, I_p. This relationship is useful to determine the required fitting parameter κ value that can be used in the shear strength function for predicting the drained shear strength of an unsaturated soil using the soil-water characteristic curve and the effective shear strength parameters (i.e., c' and ϕ'). The I_p *vs* κ relationship is proposed using the available experimental results in the literature on statically compacted specimens. The validity of using the κ value from this relationship for different types of natural soils and compacted conditions is not known.

In this paper, a simple experimental procedure is proposed for determining the fitting parameter, κ value. This procedure requires the results of conventional unconfined compressive shear strength along with the matric suction value of the unsaturated soil specimen and the effective shear strength parameters. Experimental studies were undertaken on clay till specimens compacted at three different initial water content conditions representing dry of optimum, optimum, and wet of optimum conditions to verify this procedure. The fitting parameter, κ values determined using this technique were consistent with the I_p vs κ relationship. Comparisons were also provided between the predicted and measured unconfined shear strength values. There is a reasonably good comparison (i.e., with in a percentage of accuracy of 10 to 15%) between the predicted and measured shear strength values of clay till specimens.

2. BACKGROUND

Fredlund et al. (1978) have proposed a relationship to explain the shear strength of unsaturated soils in terms of two independent stress state variables as shown below:

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [1]$$

where

τ = shear strength of unsaturated soil;
c' = effective cohesion;
ϕ' = effective angle of frictional resistance for a saturated soil;
ϕ^b = angle of frictional resistance with respect to soil suction;
$(\sigma_n - u_a)$ = net normal stress;
u_a = pore-air pressure;
u_w = pore-water pressure; and
$(u_a - u_w)$ = matric suction.

Experimental studies using modified direct shear tests and modified triaxial shear tests can be undertaken to interpret the shear strength of unsaturated soils using Equation [1] (Gan et al. 1987, Fredlund and Rahardjo, 1993). Experimental studies to determine the shear strength of unsaturated soils using modified direct shear and triaxial shear tests are however time consuming and expensive.

Vanapalli et al. (1996a) and Fredlund et al. (1996) proposed a function for predicting the shear strength of unsaturated soils. The soil-water characteristic curve and the effective shear strength parameters are required for predicting the shear strength. This function is referred to as University of Saskatchewan procedure in this paper and is given below:

$$\tau = [c' + (\sigma_n - u_a)\tan\phi'] + \left[(u_a - u_w)\left\{\left(\Theta^{\kappa}\right)(\tan\phi')\right\}\right] \qquad [2]$$

where:

κ = fitting parameter used for obtaining a best-fit between the measured and predicted values; and
Θ = normalized volumetric water content, θ_w/θ_s
θ_w= volumetric water content
θ_s= volumetric water content under fully saturated condition.

Equation [2] can also be written in terms of degree of saturation, S, or gravimetric water content, w, to predict the shear strength yielding similar results. The normalized volumetric water, Θ, is also equal to degree of saturation, S. The philosophy behind the interpretation of the shear strength of unsaturated soils using Equation [2] is similar to Equation [1]. The following relationship can be obtained by comparing Equations [1] and [2], with the realization that ϕ^b is not a single value.

$$\tan\phi^b = \left(\Theta^\kappa\right)\left(\tan\phi'\right) \quad [3]$$

Figure 1 shows the relationship between the fitting parameter, κ, and plasticity index, I_p (Vanapalli and Fredlund, 2000). This relationship is useful to determine the required fitting parameter κ value in Eqn. [2]. This relationship is developed using the shear strength test results undertaken with statically compacted specimens.

Vanapalli et al. (2000) extended Equation [2] for interpreting unconfined compression strength test results as given below:

$$c_u = \frac{\sigma_1}{2} = \frac{c'\cos\phi' + (u_a - u_w)\left\{\left(\Theta^\kappa\right)\left(\tan\phi'\right)\right\}\cos\phi'}{\left(1 - \sin\phi'\right)} \quad [4]$$

where:
c_u = unconfined compressive shear strength.

