A Normalized Function for Predicting the Coefficient Permeability of Unsaturated Soils

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ABSTRACT: A normalized functional relationship is proposed in this paper to predict the coefficient of permeability of unsaturated soils. This normalized function is a relationship between the relative coefficient of permeability, k_{rel} (defined as the ratio of unsaturated coefficient of permeability, k_{unsat}/ saturated coefficient of permeability, k_{sat}) and the (*degree of saturation*)$^{\gamma}$. The relationship is developed using the published experimental data from the literature for different soils ranging from sand (I_p = 0%) to clay (I_p = 50%). The fitting parameter γ is found to be dependent on the type of soil. A relationship has also been proposed between the fitting parameter, γ and the plasticity index, I_p of the soil. The results show that the proposed simple relationship is valid to estimate the coefficient of permeability of for different types of soils that are in a state of unsaturated condition.

1 INTRODUCTION

The coefficient of permeability can vary widely in comparison to other engineering properties of soils such as shear strength and volume change. The saturated coefficient of permeability varies about eight to ten orders of magnitude when considering soils that range from a coarse-grained soil such as gravel to a fine-grained soil such as clay (Lambe and Whitman, 1979). The coefficient of permeability of an unsaturated soil can vary significantly due to the influence of soil suction. Typically, the coefficient of permeability of an unsaturated soil can vary several orders of magnitude for the suction range of practical interest to engineers (i.e., the suction range of 0 to 1,000 kPa). Geotechnical and geo-environmental engineers are interested in understanding both the saturated and unsaturated flow behaviour such that rational procedures can be used in the design of waste management structures such as soil liners and covers.

It is a common practice to determine the saturated coefficient of permeability either in the laboratory or in the field. The procedures for determining the saturated coefficient of permeability are relatively simple and are not expensive. There are also several experimental procedures available to directly measure the coefficient of permeability of unsaturated soils; but they are time consuming and difficult, and hence costly (Fredlund & Rahardjo, 1993). As a result, much of research focus has been towards developing semi-empirical procedures to predict the unsaturated coefficient of permeability (Gardner 1958, Brooks & Corey 1964, van Genuchten 1980, Fredlund et al 1994, Leong & Rahardjo 1998).

The coefficient of permeability is conventionally predicted using the saturated coefficient of permeability and the soil-water characteristic curve. The soil-water characteristic curve is defined as the relationship between the soil suction and the water content (either gravimetric, w or volumetric, θ, or degree of saturation, S). Several parameters such as compaction water content, stress state, soil structure (or aggregation), mineralogy, texture, organic content and hysterisis influence the soil-water characteristic curve behaviour. Recent studies have shown that the initial compaction water content and the stress history can have a significant influence on the soil-water characteristics of compacted fine-grained soils (Vanapalli et al, 1999, Ng & Pang, 2000). All the parameters that influence the soil-water characteristic curve behaviour also influence the coefficient of permeability of unsaturated soils. From an engineering practice point of view, it is important to understand that there is no unique soil-water characteristic curve for a soil, particularly for compacted fine-grained soils.

A simplified functional relationship is proposed to predict the unsaturated coefficient of permeability of both coarse and fine-grained soils in this research paper. This function is a normalized relationship that is useful for estimating the unsaturated coefficient of permeability taking into account the influence of stress state and soil structure. The stress state and soil structure are the two key parameters that influence the engineering behavior of unsaturated soils.

2 BACKGROUND

Darcy's law given below is commonly used to interpret the water flow behavior in saturated soils.

$$v_w = -k_{sat} \frac{\partial h_w}{\partial y} \tag{1}$$

where v_w = flow rate of water, k_{sat} = the saturated coefficient of permeability and $\partial h_w / \partial y$ = the hydraulic head gradient in the direction of flow.

The saturated coefficient of permeability can be described as a factor relating water flow rate to the hydraulic gradient and is a measure of the ease of water movement in soil. The resistance of water movement in a saturated soil is primarily a function of soil particle size and their arrangement and distribution of pores.

Darcy's law is also valid for unsaturated soils. The unsaturated coefficient of permeability of a soil is dependent on the pore-size distribution and the amount of pore space available for water. Water can only travel through the continuous channels of soil pores that contain water. Water flows through all the channels under fully saturated conditions (S=100%). As air enters the soil (S<100%) the amount of pore space available for water to flow through decreases and results in the reduction of the coefficient of permeability.

