INFLUENCE OF VOLUME CHANGE BEHAVIOR ON THE PREDICTION OF THE SHEAR STRENGTH OF UNSATURATED SOILS

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ABSTRACT
The procedures available in the literature for predicting the shear strength of unsaturated soils do not take into account influence of the volume change behavior on the shear strength. In this paper, the influence of volume change on the shear strength behavior is studied using the Vanapalli et al. (1996) semi-empirical procedure for predicting the shear strength of an unsaturated soil and provide comparisons with the experimental results. A modified experimental procedure is suggested to simultaneously determine the shear strength, volume change and the soil-water characteristic curve behavior of unsaturated soils using a specially designed dual chamber triaxial shear test apparatus to better understand the limitations of the semi-empirical methods towards predicting the shear strength of unsaturated soils. The design details of the dual chamber triaxial shear test apparatus are presented in the paper.

RÉSUMÉ:
Les méthodes présentement disponibles pour la prédiction de la résistance de sols non-saturés ne tiennent pas compte de l'influence du changement de volume sur la résistance des sols. Dans cet article, l'influence du changement de volume est étudiée en utilisant l'approche semi-empirique de Vanapalli et al. (1996), pour prédire la résistance de d'un sol insaturé. Une approche expérimentale est proposée pour déterminer simultanément la résistance, le changement de volume et la courbe de rétention d'eau d'un sol insaturé en utilisant un appareil triaxial à double paroi spécialement conçu afin de mieux comprendre les limitations des méthodes semi-empiriques dans la prédiction de la résistance de sol non-saturés. Les détails de l'appareil et de l'approche expérimentale à être utilisés sont également présentés.

1. INTRODUCTION

Shear strength and volume change form important engineering properties in the design of geotechnical and geo-environmental structures such as earth dams, retaining walls, pavements, soil liners, and soil covers. Many of these soil structures are typically in a state of unsaturated condition and often do not attain fully saturated conditions during their design life period. Thus, the mechanical behavior (i.e., shear strength and volume change) of unsaturated soils is relevant in the design of several geotechnical and geo-environmental structures.

The shear strength of an unsaturated soil can be determined using modified direct shear or triaxial shear equipment. Experimental studies related to the determination of the shear strength of unsaturated soils are time consuming and require extensive laboratory facilities (Escario 1980, Escario and Jucá 1989, Gan et al. 1988). In recent years, several semi-empirical procedures were proposed to predict the shear strength of unsaturated soils (Vanapalli et al. 1996, Fredlund et al. 1996, Khalili and Khabbaz 1997, Oberg and Salfours 1997, Bao et al. 1998). The proposed prediction procedures use the effective shear strength parameters (c', ϕ') along with the soil-water characteristic curve data to predict the shear strength of unsaturated soils. The soil-water characteristic curve defines the relationship between the soil suction and gravimetric water content, ***w***, or the volumetric water content, θ, or the degree of saturation, *S*. The prediction procedures use the soil-water characteristic curve as a tool as there is a strong relationship between water content and the shear strength of a soil. The information about the geometry and distribution of the water in the liquid phase and the stress within the pore water is derived from the soil-water characteristic curve and used in the prediction of the shear strength of unsaturated soils.

The soil-water characteristic curves are conventionally measured using Tempe cell or the pressure plate apparatus on specimens which are typically 50 mm in diameter and 20 mm in height. The volume change of the soil specimen is not commonly measured while determining the soil-water characteristic curves. In several cases, the shear strength of an unsaturated soil is predicted using the soil-water characteristic curve measured without the application of any loading. In other words, the soil-water characteristic curve is measured without taking account influence of loading and the resulting volume change. The volume change with respect to soil suction is also ignored assuming it to be relatively small. From a practical perspective, the soil will be subjected to some loading and there will be volume change both during the loading and as well as the shearing stage. The shear strength of an unsaturated soil hence will be influenced by the volume change characteristics of the soil. None of the procedures available in the literature have addressed the influence of volume change behavior on the prediction of the shear strength of unsaturated soils.