The water content in the unconfined compression testing specimens subjected to shearing will not change during the testing period. This is due to the faster rate of shearing of the unconfined soil specimens (i.e., the soil specimen is typically sheared in a couple of minutes). The analyses of unconfined compressive strength results using Eqn. [4] is based on the assumption that the suction changes in the specimen are not significant during the shearing stage. In other words, it is assumed that the shear strength contribution due to suction is approximately the same both under drained and undrained loading conditions. This assumption is reasonably valid for fine-grained soils as suction in soils is dependent on water content and the water content in the specimens does not change during the shearing stage. More details about this simplified approach are available in Vanapalli et al. (2000).

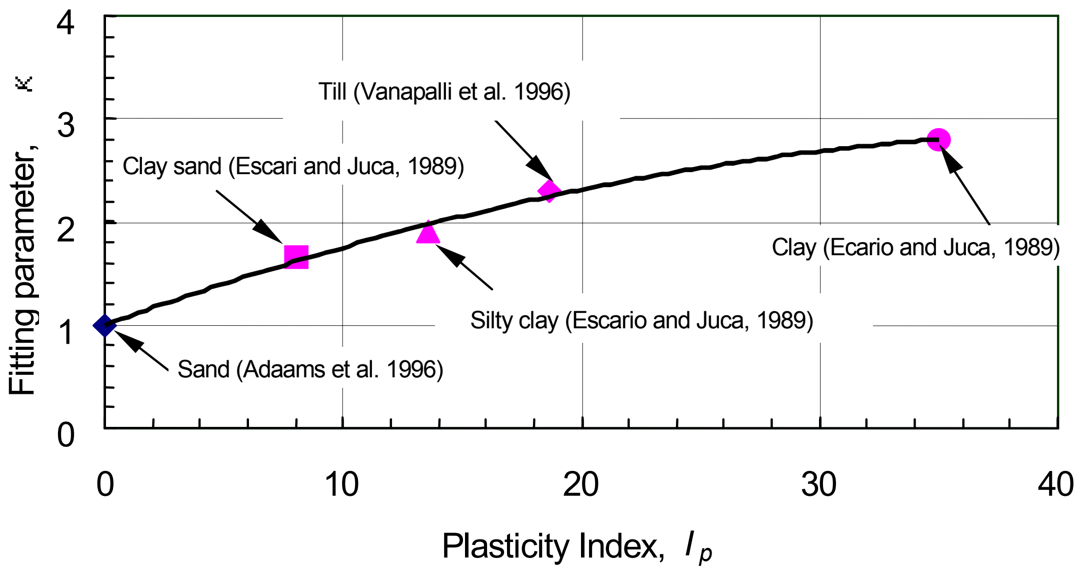

FIGURE 1. The relationship between the fitting parameter, κ, and the plasticity index, I_p. (from Vanapalli and Fredlund, 2000).

Studies undertaken on a silty soil and a clay till for a large suction range using Eqn. [4] have provided good comparisons between the predicted and measured strength values of unconfined compressive strength using Eqn. [4] (Vanapalli et al. 2000 and Remiul 2001). The fitting parameter, κ value for both these soils was estimated from plasticity index values, I_p using Figure 1.

The required data for undertaking the analysis for this approach (i.e., Eqn. 4) are the soil-water characteristic curve, effective shear strength parameters, (c' and ϕ'), initial suction values in the unsaturated soil specimens used for testing unconfined compression tests, and the unconfined compression strength value.

The shear strength of an unsaturated soil and the soil-water characteristic curve are dependent on soil structure or the aggregation, which in turn is dependent on the "initial" water content and the method of compaction (Vanapalli et al. 1996b). The laboratory preparation of specimens used for testing must closely represent the physical conditions and the stress state conditions likely to occur in the field to properly assess the shear strength characteristics. This is true for other engineering properties of the soil such as the coefficient of permeability and the volume change.

Figure 2 shows the soil-water characteristic curve along with the degree of saturation versus matric suction results of individually compacted specimens of clay till.