The unsaturated coefficient of permeability can be determined using various laboratory and field-testing procedures. There are several difficulties associated with these methods, especially at high matric suctions due to the extremely low flow rates. The procedures are also time-consuming tests and require extensive testing facilities. Most conventional soil-testing laboratories do not have the equipment, facilities or the trained personnel to undertake these tests.

For this reason much research has been focused at predicting the unsaturated coefficient of permeability. The presently available prediction techniques use the soil-water characteristic curve as a tool to estimate the coefficient of permeability of an unsaturated soil. Three different types of modeling techniques are commonly used to predict the unsaturated coefficient of permeability. These include empirical, macroscopic and statistical models.

Empirical models or equations are developed using the results of laboratory tests. Several researchers have summarized and normalized the data into a curve or function (Richards 1952, Gardner 1958).

Macroscopic models are analytical expressions that take into account the many variables that influence the flow of water through the soil (Brooks & Corey 1964).

Statistical models are presently the most widely used models for predicting the unsaturated coefficient of permeability. These models have been developed by considering the probability of liquid phase continuity between pores of various sizes in formulating the probability function. Childs (1940) demonstrated that the soil-water characteristic curve could be used as a tool to obtain the pore-size distribution of a soil.

The earlier functions developed using the statistical approach required a knowledge of the *residual degree of saturation* for the soil (i.e., where an increase in soil suction does not result in a any more significant reduction in water content). These include the van Genuchten (1980) and the Fredlund & Xing (1994) equations.

The equation proposed by van Genuchten (1980) is useful for predicting the unsaturated coefficient of permeability for a suction range between 0 to 1500 kPa. Fredlund & Xing (1994) proposed a new method for predicting the unsaturated coefficient of permeability over the entire suction range. (0 to 10^6 kPa). This method is useful for predicting the unsaturated coefficient of permeability for all types of soils.

A more recent equation for predicting the unsaturated coefficient of permeability is by Leong and Rahardjo (1997). This formulation relates permeability to soil suction and relies on a fitting parameter p, which must be established for each specific soil. The idea of using a fitting parameter was further researched by Fredlund et al (2001). Approximately 300 sets of permeability data were used to determine typical values for the fitting parameter. Their research includes a statistical analysis of the fitting parameter based on soil type. The results of the statistical analysis show increased scatter in fitting parameters for fine-grained soils.

The present methods available to predict the unsaturated coefficient of permeability provide reasonable results for most soils. However, the laboratory investigations related to the measurement of soil-water characteristic curves taking into account all the parameters that influence their behaviour can be costly and time consuming. This research paper presents a simplified permeability function that relies on conventional soil properties, which can be determined from simple laboratory tests. The results of this study suggest that the coefficient of permeability of an unsaturated soil can be predicted with a reasonable degree of accuracy for most soils.

3 NORMALIZATION TECHNIQUE

The experimental data available in the literature for various soils representing both coarse and fine-grained soils has been correlated to show a relationship between the *degree of saturation*, S and the *relative coefficient of permeability*, k_{rel}. Figure 1(a) shows the relationship between k_{rel} and S for two soils, namely, sand and clay loam. The experimental results of the sand and clay loam were undertaken by Brooks & Corey (1964) and Elrick & Bowman (1964) respectively. The variation of S and k_{rel} are both plotted on a logarithmic scale

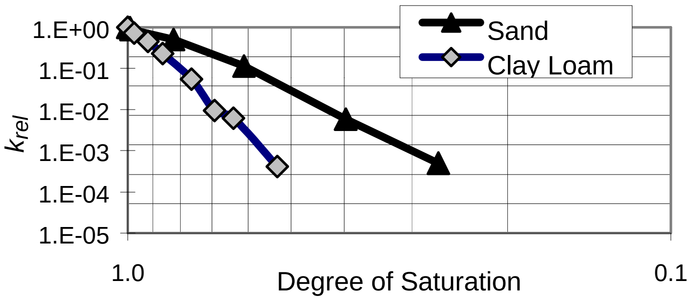

Figure 1(a). Comparison of k_{rel} and S for two typical soils

The results suggest that the relationship is a relatively straight line on a logarithmic scale. Similar studies on different types of soils have shown that this relationship is a straight line for a large suction range (i.e., 0 to 1,000 kPa). Furthermore, investigation studies also suggested that the relationship between the k_{rel} and S follows a defined trend line for different soils provided the degree of saturation is raised to a numerical value, which is defined as a fitting parameter, γ. This concept is illustrated in Figure 1(b). Note that a fitting parameter of 0.7 and 1.4 was used for the sand and clay loam respectively.