In this paper, the influence of volume change behavior on the prediction of the shear strength using the information of the soil-water characteristic curve and the saturated shear strength parameters, (c', ϕ') was studied for a compacted Dhanauri silty clay. The results of the present study suggest that the volume change behavior during the loading and shearing stage does not significantly influence the predicted shear strength of the compacted soil for a suction range of practical interest (i.e., 0 to 500 kPa). The reasons for such a behavior are discussed in the paper. However, to better understand the influence of volume change behavior on the shear strength of different unsaturated soils, it is suggested to undertake experimental studies to simultaneously determine the shear strength, volume change and the soil-water characteristic curve and provide comparisons between the predicted and measured shear strength values. The design details of a newly designed dual chamber triaxial test apparatus are presented in the paper that facilitates to simultaneously determine the shear strength, volume change and the soil-water characteristic curve.

Experimental results on different soils using the dual chamber triaxial test apparatus would be of value to understand the influence of volume change on the shear strength behavior of unsaturated soils. Also, limitations, if any, associated with the present semi-empirical methods used for predicting the shear strength of unsaturated soils can be better understood.

2. BACKGROUND

Fredlund et al. (1978) proposed an equation for interpreting the shear strength of unsaturated soils in terms of two independent stress state variables, net normal stress, $(\sigma - u_a)$ and matric suction, $(u_a - u_w)$.

$$\tau_f = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [1]$$

where:

τ_f = the shear strength of an unsaturated soil
c' = the effective cohesion of the soil
ϕ' = the effective angle of shearing resistance for a saturated soil
σ = the total stress
u_a = the pore air pressure
u_w = the pore water pressure
ϕ' = the angle of internal friction
ϕ^b = the angle of internal friction with respect to matric suction.
$(u_a - u_w)$ = matric suction
$(\sigma - u_a)$ = net normal stress

Equation [1] extends the traditional Mohr-Coulomb failure criterion for interpreting the shear strength of unsaturated soils and is valid for both linear and non-linear shear strength failure envelopes (Gan et al. 1988; Escario and Jucá 1989).

Vanapalli et al. (1996) proposed a general, nonlinear function for predicting the shear strength of an unsaturated soil using the soil-water characteristic curve and the saturated shear strength parameters (i.e., c' and ϕ'). The shear strength function is given below:

$$\tau = [c' + (\sigma_n - u_a)\tan\phi'] + [(u_a - u_w)\{(\Theta^{\kappa})(\tan\phi')\}] \qquad [2]$$

where:

κ = fitting parameter used for obtaining a best-fit between the measured and predicted values
Θ = normalized water content, θ_w/θ_s (θ_w = volumetric water content and θ_s is the saturated volumetric water content).
τ_{us} = the shear strength contribution due to suction,

The first part of the equation is the saturated shear strength, when the pore-air pressure, u_a, is equal to the pore-water pressure, u_w. This part of the equation is a function of normal stress as the shear strength parameters c' and ϕ' are typically constant for a saturated soil for the range of practical loadings commonly encountered in engineering practice. Hence, for a particular value of net normal stress, $(\sigma - u_a)$, the first part of the equation is a constant value. The second part of the equation is the shear strength contribution due to suction, which can be predicted using the soil-water characteristic curve and the effective shear strength parameters.

Vanapalli and Fredlund (2000) have shown comparisons between the measured and predicted values of shear strength for several statically compacted soils using Equation [2]. The analyses of the results have shown that that the shear strength of unsaturated soils can be predicted with a reasonable degree of accuracy for a large suction range (i.e., 0 to 10,000 kPa). A relationship between the fitting parameter, κ, and the plasticity index, I_p was also proposed based on this study (Figure 1). This relationship is useful to estimate the value of fitting parameter, κ and substitute in Equation [2] to predict the shear strength of unsaturated soils.