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The initial void ratios for all individually compacted specimens are the same. The initial water contents are however different. The matric suction in each of these specimens was determined using a null pressure plate apparatus using axis translation technique (1956). The soil-water characteristic curve was determined using a pressure plate apparatus using a specimen that is compacted with initial water content equal to 16.3% (Vanapalli et al. 1999).

It is of interest to note that the soil-water characteristic curve lies below the results of individually compacted specimens in the region of 0 to 150 kPa matric suction. The initial water content in individually compacted specimens for this range of matric suction is higher than 16.3% (i.e., degree of saturation greater than 86%). With the increased water content in the individually compacted specimens, the soil aggregations differ and result in a different soil structure. As the water content in the individually compacted specimens decreases, the values of matric suction fall below the soil water characteristic curve. This discrepancy is expected because the specimens with lower water contents exhibit a different soil structure based on their aggregation, which in turn depends on the water content. At a degree of saturation around 86% (i.e., at 16.3% water content), both individually compacted specimens and the specimen used for the soil-water characteristic curve show similar matric suction values. This correspondence is possible as both the structure and density in two specimens is approximately same and they behave as "identical" specimens. Similar results were obtained for specimens compacted with initial water contents representing dry of optimum and wet of optimum conditions.

These discussions suggest that the influence of the soil structure should be taken in account while interpreting the engineering behavior of unsaturated soils. Vanapalli et al. (1999) provide more information and experimental data and discuss the influence of soil structure and stress state on the engineering behavior of. unsaturated soils.

Equation [2] and Eqn. [4] are respectively used for predicting the shear strength under drained and undrained loading conditions. The discussions provided using Figure 2 suggest that the shear strength contribution due to suction would be approximately the same for a particular soil under similar initial conditions with respect to stress state and soil structure. In other words, Eqn. [4] can be used to determine the fitting parameter, κ from simple unconfined compression test results and used in predicting the drained shear strength of an unsaturated soil using Eqn. [2].

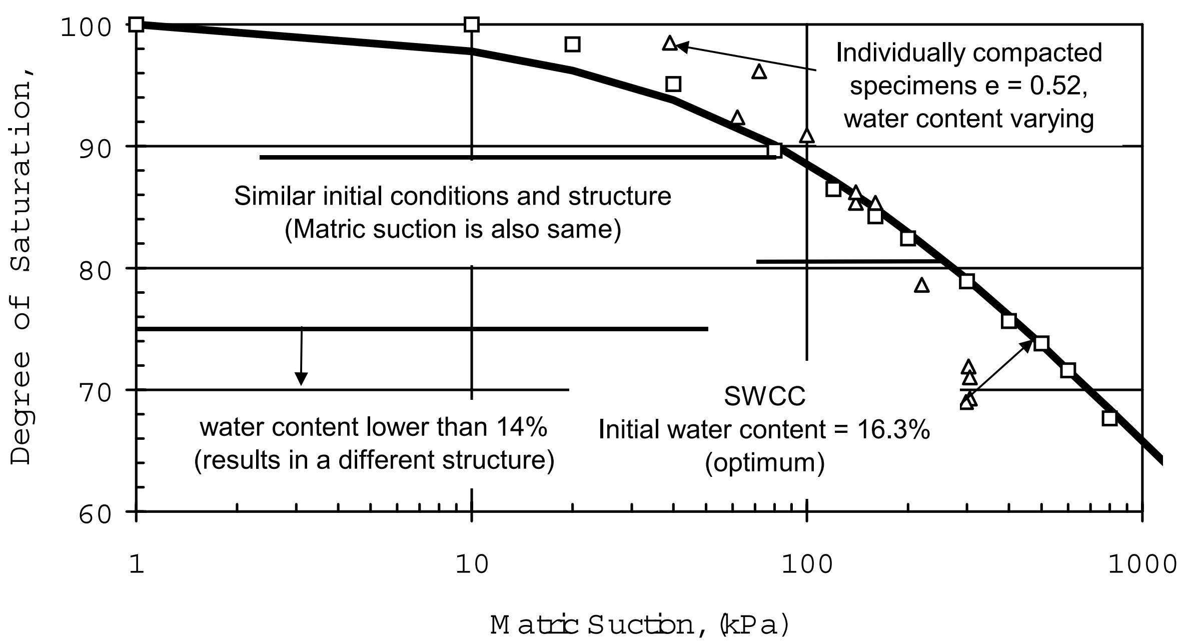

FIGURE 2. Comparison of soil-water characteristic curves for specimens compacted at optimum water content and those compacted at the same initial void ratio.