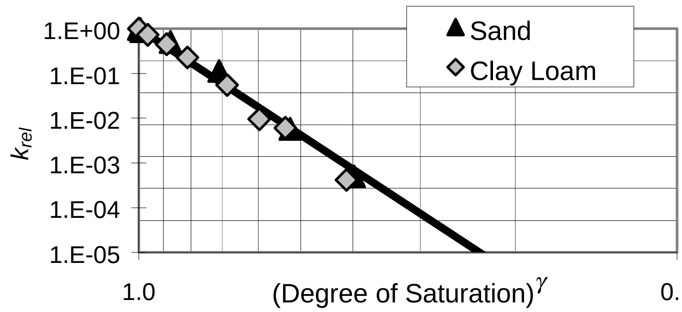

Figure 1(b). Comparison of k_{rel} and S for two typical soils using separate γ values

Using the technique illustrated in Figures 1(a) and 1(b), experimental data for various soils ranging from sand to clay was normalized. Figure 2 presents normalized data for four of these soils.

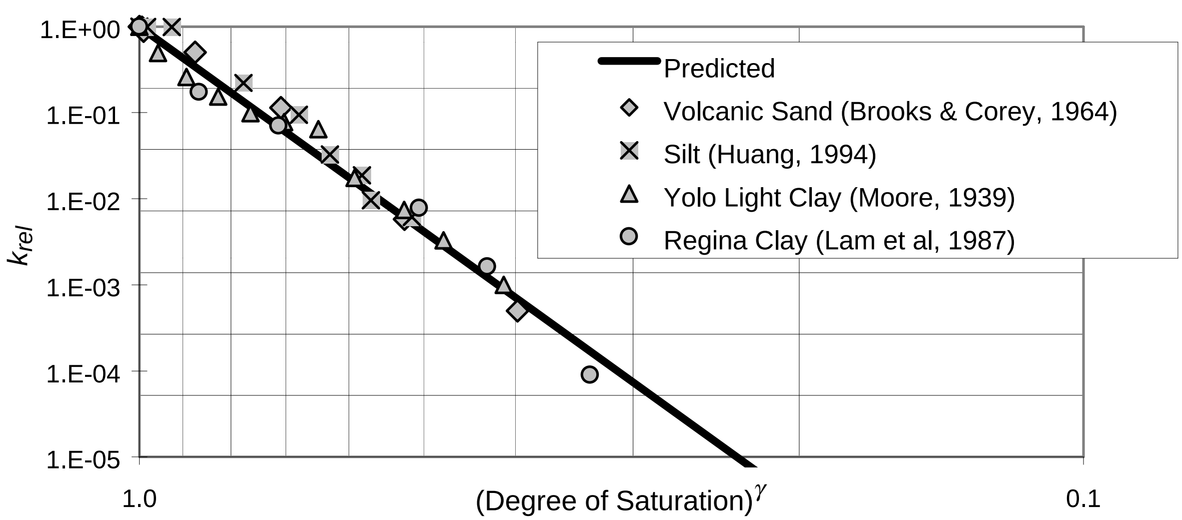

Figure 2. Normalized data for four soils

4 INFLUENCE OF STRESS STATE AND THE SOIL STRUCTURE ON THE NORMALIZED RELATIONSHIP

In addition to the normalization of experimental data into a function, the effects of stress state and soil structure on the relative coefficient of permeability were studied. The different soils representing fine and coarse-grained soils (i.e., a sand, silt and clay till) were used in the study.

The influence of applied loading on the relative coefficient of permeability, k_{rel} and (*degree of saturation*)$^{\gamma}$ was analyzed for a silty soil using the experimental results of Huang (1994). The data is presented in the following Figure 3. A single fitting parameter, γ, of 1.00 has been used in order to adjust the experimental data to the proposed trend line.