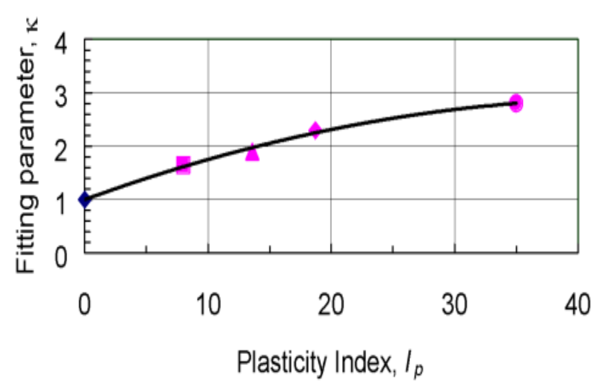

Figure 1. Relationship between the fitting parameter κ and the plasticity index of a soil (from Vanapalli and Fredlund, 2000)

Several parameters can influence the semi-empirical procedures used for predicting the shear strength of unsaturated soils using the soil-water characteristic curve. One of the key parameters is the volume change behavior of unsaturated soils. The soil volume changes both during the loading and the shearing stage in unsaturated condtions. The stress history and pore pressure parameters are the other parameters which are related to the volume change behavior of unsaturated soils.

Fredlund and Rahardjo (1993) have shown that the changes in the pore-air pressure, du_a and pore-water pressure, du_w under triaxial loading conditions can be expressed as given below:

$$du_a = B_a \left[d\sigma_3 + A_a \, d(\sigma_1 - \sigma_3) \right] \quad [3]$$

$$du_w = B_w \left[d\sigma_3 + A_w \, d(\sigma_1 - \sigma_3) \right] \quad [4]$$

The pore pressure parameters for unsaturated soils, B_w and B_a are similar to Skempton's B parameter relating the pore-water pressure changes to the changes in confining pressure in saturated soils. Similarly, the pore pressure parameters, A_w and A_a are pore-water and pore-air parameters relating the changes in pore pressures to the changes in shear stresses. For the specific case of an isotropic and elastic soil, the A pore pressure parameters (i. e., both A_w and A_a) will equal to 1/3. When the A parameter is not equal to 1/3, the pore pressure response will be influenced by the shear stress (Skempton, 1954).

Unsaturated soils are frequently dense due to desiccation, and the net result is a low A pore pressure parameter (Fredlund and Rahardjo, 1993). A low value of the A parameter means that the soil will have a low pore pressure response to changes in deviator stresses which in turn translates into smaller volumetric strains. Extending this argument, it can be stated that the soil-water characteristic curve behavior may not be significantly influenced in some compacted soils. Due to this reason, it is likely the predicted shear strength using the soil-water characteristic curve measured without taking account volume change may provide reasonable comparisons with the measured shear strength values for compacted soils in the low suction range. There are however no experimental studies available in the literature to support these observations.

3. DATA ANALYSIS

In this paper, the shear strength behavior of an unsaturated soil is predicted based on two different approaches to understand the influence of volume change behavior. Comparisons are provided between the measured and predicted values of shear strength. More details with respect to the *two different approaches* are discussed later in the paper. Triaxial shear strength results undertaken by Satija (1978) on Dhanauri clay are used for the analysis.

Dhanauri clay is a naturally occurring transported soil from the banks of the Dhanauri River situated about 7 km from Roorkee (U.P.), in India. The soil contains 5% sand, 70% silt and 25% clay size particles. The plasticity index, I_p, of the soil is 23.5%. The soil can be classified as silty clay. The effective shear strength parameters of statically compacted Dhanauri clay at an initial density of 15.9 kN/m³ are: c' =37.3 kPa and ϕ' = 28.5°.

The experimental program was undertaken using a standard triaxial shear strength testing equipment in which a high air-entry disk was used. This disk allows water, but not air, to circulate at the base of the specimen. The air pressure was applied at the top of the specimen through a coarse porous stone. Two fabric discs, with excellent water repellent properties, but highly pervious to air were used to ensure that no upward migration of water would take place from the specimen (Satija, 1978). A desired value of matric suction was achieved in the initially saturated soil specimens in the triaxial shear equipment using the axis translation technique (Hilf, 1956) to determine the shear strength at various values of matric suction.