3 TEST PROGRAM, RESUTLS AND ANALYSIS.

A clay till obtained from Indian Head, Saskatchewan was used for the study. The liquid limit, w_L, and the plastic limit, w_P are 35.5% and 16.8% respectively. The percentages of sand, silt, and clay are respectively 28, 42 and 30%. The AASHTO standard compacted density was 1.80 Mg/m^3 and the optimum moisture content was 16.3%. The specific gravity of the soil was 2.73. The soil is classified as a CL.

Three initial water contents were selected for preparing soil specimens representing dry of optimum, optimum and wet of optimum conditions. The pre-calculated water content was added to the soil mass and wrapped in a plastic bag. The prepared soil sample was placed in a humidity-controlled room for a period of 24 hours to achieve uniform water content conditions. The specimens for testing unconfined compression strength values were statically compacted in six equal layers using specially designed molds with spacers The compacted specimens were 50 mm in diameter and 100 mm height. More details of the preparation of the specimens is available in Vanapalli et al. 2000.

The initial matric suction value in the compacted specimens was determined using null pressure plate using axis translation technique (Hilf, 1956). Figure 3 shows typical results of unconfined compressions tests conducted on specimens compacted with an initial water content equal to 16.3%.

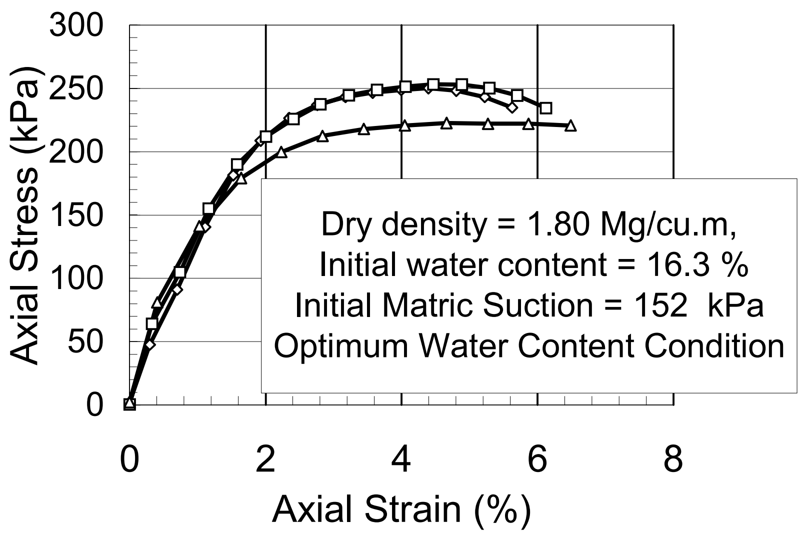

FIGURE 3. Stress versus strain relationships from unconfined compression tests.

The specimens were sheared in a conventional unconfined compression testing loading frame at a rate of 1.2 mm/min. Three tests were conducted and the average unconfined compressive strength value was used in the analysis. All the specimens after testing were cut into two halves at approximately mid height of the specimen and the water content was determined. There was negligible or no variation in the water content of both halves of the same specimen. Hence, it can be assumed that the suction value in the specimen is uniform throughout the specimen.