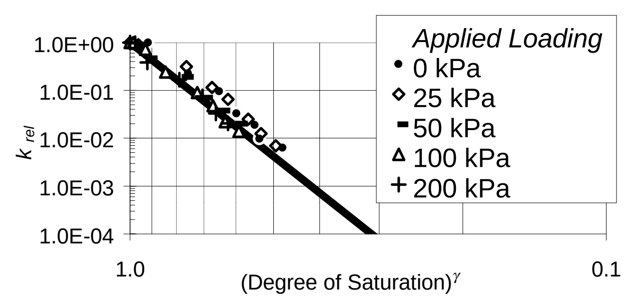

Figure 3. Effect of stress state on silt

The results suggest that the applied load does not significantly influence the relative coefficient of permeability, k_{rel}, and (*degree of saturation*)$^{\gamma}$ relationship for estimating the unsaturated coefficient of permeability for silt.

The effect of stress state (i.e., loading) was also analyzed for a clay till using the results of Vanapalli et al (1997). Experimental data for the measure values of unsaturated coefficient of permeability for the clay till are not available. However, the analyses are based on predicted values of the unsaturated coefficient of permeability using the soil-water characteristic curve data and the saturated coefficient of permeability. SoilVision™ 2.04 was used for predicting the variation of the unsaturated coefficient of permeability.

Figure 4 presents the relationship between k_{rel} and (*degree of saturation*)$^{\gamma}$ for a clay till specimens compacted at an initial water content representing *dry of optimum* conditions and subjects to different stress states.

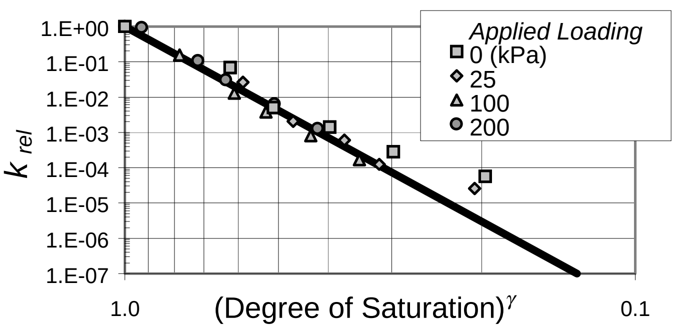

Figure 4. Effect of stress state on clay till compacted *dry of optimum*. (Vanapalli et al, 1997)

The results presented in Figure 4 show that the applied loading does not affect the unsaturated coefficient of permeability of the Indian Head till.

The effects of soil structure on the relative coefficient of permeability were also studied as part of this research paper. The term 'soil structure' used in this paper may be defined as the arrangement of soil particles in a compacted fine-grained soil. The compaction water content and the compactive effort have a significant influence on the resulting soil structure in a fine-grained soil (Lambe, 1958).

The following Figure 5 summarizes predicted data points for clay till compacted using different initial water contents (i.e., dry of optimum, optimum and wet of optimum conditions).

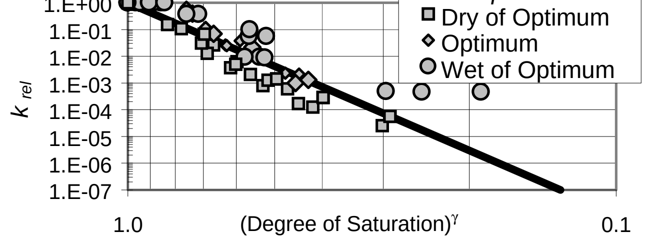

Figure 5. k_{rel} versus S^{γ} for Indian Head Till using a single fitting parameter (γ)

Figure 5 shows the closest possible relationship between the k_{rel} versus S^{γ} with the trend line using the fitting parameter γ equal to 1.90. The relationship shown in the figure demonstrates that the relative coefficient of permeability, k_{rel} is dependent on the soil structure (i.e., compaction water content) for fine-grained soils.