The triaxial cell was pre-calibrated in order to determine the magnitude of the cell expansion and creep characteristics when subjected to different levels of loading. Preliminary tests showed the necessity to reinforce the cell circumferentially by pressure-wrapping a permanent glue coated wick cloth at three locations along the height of the cell (Satija, 1978). Volume measuring devices were used to monitor the volume of water entering and leaving the specimen from the triaxial cell with a precision of 0.05 cc. This information was useful in the calculation of the water content, w and the degree of saturation, S of the specimen both prior to determining the shear strength and at failure conditions of the test.

In the *first approach* the shear strength prediction is based on the information of the effective shear strength parameters (c', ϕ') and the soil-water characteristic curve. The information related to the soil-water characteristic curve (i.e., water content versus matric suction) data comes from different triaxial tests that were tested under a common confining stress condition but subjected to desaturation by applying different values of matric suction to the soil specimens. All the specimens satisfy the criteria of “identical” specimens (i.e., the initial density and compaction water content are the same for all the specimens). The soil-water characteristic curve derived as detailed in this *first approach* is referred as the pre-shear soil-water characteristic curve in the remainder of the paper. This procedure is different from the conventional technique of the measurement of soil-water characteristic curve using a single specimen in Tempe cell or pressure plate apparatus.

In the *second approach*, the soil-water characteristic curve information is derived from shear strength data of triaxial tests at failure conditions. In other words, in this approach, the derived soil-water characteristic curve information from the shear strength test data takes account influence of volume change of the specimen due to loading and shearing. The soil-water characteristic curve derived as explained in this *second* *approach* is referred as the post-shear soil-water characteristic curve in the remainder of the paper.

Comparisons are provided between the above two described approaches using the semi-empirical prediction equation proposed by Vanapalli et al (1996). For the purpose of analysis, comparisons will be provided between the measured and predicted values of the component of shear strength contribution due to matric suction. Only the second part of Equation [2], shown in Equation [5] is required for predicting the shear strength contribution due to matric suction:

$$\tau_{suction} = (u_a - u_w)\left[\left(\Theta^{\kappa}\right)(\tan\phi')\right] \qquad [5]$$

The experimental setup facilitated to control matric suction, $(u_a - u_w)$ in the soil specimen at all stages of testing. Since the information of both matric suction, $(u_a - u_w)$ and degree of saturation, S both at pre-shear and at failure conditions (i.e., post-shear) was available, the soil-water characteristic curve information related to pre-shear and post-shear conditions were derived as detailed earlier. This information was useful to study the influence of volume change behavior on the prediction of the shear strength of the tested Dhanauri clay.

The plasticity index, I_p of the tested soil, Dhanauri clay was equal to 23.5% and therefore, a κ parameter of 2.48 was estimated from Figure 1 and used for predicting the shear strength contribution with respect to suction using Equation. 5. The same value of κ was used for predicting the shear strength using both pre-shear and post-shear soil-water characteristic curves.

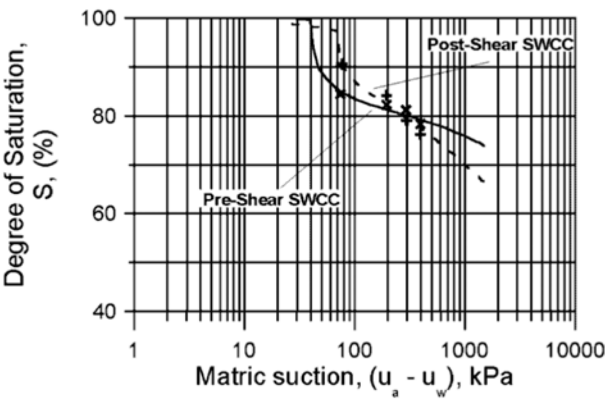

Figure 2. Soil-water characteristic curve data derived from the triaxial shear test results undertaken on Dhanauri clay specimens tested under a confining pressure of 96 kPa.