Table 1
Measured and Predicted Values of Unconfined Compressive Strength using Eqn. 4

Water Content (%)	Water Content Relative to Optimum	Matric suction (u_a-u_w) (kPa)	Effective cohesion (kPa)	Friction angle, ϕ' (degrees)	Unconfined compressive strength, (kPa) (Measured)	Fitting parameter (κ)	Unconfined compressive strength (kPa) (Predicted using Equation 4))
16.3	Optimum	152	20	23.5	120.8	2.3	101.7
13.0	Dry of Optimum	368	20	23.5	180.5	2.3	172.0
19.2	Wet of Optimum	68	20	23.5	64.3	2.3	71.5

Table 1 provides a summary of the measured and predicted values of shear strength values using Eqn. 4. The values of fitting parameter, κ are also shown in the table. There is a reasonably good comparison (i.e., with in a percentage of accuracy of 10 to 15%) between the predicted and measured shear strength values of clay till specimens tested with different initial water content conditions representing dry of optimum, optimum, and optimum conditions. The results of this study suggest that fitting parameter, κ value is reasonably independent of initial water content conditions. The fitting parameter κ may however be influenced by other parameters such as the nature of soil (i.e., slurry consolidated, dynamically compacted, statically compacted or natural state).

In an earlier study, Vanapalli et al. (1996a) have undertaken studies to show comparisons between the measured drained shear strength results using modified direct shear results and predicted shear strength test results using the soil-water characteristic curve and the effective shear strength parameters. The same clay till used in the present research program was used for the earlier study. Equation 2 was used for providing comparisons in this study. Typical results are shown in Figure 4. The fitting parameter, κ value between 2.2 to 2.5 was required to provide good comparisons between the measured drained shear strength values and predicted shear strength results (results in Figure 5 using κ equal to 2.3). It is of interest to note that approximately the same fitting parameter value was required using both the approaches (i.e., Equation 2 and Equation 4). These results suggest that the assumptions used in the analysis are valid and the unconfined compression test results can be used for determining the fitting parameter, κ.

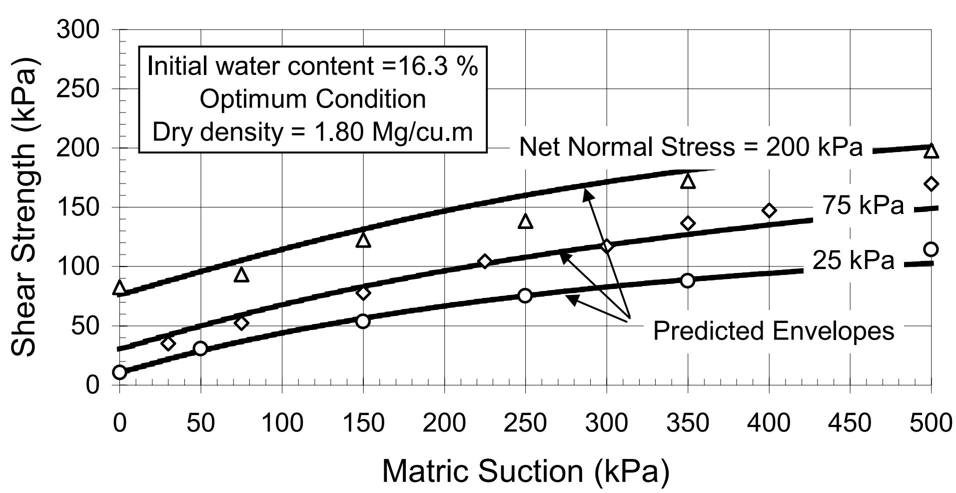

FIGURE 4. Comparison between the predicted and measured drained shear strength for specimens compacted at optimum water content using Equation 2.