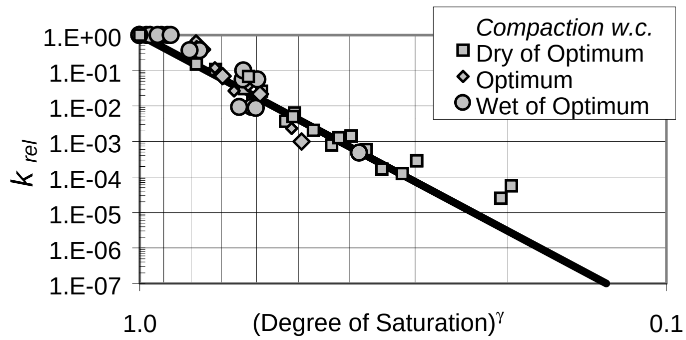

Figure 6. k_{rel} versus S^{γ} for Indian Head Till using different fitting parameters (γ)

Figure 6 shows a better fit for the three different compaction water contents. Different fitting parameters, γ values were used for different compaction water contents conditions. Table 1 summarizes the fitting parameters used for different compaction water contents of the Indian Head till.

Table 1. Fitting parameters, γ values used for Indian Head till

Compaction Water Content		Fitting Parameter
General	w (%)	(γ)
Dry of Optimum	12.5	2.50
Optimum	16.3	1.70
Wet of Optimum	19.2	1.50

The results show significant variations in the relative coefficient of permeability and (degree of saturation)$^{\gamma}$ relationship for the Indian Head till compacted with different initial water content conditions.

5 SIMPLIFIED COEFFICIENT OF PERMEABILITY FUNCTION

A simplified coefficient of permeability function is proposed using the published results from the literature. The function is a normalized relationship between the *relative coefficient of permeability*, k_{rel} defined as the ratio between the unsaturated coefficient of permeability, k_{unsat} and saturated coefficient of permeability, k_{sat} versus (*degree of saturation*)$^{\gamma}$. This relationship is useful to estimate coefficient of permeability of an unsaturated soil, k_{unsat}, from the information of saturated coefficient of permeability, k_{sat}, degree of saturation, S and the plasticity index, I_p.

The proposed function takes into account various parameters such as the soil texture, stress state and the soil structure that influence the coefficient of permeability of an unsaturated soil. The development of this function is presented in the following sections.

The relative coefficient of permeability, k_{rel} versus the (*degree of saturation*)$^{\gamma}$ relationships were developed for various soils. Table 2 summarizes the results of various soils including the fitting parameter, γ and the plasticity index, I_p values.

Table 2. Plasticity Indices, I_p and Fitting Parameter γ for Normalized Soils

Soil	γ	I_p (%)	Reference
Lakeland Sand	0.75	0.0	Elzeftawy & Cartwright, 1981
Superstition Sand	0.75	0.0	Richards, 1952
Volcanic Sand	0.70	0.0	Brooks & Corey, 1964
Columbia Sandy Loam	0.80	5.0*	Brooks & Corey, 1964
Fragmented Mixture	0.75	5.0*	Brooks & Corey, 1964
Silt	1.00	5.6	Huang, 1994
Touchet Silt Loam	0.80	6.0*	Brooks & Corey, 1964
Guelph Loam	1.40	6.0*	Elrick & Bowman, 1964
Madrid Clay Sand	2.00	8.0	Jucá & Frydman, 1996
Yolo Light Clay	1.20	10.0*	Moore, 1939
Red Silty Clay	5.00	13.6	Jucá & Frydman, 1996
Indian Head Till	1.70	19.0	Vanapalli et al, 1997
Regina Clay	9.00	50.0	Lam et al, 1987

* Estimated plasticity index values from clay content

The following Figure 7 presents the fitting parameter, γ variation with the plasticity index, I_p using the summarized results from Table 2.

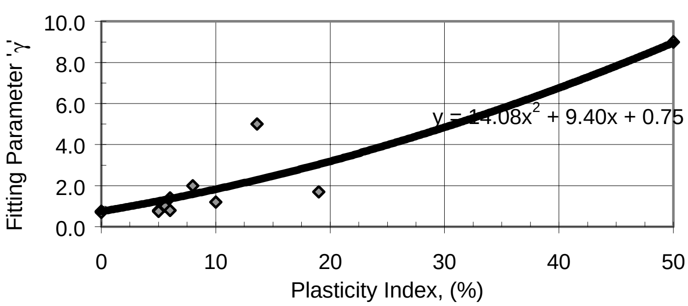

Figure 7. Relationship between fitting parameter, γ and plasticity index

The relationship given in Figure 7 allows a fitting parameter γ to be selected for any soil based on the plasticity index, I_p value. An estimation of k_{rel} may be obtained for any soil using the fitting parameter from Figure 7. The proposed permeability function is plotted in the following Figure 8.