Figure 2 shows the soil-water characteristics for the pre-shear and post-shear (i.e., at failure) conditions for Dhanauri clay tested under a net confining pressure of 96kPa. Figure 3 shows the variation of the predicted shear strength with respect to matric suction using Equation [5]. The measured shear strength values are shown as symbols in Figure 3. There is a reasonably good agreement between the measured and predicted values of shear strength for the matric suction range considered using both the pre-shear and post-shear soil-water characteristic curves. However, the curvature of the predicted shear strength function using the post-shear soil-water characteristic appears to follow more closely the apparent curvature of the measured shear strength values. This behavior suggests that the influence of volume change characteristics on the prediction of shear strength may be negligible in the range of suction values tested (i.e., 0 to 400 kPa). Significant differences can be observed in predicted shear strength values when a large suction range (i.e., 0 to 1,500 kPa) is considered using both the approaches (Figure 4).

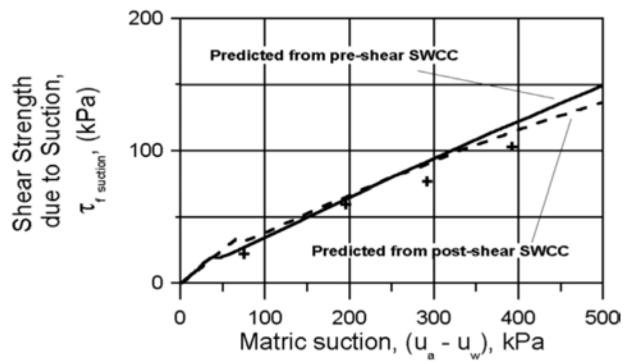

Figure 3. Measured and predicted shear strength of Dhanauri clay for a suction range of 0 to 400 kPa and a net confining pressure of 96kPa.

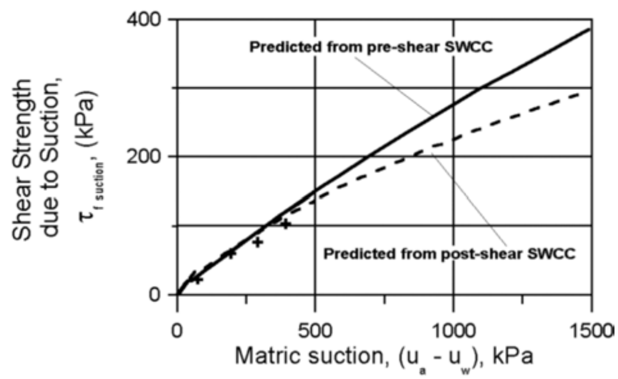

Figure 4. Measured and predicted shear strength of Dhanauri clay for a net confining pressure of 96 kPa.

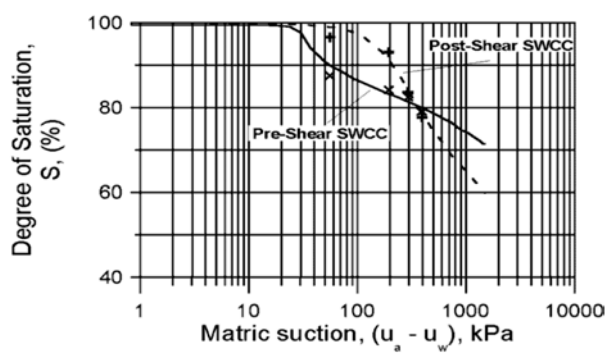

Figure 5. Soil-water characteristic curve data derived from the triaxial shear test results undertaken on Dhanauri clay specimens tested under a net confining pressure of 386 kPa.

Figure 5 shows the derived pre-shear and post-shear soil-water characteristic curve data for Dhanauri clay tested under a net confining pressure of 386kPa. The predicted and measured values of shear strengths are shown in Figure 6 (low suction range of 0 to 500 kPa) and Figure 7 (high suction range of 0 to 1,500 kPa). Similar to the observations in Figure 4, there is a reasonably good comparison in the low suction range. However, significant differences can be observed between the predicted shear strength values using the pre-shear and post-shear soil-water characteristic curve for the high suction range (Figure 7). As the experimental results are not available for the high suction range (i.e., greater than 400 kPa), it is difficult to comment on the relative accuracy of the predicted shear strength behavior in the high suction range.