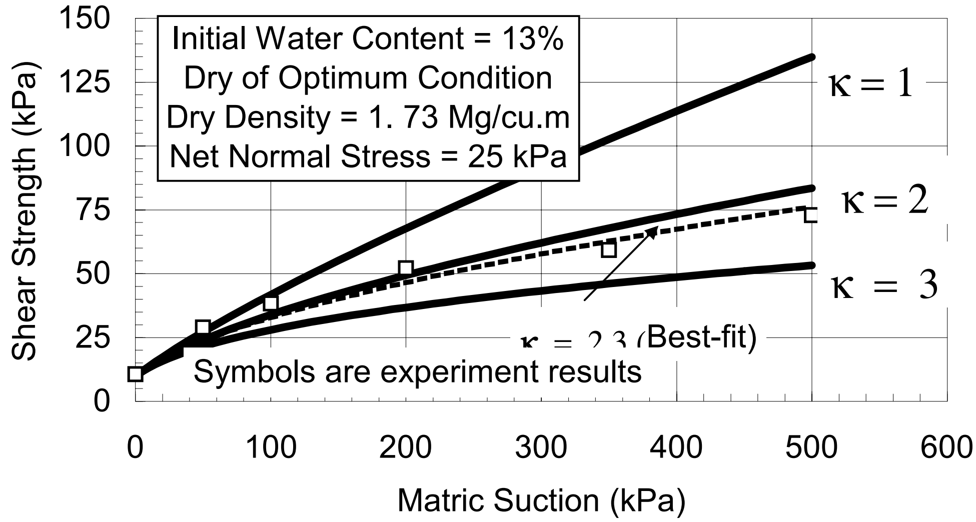

FIGURE 5. Variation of shear strength with matric suction using various values of κ

4. THE FITTING PARAMETER, κ

Figure 5 shows comparison between the measured shear strength values for a specimen compacted with dry of optimum conditions using modified direct shear testing equipment and predicted shear strength values using different values of fitting parameter, κ using Eqn. [2]. In the low suction range (i.e., 0 to 50 kPa) fitting parameter values ranging from 1 to 3 provide reasonably good comparisons between the predicted and measured values. In fact, any fitting parameter, κ value provides good comparisons up to the air-entry value of the soil. However, it is important to note that the fitting parameter, κ is a value that provides good comparisons between the predicted and measured shear strength values for a large suction range.

The studies undertaken in this research program and other research programs by Vanapalli et al. (2000), Vanapalli and Fredlund (2000) and Remiul (2001) suggest that the fitting parameter, κ is a unique value for a soil and is strongly dependent on the plasticity index, I_p value.

5. SUMMARY AND CONCLUSIONS

Several procedures are available in the literature to predict the shear strength of unsaturated soils using the soil-water characteristic curve and the effective shear strength parameters (Vanapalli et al. 1996a, Fredlund et al. 1996, Oberg and Sallfours 1997, Khallili and Khabbaz 1998 and Bao et al. 1998). Recent studies suggest that the University of Saskatchewan procedure (i.e., Equation 2) provides reasonably good comparisons between the measured drained shear strength and predicted shear strength values both for low and large suction ranges for various soils (Vanapalli and Fredlund 2000). A fitting parameter, κ is necessary in this procedure to provide comparisons between the predicted and measured shear strength values. Vanapalli and Fredlund (2000) have recently provided a relationship between the fitting parameter, k and the plasticity index, I_p. The validity of this relationship for various other parameters such as the nature of soil (i.e., slurry consolidated, dynamically compacted, statically compacted or natural state) is not known.

A simple procedure is presented in this paper to determine the fitting parameter κ value. The effective shear strength parameters, c' and ϕ', the unconfined compressive strength along with the soil suction value are required for determination of the fitting parameter, κ using Eqn. [4]. Once the fitting parameter value is estimated from simple unconfined compression tests; the drained shear strength of an unsaturated soil can be predicted using Eqn. [2.] The required information for predicting the drained shear strength includes the soil-water characteristic curve and the effective shear strength parameters.

The approach presented in this paper was successfully used to determine the fitting parameter values for clay till specimens compacted at three different initial water content conditions representing dry of optimum, optimum, and wet of optimum conditions. It is important to take into account of the influence of both the stress state and the soil structure such that the fitting parameter, κ. obtained from unconfined compression tests would be valid for predicting the drained shear strength of unsaturated soils using Eqn. [2].

The analysis of the results presented in this paper are based on the assumption that the suction changes in the specimen are not significant during the shearing stage of unconfined compression tests for clay till used in the study. This assumption should be reasonably valid good for other fine-grained soils. The proposed simple approach is useful for the determination of the fitting parameter, κ and can be used for compacted and natural soils. More studies are necessary on different types of soils to further validate the results presented in the paper.

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