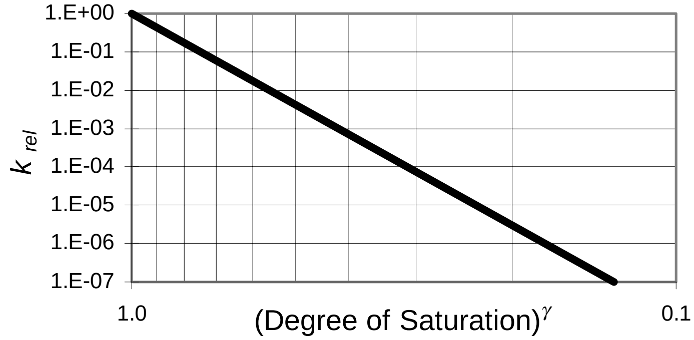

Figure 8. Proposed Permeability Function

The relationship shown in Figure 8 between the relative coefficient of permeability, k_{rel} and the adjusted degree of saturation, S^{γ} is given below:

$$k_{rel} \approx 10^{(7.9 \log S^{\gamma})} \tag{2}$$

where S^{γ} is the adjusted degree of saturation and $k_{rel} = k_{unsat}/k_{sat}$.

Furthermore, the mathematical function represented by the trend line given in Figure 8 provides an estimation of the fitting parameter as given below:

$$\gamma = 14.08\,(I_p)^2 + 9.4\,(I_p) + 0.75 \tag{3}$$

where I_p is the plasticity index of the soil.

There is a reasonably well-defined relationship between the fitting parameter, γ and the plasticity index, I_p (Figure 7). However, there is scatter in the relationship for plasticity index, I_p values greater than 20%. Such a behavior could be attributed to the influence of soil structure on the coefficient of permeability of unsaturated soils or due to the limited available data analyzed.

6 SUMMARY AND CONCLUSIONS

The determination of the coefficient of permeability of an unsaturated soil is time consuming and expensive. Hence, it has become a conventional engineering practice to predict the unsaturated coefficient of permeability through the use of the soil-water characteristic curve and the saturated coefficient of permeability. The coefficient of permeability of soils both in saturated and unsaturated conditions is influenced by various parameters such as the compaction water content, compactive effort and the stress state.

To obtain reliable predictions of the coefficient of permeability, it is important to take into account the factors such as the stress state and soil structure while measuring the soil water characteristic curves. In other words, a rigorous analysis of predicting the unsaturated coefficient of permeability should involve a range of soil-water characteristic curves.

In this research paper, a normalized functional relationship is proposed to estimate the unsaturated coefficient of permeability. This normalized function is a relationship between the relative coefficient of permeability, k_{rel} (defined as the ratio of k_{unsat}/k_{sat}) and the *(degree of saturation)*$^{\gamma}$.

The results of the present study suggest that the stress state or applied loading does not have significant influence on the proposed relationship (i.e., the relative coefficient of permeability, k_{rel} and the (*degree of saturation*)$^{\gamma}$. However, the results suggest that the soil structure has a significant influence on the proposed relationship.

The coefficient of permeability data presented in this study has been normalized using logarithmic scales into a single function with reasonable degree of success. The function relies on a fitting parameter γ. This fitting parameter, γ is related to the plasticity index, I_p of the soil. Figure 7 demonstrates a distinct trend of this relationship. A mathematical function has been extrapolated to estimate the value of γ based on the plasticity index, I_p.

The numerical results show good agreement with experimental data for sandy and silty soils. While experimental data is readily available for sandy/silty soils, the available experimental data for fine-grained soils is limited in the literature. More experimental data is required at close intervals over a wide range of the plasticity index, I_p values to propose a well-defined relationship.

The simple approach presented in this research paper to estimate the unsaturated coefficient of permeability based on water content or degree of saturation, plasticity index, I_p and saturated coefficient of permeability, k_{sat} is valuable for practicing engineers. The proposed method would allow the practicing engineer to estimate the unsaturated coefficient of permeability with data that is typically readily available. A more rigorous design approach can be undertaken using the simple functional relationship proposed in this paper towards the analysis of earth dams, slope, roadbed, foundation or other waste management structures.

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