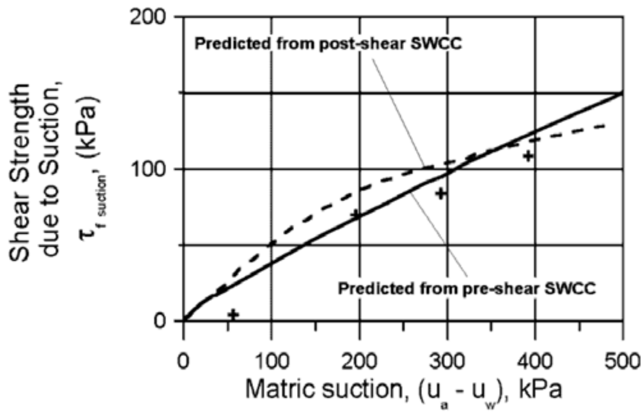

Figure 6. Measured and predicted shear strength of Dhanauri clay for a suction range of 0 to 400 kPa and a net confining pressure of 386kPa

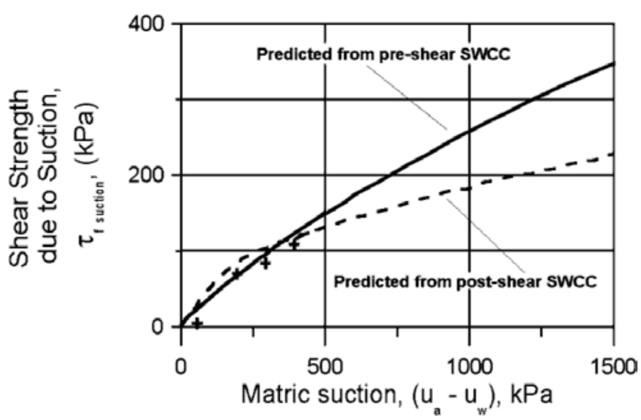

Figure 7. Measured and predicted shear strength of Dhanauri clay for a net confining pressure of 386kPa

The analysis of shear strength undertaken on Dhanauri compacted clay suggests that the soil-water characteristic curve determined at a density and compaction water content consistent with the soil's original pre-shear condition may not be representative of the soil-water characteristics once it is subjected to loading and shearing (see Figure 2 and 5).

The degree of saturation of post-shear soil-water characteristic curve is higher in the suction range of 0 to 300 kPa (see Figures 2 and 5). Such a behavior suggests that the specimens consolidated (i.e., volume decrease) during the shearing stage and results in an increase in the degree of saturation, *S* of the soil specimen. However, when the specimens were subjected to shearing at higher suction values, the soil specimens exhibit relatively higher desaturation characteristics (see Figures 2 and 5). The changes in the soil-water characteristics are more predominant in higher suction range in comparison to low suction range. The significant changes in volume change occurring during the shearing stage in the higher suction range can be attributed to dilation (i.e., volume increase) which contributes to a decrease in the degree of saturation, *S*. Due to the resulting changes in the specimen volume (and degree of saturation), the shear strength prediction using the post-shear soil-water characteristic curve is relatively higher in comparison to shear strength prediction using the pre-shear soil-water characteristic curve in the low suction range (see Figure 6) and lower in the high suction range (see Figures 4 and 7). Colmenares and Ridley (2002) reported similar observations of soil-water characteristic curve behavior (as in Figure 2 and 5) from their experimental studies.

Shear strength predictions using both the pre-shear and post-shear soil-water characteristic curves are approximately the same as significant differences are not observed in their soil-water characteristic curve behavior in the low suction range. More discussions cannot be offered with respect to the predicted shear strength behavior in the high suction range as experimental results of measured shear strength values are not available. However, there are significant differences between the pre-shear and post-shear soil-water characteristics in the high suction range which suggests the influence of volume change behavior on the shear strength of unsaturated soils should be more carefully analyzed.

The presented analysis suggests that there is a need to use test equipment that is capable of monitoring both the water content and volumetric changes of the specimen during shearing stage along with the measurements of shear strength and soil-water characteristics to better understand the limitations of the semi-empirical procedures used for predicting the shear strength of unsaturated soils. Also, it will be useful to determine the pore pressure parameters (i.e., *A* parameter) as they provide valuable information with respect to understanding the influence of volume change behavior on the shear strength of unsaturated soils.

4. MODIFIED TRIAXIAL SHEAR STRENGTH TESTING APPARATUS

The design details of a new, modified triaxial test set up is described in this section that is useful to simultaneously measure the shear strength, volume change and the soil-water characteristic curve. Several modifications have to be introduced in comparison to conventional triaxial shear test apparatus. One of the key modifications includes the use of a dual cylinder triaxial cell (Figure 8). Wheeler (1988) earlier used a dual cylinder to determine the shear strength of soils containing large gas bubbles. The soil specimen to be sheared is to be placed and tested in the internal cylinder. The internal cylinder is similar to conventional triaxial cell used for measuring the shear strength of an unsaturated soil.

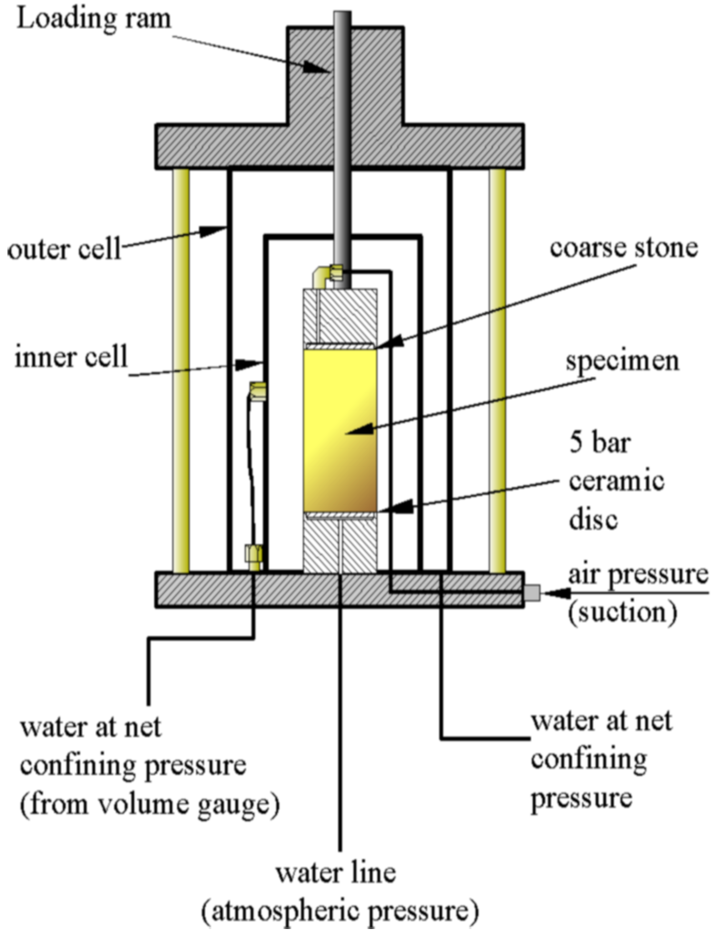

Figure 8. Modified triaxial cell for determining the shear strength of unsaturated soil specimens.

The objective of using the outer cylinder is to facilitate measurements of volume change of the unsaturated soil specimen. This is achieved by filling both the internal and external cells with water and subjecting them to the same pressure while determining the shear strength of an unsaturated soil. Because the inner and outer triaxial cells are filled with water and subjected to identical stress conditions, there will not be any volume change in the inner chamber, due to the stretching of the cell wall. All the measured water movement from the internal cell can be attributed to the volume changes of the specimen and to the compression of the inner wall, which is relatively small and can be taken into account by calibrating the device.

Provisions are also included in the design to measure the volume of water leaving or entering the specimen as well as the volume of water entering or leaving the inner triaxial cell. This arrangement allows the measurement of the specimen's water content change and specimen's volume change to be monitored independently. The volume change of the soil specimen can be determined at different values of matric suction. In other words, information related to water content and matric suction (i.e., soil-water characteristic curve data) can also be obtained from the test set up. The remainder of the testing technique is similar to conventional testing procedures used in the determination of the shear strength parameters for an unsaturated soil.

It is necessary to include in the design of the triaxial testing system a mechanism to flush out the air bubbles because air dissolved in the water may come out of solution below the ceramic disk. In order to keep proper track of the volume of water coming out of the specimen, a flushing system as shown in Figure 9 is designed as a closed system. As the volume of water rises in the water column, the pore pressure rises in the system, and is measured by the sensitive pressure transducer attached to it. Additionally two water reservoirs are provided for the entry and evacuation of air from the water column. These reservoirs ensure that the air entering the system is saturated so that the circulation of air will not reduce the amount of water present in the system.

The above designed modified dual cell triaxial shear testing equipment is presently being used to determine the shear strength and soil-water characteristic curves of Indian Head Till while simultaneously monitoring the volume changes of the specimen at the University of Ottawa, Geotechnical Engineering Labs. The triaxial shear tests are undertaken under consolidated drained conditions. The soil-water characteristic curve will also be determined using conventional apparatus such as the Tempe cell. Comparisons will be provided between the soil-water characteristic curve behavior determined using both of these methods. Comparisons will be provided between the predicted and measured shear strength values taking into account the influence of the volume change behavior. The pore pressure parameters (i.e., *A*) values will also be determined.

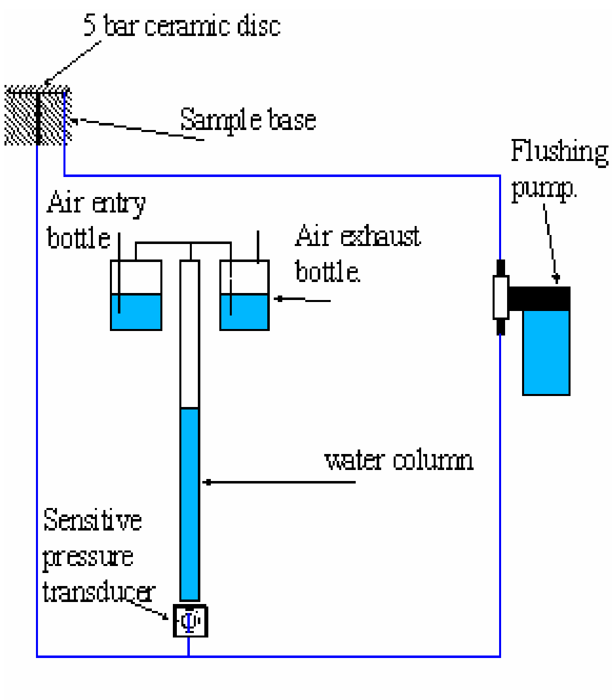

Figure 9. Detail of the flushing system.

5. SUMMARY AND CONCLUSIONS

The unsaturated shear strength of Dhanauri clay is predicted using two different approaches. In the *first approach,* the shear strength prediction is based on the information of the effective shear strength parameters (c', ϕ') and the pre-shear soil-water characteristic curve (i.e., using the soil-water characteristic curve determined without taking into account influence of the volume change). In the *second approach*, the post-shear soil-water characteristic curve is used taking into account influence of volume change of soil due to loading and shearing to predict the shear strength. The results of the study suggest that there is a reasonably good comparison between the predicted and measured shear strength values using the two different approaches in the suction range of 0 to 400 kPa. However, there were significant differences in the predicted shear strength values in the high suction range using the two different approaches.

The information of the soil-water characteristic curve; shear strength and volume change behavior cannot be simultaneously obtained using conventional experimental techniques. All the above information is necessary to be determined using triaxial shear testing equipment to understand the influence of volume change behavior on the prediction of the shear strength of unsaturated soils. Several modifications have to be introduced into the conventional testing techniques to experimentally determine the above information. The design details of a modified dual cell triaxial shear strength testing equipment capable of determining all the above data is presented in the paper. Experimental studies are presently under progress on Indian Head till at the University of Ottawa using the equipment to better understand the limitations of the semi-empirical methods towards predicting the shear strength of unsaturated soils.

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